TRANSITION TO CHAOS THROUGH AN INFINITE SERIES OF PHASE TRANSITIONS IN SYSTEMS MODELED WITH NONLINEAR TRANSFORMATIONS

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Abstract:

The results, we have obtained, propose that in modelled system that we are concerned with, the transition to chaos occurs through infinite series of phase transitions determined by parametar b *in the potential f (z, b). Particular phases of the system correspond to particular parts of bifurcative caracteristics. At the same time, each phase distinguishes itself with determined symmetry, which in a process of phase transitions changes and finally the system gets disordered phase.*

1. The Feigenbaum's theory of universality gives strong arguments about the transition of system from regular to chaotic modes as a function of an external parametar [1, 2]. It proved to be that iterative functions which are qualitatively alike display the same quantitative behaviour. So to speak if the mapping doubles the period, then theory of universality gives exact quantitative prediction, no matter of the concret form of the mapping. This conclusion is formally similar to the correspondent from the theory of phase transitions where several qualitative caracteristics of the system which displays phase transition, especially dimensionality, predetermine universal critical exponents.

In this paper, the theory of universality is applied to a set of plane curves, the equations of which can be rewritten, formally, in terms of thermodynamical state equations [3].

2. Let Γ be finite, rectifiable, plane curve. If the length of the curve, $|\Gamma|$ is identified with its „volume" V and the length of the frontier of the convex hull of Γ with inverse „pressure", for the curve Γ one can write an „equation of state" [3]

$$2\,PV = \frac{1}{1 - \exp(-1/T)}\ , \tag{1}$$

where T is „temperature"$^{x)}$. At high temperature from (1) one can get $PV = T/2$ which proves that at high temperature, a curve behaves like perfect gas ($R = 1/2$).

Let $\rho_o(\theta)$ denotes the equation of such curve Γ_o, in polar coordinates (ρ, θ). Let us consider a set of curves $\{\Gamma\}$ which originate from Γ_o with the help of the mapping f: $\rho_n \to \rho_{n+1}$ ($n \in N^o$) i.e.

$$\rho_{n+1}(\theta) = f(\rho_n(\theta)), \qquad \rho_{n+1} = f(\rho_n) \tag{2}$$

It's obvious that

$$\rho_{n+1}(\theta) = f(\rho_n(\theta)) = f(f(\rho_{n+1}(\theta))) = \ldots = f^{n+1}(\rho_o) \tag{3}$$

where we use

$$f^n(z) \stackrel{def}{=} f(f(f \ldots f(z)) \ldots); \qquad f^o(z) \stackrel{def}{=} z \tag{4}$$

In relation to the mapping (2) we suppose that curves belonging to set Γ_i we are finite rectifiable curves.

According to (1), the curve $\{\Gamma_i\}$ (i = 1, 2, 3, . . .) can be joined by three thermodynamical parameters (V_i, P_i, T_i) by means of which the i-th equilibrium state of the thermodynamical system is obtained. Because $f^i : \Gamma_o \to \Gamma_i$, i.e. $f^i : \rho_o(\theta) \to \rho_i(\theta)$ holds $f^i : (V_o, P_o, T_o) \to (V_i, P_i, T_i)$.

So, the mapping (2) models the thermodynamical state of the system. The function f(z), which determines mapping, can be denominated as a potential of the thermodynamical system. The state (V_i, P_i, T_i) depends on initial state (V_o, P_o, T_o) as well as on potential f(z).

If the potential f depends on one (for reasons of simplicity) parametar b, then the i-th state of the system depends on b also, i.e. the i-th state is determined by $(V_i(b), P_i(b), T_i(b))$. This result can be generalized with more parametars.

3. Let the potential f be given in a form

$$f(z, b) = 4bz(1 - z) \tag{5}$$

and the initial state (curve Γ_o), with equation

$$\rho_o(\theta) = 70 \sin(4\theta) \tag{6}$$

Let's inspect this experiment. The potential (5) acts on initial state (6) during some time t, and gives some final state Γ_i. We perform a series of such experiments, always starting with the same initial state. In conditions of real experiment the time interval t is realized with an accuracy $\Delta t << \langle t \rangle$. In other words, each measuring which is numerically modelled, finds the system in state ($\underline{N}$ + $\underline{k}$) ($\underline{k}$ is accidental whole number (| $\underline{k}$ | << $\underline{N}$), and $\underline{N}$ is discrete time.) With ($\underline{N}$ + $\underline{k}$) the number of iterations is determined, as a result of which, one can estimate the final state starting with (6) with help of (5). In one series we perform 20 repetitions for $\underline{N}$ = 1000 and $0 \leqslant |\underline{k}| \leqslant 10$ so we can estimate average values $\langle V \rangle$, $\langle T \rangle$. From graphic presentation (Fig. 1) one can see that function $\partial \langle V \rangle / \partial b$ has leaps at points b_i; i = 1, 2, . . . for which holds

$$\lim_{i \to \infty} \frac{bi + 1 - bi}{bi + 2 - bi + 1} = \delta$$

where δ = 4,6692016. . . is universal constant of Feigenbaum.

Similar behaviour has function $\partial \langle T \rangle / \partial b$ also.

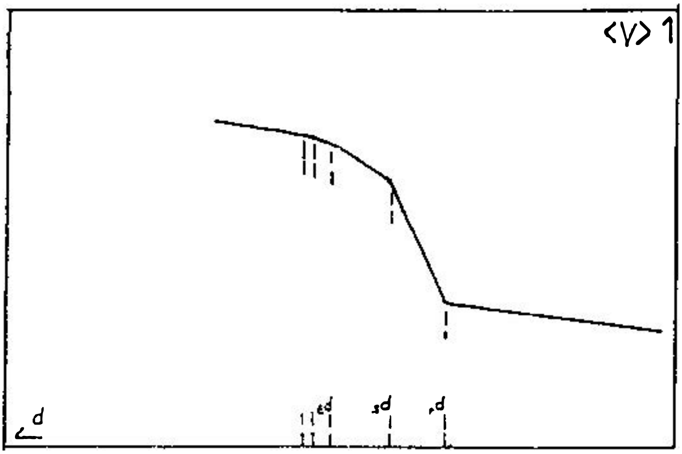

Fig. 1.

4. Such behaviour of functions $\partial < V > /\partial b$ and $\partial < T > /\partial b$ in conditions where the potential (5) operates, can be considered in terms of phase transitions which occur during the change of the parametar b; the points b_i ($i = 1, 2, \ldots$) where one can find leaps, are points of phase transition. According to the general theory of universality, this conclusion holds for any other potential which posses property to double his period.

The results, we have obtained, propose that in modelled system that we are concerned with, the transition to chaos occurs through infinite series of phase transitions determined by parametar b in the potential f (z, b). Particular phases of the system correspond to particular parts of bifurcative caracteristics [2]. At the same time, each phase distinguishes itself with determined symmetry, which in a process of phase transitions changes and finally the system gets disordered (chaotic) phase.

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x) Having in mind this, we don't write quotation marks further on.