

BARION AXIAL-VECTOR COUPLINGS AND SU(3)-SYMMETRY
BREAKING IN CHIRAL QUARK MODELS

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ABSTRACT

We investigate the possibility of explanation of the $g_{A/S} / g_A^{np}$ ratio using the chiral bag model with skin. This ratio has been estimated from the analysis of the deep inelastic scattering of polarized electron on polarized protons. With the same model we reproduce experimental values for axial vector coupling constants, g_A , for semileptonic decays.

In the MIT bag model the quantity $g_{A/S}$, axial vector isoscalar coupling constant, is defined by isospin matrices appearing in the definition of axial vector coupling constant for the beta decay [1] (g_A^{np}) with unit SU(2) matrix. Experimentally it is not very well established. Using some theoretical extrapolations [2] to the experimental results [3] it was found that the value of the $g_{A/S}/g_A^{np}$ ratio is in the range 0.41-0.55 ($g_A^{np} = 1.239 \pm 0.009$).

We investigated the possibility of achieving good values for $g_{A/S}/g_A^{np}$ and g_A^{np} in some quark models. Non-relativistic models give a too high value for g_A^{np} , 1.67. All bag models which satisfy the SU(3) flavor symmetry give $g_{A/S}/g_A^{np} = 0.6$, a too high value, so we considered breaking of that symmetry. In the MIT bag model it can be broken using nonequal quark masses: one can reproduce good values for $g_{A/S}/g_A^{np}$ and g_A^{np} ; however, too big u and d quark masses have to be used [3]. Another possibility to break the symmetry is to extend the model by introducing mesons (as is done in the chiral bag models) with their experimental mass values. In the cloudy chiral bag model [5] there is no meson contribution to the g_A^{np} and $g_{A/S}$ so the MIT bag model results are obtained. Brown-Rho chiral bag model [4] can reproduce $g_{A/S}/g_A^{np}$ value within given experimental range, but it gives a too high g_A^{np} value, 1.565. This led us to investigate whether these quantities can be reproduced in chiral bag model with skin [6] (CBMS).

In the CBMS the Wigner phase region (region where chiral symmetry is not spontaneously broken, i.e. where

mesons do not exist) is smaller than the confinement region (region where quarks exist), that is, for spherical model, chiral radius R_{CH} is smaller than the confinement radius R . In the model we include the whole nonet of pseudoscalar mesons, so the Lagrangian describing it is $U(3) \times U(3)$ chiral symmetric [3]. From the Lagrangian one gets highly nonlinear equations for mesons and quarks and the expression for axial vector current. To solve these equations we made the following assumptions:

- (1) we assumed that meson-quark coupling constant, $1/f$, is small and retained only zeroth and first order terms in Jaffe's expansion.
- (2) we assumed that there is no meson-quark coupling in the $R_{CH} < r < R$ region, where quarks and mesons coexist, so quark-meson interaction is retained only on confinement boundary.
- (3) we assumed that the meson term can be neglected in the linearized quark boundary condition.
- (4) gluonic degrees of freedom were neglected.

Using these approximations we obtained the following equations for quarks and mesons

$$i \not{\partial} q = 0 \quad r < R \quad (1)$$

$$i \not{\partial} q = g \quad r = R \quad (2)$$

$$\partial^2 \phi^a + \mu^2 \phi^a = 0 \quad r > R_{CH} \quad \mu^2 = \text{meson mass} \quad (3)$$

$$n \partial \phi^a = 0 \quad r = R_{CH} \quad (4)$$

and the expression for axial vector currents

$$A_\mu^a = \bar{q} \gamma_\mu \gamma_5 \frac{\lambda^a}{2} q \theta(r-R) - f q \phi^a \theta(r-R_{CH}) \quad (5)$$

To obtain the values of $g_{A/S}/g_A^{np}$ and of $g_A^{B'B}$ for semi-leptonic decays we need matrix elements of the space integrals of the axial vector currents. Quark contribution, as is seen from eq.-s (1),(2) and (5) is the same as in the MIT bag model because (1) and (2) are the usual MIT bag model equations for quarks. Meson contribution can be found by solving equations (3) and (4) and by using continuity of the axial vector current at the confinement radius. Putting all together one obtains the following expression

$$\langle B' | \int d^3x \bar{A}^a(x) | B \rangle = \chi_{B'}^\dagger \frac{-\lambda^a}{2} \chi_B O(B', B) g_A^{Q, B'B} \left(1 - \frac{R_H^2}{R} \Delta_1^a(R_H, R, \mu_0) \right) \quad (6)$$

where

$$\Delta_1^a(R_H, R, \mu_0) = -\frac{R_H}{2R^2} \frac{(1 + \mu_0 R) \bar{E}^a(R - R_H)}{1 + \mu_0 R + 0.5(\mu_0 R)^2}$$

$$g_A^{Q, B'B} = \int_{r < R} d^3x \left(u_{B'}(r) u_B(r) - \frac{1}{3} v_{B'}(r) v_B(r) \right)$$

χ are Pauli spinors, λ^a $a=0, \dots, 8$ are U(3) generators [7] $O(B', B)$ are factor coming from SU(6) structure of barion wave functions [3] and u and v are up and down component of quark wave functions for spherical bag. The first term in the brackets in (6) corresponds to quark contribution while the second comes from mesons.

The expressions for $g_A^{B'B}$ for a specific $B \rightarrow B' e \bar{\nu}$ semileptonic decay is obtained by taking the matrix element of space integral of combination of axial vector currents corresponding to that transition between the barion states. The mesonic contribution comes from the meson which has the same quantum numbers as the semileptonic decay matrix element.

In the case of $g_{A/S}$ two mesons have the same quantum numbers as the matrix element, η and η' so they both can give contribution. Proper combination of axial vector currents corresponding to η and η' was extracted from the expression for $g_1(\xi)$ function [2] appearing in the expression for the cross section of polarized electrons on polarized protons in the Björken limit, from which $g_{A/S}$ is defined [3]

$$g_{A/S} \chi^\dagger \bar{\sigma} \chi = \int d^3x \langle p | \frac{\sqrt{3}}{5} \bar{A}_8 - \frac{\sqrt{24}}{5} \bar{A}_0 | p \rangle \quad (7)$$

From the above discussion and using expressions (6),(7) one obtains expressions for $g_{A/S}$, g_A^{np} and $g_A^{B'B}$:

$$g_{A/S} = \hat{g}_A^{\omega, pp} \left(1 - \frac{R_{CH}^2}{R} \left(\frac{1}{5} \Delta_1^8 (R_{CH}, R, \mu_\eta) - \frac{4}{5} \Delta_1^0 (R_{CH}, R, \mu_\eta) \right) \right) \quad (8)$$

$$g_A^{np} = \frac{5}{3} \hat{g}_A^{\omega, pp} \left(1 - \frac{R_{CH}^2}{R} \Delta_1^3 (R_{CH}, R, \mu_\pi) \right) \quad (9)$$

$$g_A^{B'B} = O(B', B) \hat{g}_A^{\omega, B'B} \left(1 - \frac{R_{CH}^2}{R^2} \Delta_1 (R_{CH}, R, \mu) \right) \quad (10)$$

From (8) and (9) we have

$$\frac{g_{A/S}}{g_A^{np}} = \frac{3}{5} \frac{1 - \frac{R_{CH}^2}{R} \left(\frac{1}{5} \Delta_1^8 + \frac{4}{5} \Delta_1^0 \right)}{1 - \frac{R_{CH}^2}{R} \Delta_1^3} \quad (11)$$

For SU(3) symmetric case (when all μ_a are equal) the ratio $g_{A/S}/g_A^{np}$ is equal to 0.6. The cloudy chiral bag model ($R_{CH} = 0$) leads to the same value. But when experimental meson masses are used in eq. (11), the ratio becomes smaller than 0.6 because η and η' masses are larger than the $\bar{\pi}$ mass (see expression for Δ_1 , eq. (6). Numerical studies showed that both $g_{A/S}/g_A^{np}$ and g_A^{np} hardly change when R_{CH}/R ratio

was fixed and R was changed by $\sim 70\%$. Values for them are $g_A \sim 1,23$ and $g_{A/S}/g_A^{np} \sim 0.55$ i.e. experimental values are reproduced. It should be noted that the same model reproduces very well the values for semileptonic decays as can be seen from the following table

SL decay	CBMS	EXP ⁺
$\Sigma^- \rightarrow \Lambda e \bar{\nu}$	0.599	0.595
$\Lambda \rightarrow p e \bar{\nu}$	0.937	0.857
$\Xi^- \rightarrow \Sigma^0 e \bar{\nu}$	0.902	0.891
$\Xi^- \rightarrow \Lambda e \bar{\nu}$	0.312	0.340
$\Sigma^- \rightarrow n e \bar{\nu}$	0.255	0.310
$n \rightarrow p e \bar{\nu}$	4.223	4.223

It is interesting that CBMS confirms both assumptions that Ellis and Jaffe made [2] in deriving the expression for sum rules for deep inelastic scattering of polarized electrons on polarized protons. The first is that sea quarks do not contribute in matrix elements of currents which yields $\langle A_0 \rangle \sim \sqrt{2} \langle A_g \rangle$ ($\langle \rangle$ meaning barion matrix element) and the second is the assumption of SU(3) symmetry of currents which together with the first assumption gives $g_{A/S} = 3F - D$ (F and D are usual constants appearing in SU(3) symmetric expressions for axial vector currents [7]). Taking experimental values for F and D [8] one obtains value for $g_{A/S}$ very near to ours ($g_{A/S}^{exp} = 0.675$).

In conclusion, using the quoted approximations we were able to reproduce good values for $g_{A/S}$ and $g_A^{B'B}$ in CBMS. We haven't taken into account recoil and center of mass corrections. It would be interesting to see what

⁺ Experimental results taken from ref. [8]

values for $\bar{g}_{A/S}$ would yield other quark models. Also a more precise measurement of that quantity is desirable because it would be a good test for hadron models.

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