ON THE THEORY OF LEVEL SPACING DISTRIBUTION IN TYPICAL QUANTUM SYSTEMS

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In ref. [1], as well as in the other papers treating this subject, the energy level spacing distribution is studied on systems which can be treated quasyclassically. Newertheless, there exists a great number of systems for which the quasyclassical approximation is not applicable, even when the high quantum number energy levels are treated.

A typical quantum system is treated in ref. [2], where the level spacing distribution of a piece-vice linear oscillator, with semitransparent barrier between the linear parts, i.e. an „oscillator" whose potential energy is

$$V(z) = a\delta(z) + \begin{cases} m\omega_1^2 z^2/2 & , \quad z < 0 \\ m\omega_2^2 z^2/2 & , \quad z > 0 \end{cases} \tag{1}$$

where m is the mass, ω_1 and ω_2 are the frequencies, $\delta(z)$ is the Dirac delta function, is investigated. In the same paper it has been shown that for sufficiently high values of the coefficient a, there exists a region of nonuniform level distribution, where it is possible to define a level spacing distribution, and a quasyclassical region with uniformly distributed energy levels. As a distinction to the twodimensional harmonic oscillator, for this case there exists a distribution for both rational and radional frequency ratio, and in both cases the distributions have essentially different behavior.

In this paper we study a particle of mass m moving in the field of potential (1) and in addition a constant homogeneous magnetic field $\vec{B} = \{0, 0, B\}$ is present.

Introducing the wavefunction $\psi(x, y, z) = \psi_1(x, y)\,\psi_2(z)$, we separate the variables, and it is possible to obtain the equations for two independant „oscillators". The first one is described by a onedimensional Schroedinger eqation in which the potential V(z) is given by (1). For the energies $E_2 = E_{2m}$ we have the following equation:

$$\frac{1}{\Gamma(\frac{3}{4}-\frac{r}{2})\,\Gamma(\frac{1}{4}-\frac{rq}{2})} - \sqrt{q}\,[\frac{1}{\Gamma(\frac{3}{4}-\frac{rq}{2})\,\Gamma(\frac{1}{4}-\frac{r}{2})} + \frac{a^*}{\Gamma(\frac{3}{4}-\frac{r}{2})\,\Gamma(\frac{3}{4}-\frac{rq}{2})}] \tag{2}$$

where we have used the following notations

$$r = \frac{E_{2m}}{\hbar\omega_1} \quad , \qquad q = \frac{\omega_1}{\omega_2} \quad , \qquad a^* = \sqrt{\frac{m}{\hbar^2 \omega_1}}\, a \tag{3}$$

The second „oscillator" concerns the particle of mass m in a constant homogeneous magnetic field [3]. The eigenvalues E_{1n} determine the well known Landau levels

$$E_{1n} = \hbar\omega_c \,(n + 1/2) \tag{4}$$

where $\omega_c = eB/m$ is the corresponding cyclotron frequency.

The eigenvalues of the Hamiltonian of the system E_{mn}, are found by

$$E_{mn} = E_{1n} + E_{2m} \tag{5}$$

The spacing distribution of the energy levels given by (5), was numerically calculated by employment of the relations (2) and (5). We got the results for the ratios ω_1/ω_2 and ω_c/ω_1 for different number of levels. For all cases we took $a^* = 100$. At such choise of the coefficient a^* ($a^* = \sqrt{m/\hbar^3 \omega_1}\; a$) the number of the levels in the region of unhomogeneous distribution (where the quasyclassical approximation does not hold) is of order 10^6 [2]. The obtained results indicate that there exists a more complicated dependence of the distribution function P(s) on the energy level spacins s, than that reffered in [1]. This can be seen from the histograms given in fig. 1. On fig. 1a) the distribution function is obtained by addition of 500 levels, while those on fig. 1b) concerns 1000 levels, on fig. 1c) — 2000 and on fig. 1d) the distribution function corresponds to 5000 levels.

The situation is different when $\omega_1/\omega_2 = (\sqrt{5} - 1)/2$, $\omega_c/\omega_1 = 1/\sqrt{2}$ and when $\omega_1/\omega_2 = (\sqrt{5} - 1)/2$, $\omega_c/\omega_1 = 1/\pi$.

From our point of wiev, the most important fact is that the transformation of the histograms, obtained by the increase of the energy levels taking part in the distribution, are highly sensitive to the frequency ratios.

Fig. 2. Level spacing distributions for $\omega_1/\omega_2 = (\sqrt{5} - 1)/2$, $\omega_c/\omega_1 = 1/\pi$ and a) 2000 levels, b) 5000 levels.

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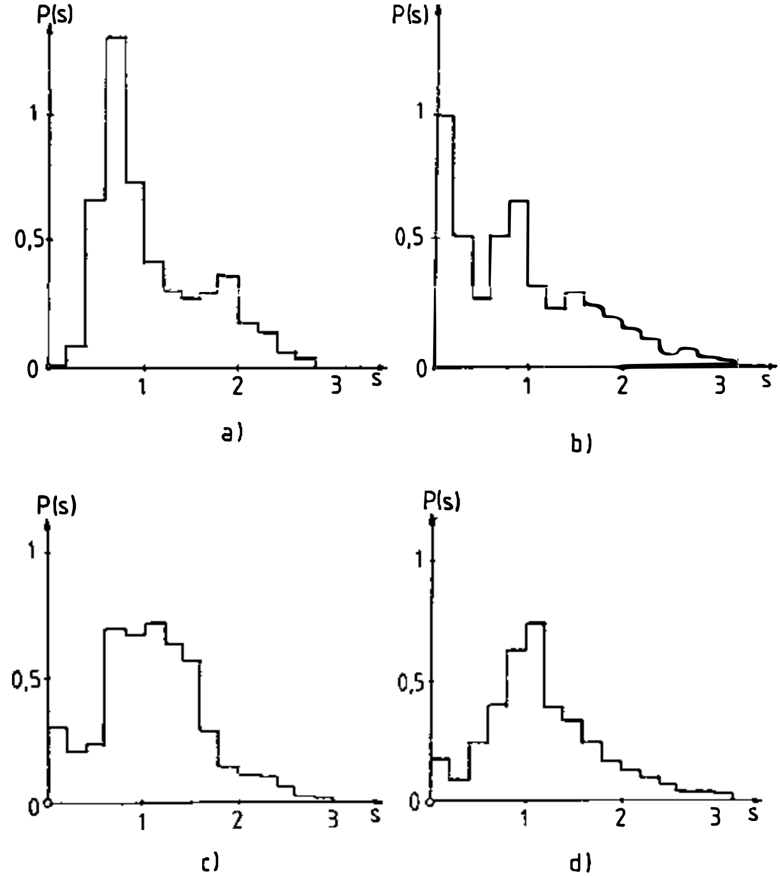
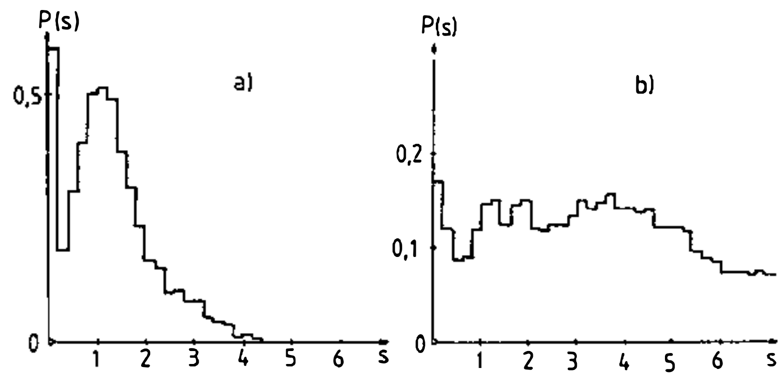


Fig. 1. Level spacing distributions for the lowest
a) 500; b) 1000; c) 2000; d) 5000
levels at the following frquency ratios:
$\omega_c/\omega_1 = 1/\sqrt{2}$ and $\omega_2/\omega_1 = 1/\delta_F$ (Number of Feigenbaum [4]).



On fig. 2 we have the histograms concerning $\omega_1/\omega_2 = (\sqrt{5} - 1)/2$ $\omega_c/\omega_1 = 1/\pi$ and 2000 levels (fig. 2a); 5000 levels (fig. 2b).

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