SOLITONS IN 1-d HEISENBERG FERROMAGNET WITH SPIN-PHONON INTERACTION IN HOLSTEIN-PRIMAKOFF REPRESENTATION

S. Stojanović, M. J. Škrinjar and D. Kapor
Institut za fiziku PMF, Novi Sad

INTRODUCTION

The problem of the existence of the localized excitations-solitons in the magnetic systems has received large attention during the last years [1, 2, 3]. We wish to study here the case of the one-dimensional compressible isotropic Heisenberg model, in order to test the influence of the spin-phonon interaction on the creation of solitons. The system can be described by the Hamiltonian (with the ground state energy subtracted):

$$\hat{H} = -\mu h \sum_{j} (\hat{S}_j^z - S) - J \sum_{j} (\vec{S}_j \cdot \vec{S}_{j+1} - S^2) - g \sum_{j} (u_{j+1} - u_j) \vec{S}_j \cdot \vec{S}_{j+1} + + \sum_{j} [\frac{p_j^2}{2m} + \frac{m v_0^2}{2 a^2} (u_{j+1} - u_j)^2] \quad (1)$$

We consider the chain with the period a in an external magnetic field h in z-direction, with an ion of the spin S_i at i-th site. Ion's displacement is u_i, corresponding momentum p_i, J is the exchange integral, $g > 0$ is the spin-phonon coupling and v_0 is the sound velocity in the chain. One can also introduce the „spring constant" $\mathcal{H}$ through the relation $\mathcal{H} a^2 = m v_0^2$.

The effective hamiltonian.

The first step is to apply Holstein-Primakoff boson representation for the spin operators [4]:

$$\hat{S}_j^z = S - \hat{B}_j^+ \hat{B}_j \qquad \hat{S}_j^+ = (g_j^-)^+$$

$$\hat{S}_j^- = \sqrt{2S}\, \hat{B}_j^+ \sqrt{1 - \frac{\hat{B}_j^+ \hat{B}_j}{2s}} \approx \sqrt{2s}\, \hat{B}_j^+ (1 - \frac{\hat{B}_j^+ \hat{B}_j}{4s}) \quad (2)$$

where B_j, B_j^+ are Bose-operators, and we limit ourselves to the approximation with terms up to four Bose operators in the Hamiltonian.

Following the standard procedure [3], one averages the Hamiltonian over the wave-function (coherent state)

$$|\psi> = \prod_j |a_j> \tag{3}$$

and this average value is treated as the classical Hamilton's function of the system. The next step is to perform the continuum transition:

$$\sum_j \rightarrow \frac{1}{a} \int dx \qquad a_j(t) \rightarrow a(x, t)$$

$$a_{j\pm1}(t) \rightarrow a(x, t) \pm a a_x + \frac{a^2}{2} a_{xx} \pm \frac{a^3}{6} a_{xxx} + O(a^4) \tag{4}$$

(subscript denotes corresponding partial derivative). The boundary conditions allow bell-shaped solution for $a(x, \epsilon)$: $a(x = \pm\infty) = 0, a_x = a_{xx} = \ldots = 0 \quad x = \pm\infty$ and kin--solution for displacement: $u(+\infty) \neq u(-\infty), u_x = u_{xx} = \ldots = 0 \quad x = \pm\infty$.

With these conditions one arrives to the following Hamiltonian density with terms up to a^3 and a^4 :

$$\mathcal{H} = \mu h |a|^2 + JSa^2 |a_x|^2 + \frac{1}{4} Ja^2 (a_x^2 a^{*2} + a^2 a_x^{*2}) - gaS^2 u_x + \qquad gaS^2 u_x +$$

$$+ ga^3 u_x [S |a_x|^2 + \frac{1}{4} (a^{*2} a_x^2 + a^2 a^{*2}) + \frac{p^2}{2m} + \frac{1}{2} \mathcal{H} a^2 u_x^2 \tag{5}$$

and corresponding Lagrangian density:

$$\bar{L} = i\hbar c^* c_t + m u_t^2 - \mathcal{H} \tag{6}$$

Now one can write down the equation of motion for u (x, t). The usual assumption of the existence of the stationary state $u(x, t) = u(x - \nu t)$ where ν is the soliton velocity, gives after integration

$$u_x = - \frac{ga^3}{mv_o^2 (1 - \nu^2)} [S |a_x|^2 + \frac{1}{4} (a^2 a_x^{*2} + a^{*2} a_x^2)] \qquad \nu = \frac{v}{v_o} \tag{7}$$

Introducing u_x into L we obtain L_{eff} up to a^2

$$L_{eff} = i\hbar a^* a_t - \mu h |a|^2 - J [1 + \frac{g^2 S^2}{J \mathcal{H} (1 - \nu^2)}] |a_x|^2 - \frac{1}{4} Ja^2 [1 + \frac{g^2 S^2}{J \mathcal{H} (1 - \nu^2)}] \times$$

$$\times (a^2 a_x^{*2} + a_x^2 a^{*2}) \tag{8}$$

giving

$$\mathcal{H}_{eff} = \mu h\,|a|^2 + \tilde{J}\,|a_x|^2 - \frac{1}{4}\,\tilde{J} a^2\,(a^2 a_x^{*2} + a_x a^{*2}) \tag{9}$$

with

$$\tilde{J} = J\;[1 + \frac{g^2\,S^2}{J\,\mathcal{H}\,(1-\nu^2)}\,] \tag{10}$$

It is important to notice that the term mu_t^2 is of order a^4 so it was neglected.

RESULT AND DISCUSSION

Our conclusion is that we have obtained the Hamiltonian density which is equal to the density of the system with no spin-phonon interaction, [3] but with renormalized exchange interaction.

The system has the soliton solution of the form:

$$a\,(x,t) = \varphi\,(5)\;e^{\,i\;[\theta\;(\zeta) + \Omega t]} \qquad\qquad \zeta = x - vt$$

$$\varphi = \frac{\varphi_o}{ch\,(\gamma_o^{\,1/2}\beta\zeta)} \qquad\qquad \varphi_o^{\,2} = 2S\beta^2 \tag{11}$$

$$\theta = \frac{V}{2}\,\zeta + \text{arc tg}\,(\sqrt{\frac{\beta^2}{1-\beta^2}}\quad th\,\beta\gamma_o^{\,1/2}\zeta) \tag{12}$$

with

$$\beta^2 = 1 - \frac{V^2}{4\,\gamma_o} \qquad\qquad \gamma_o = \frac{\Omega + \mu h}{\tilde{J}\,S} \qquad\qquad V = \frac{v}{JS}$$

The soliton characteristics are ($\hbar$= 1), (a = 1)

$$E = 4\,\tilde{J}\,S^2\;\beta\,\gamma_o^{\,1/2} + 4\,S\,\beta\;\frac{\mu\,h}{\gamma_o^{\,1/2}}$$

$$\sin\,\frac{P}{4\,S} = \beta \tag{13}$$

$$Mz = \frac{4\,S\,\beta}{\gamma_o}$$

or

$$E = \frac{8\tilde{J}S^3}{M_z}\left(1 - \cos\frac{P}{2S}\right) + \mu h M_z \tag{14}$$

It is important to note that due to the renormalization of J, the relation $v = \frac{\partial E}{\partial P}$ is fulfilled only approximately, for $v \ll v_o$.

REFERENCES

1. J. Tjon and J. Wright: Phys. Rev. B 15, 3470 (1977)
2. D. I. Pushkarov and Kh. I. Pushkarov: phys, stat. sol. (b) 81, 703 (1977)
3. L. G. de Azevedo, M. A. de Moura, C. Cordeiro and B. Žekš: J. Phys. C 15, 7391 (1982)
4. T. Holstein and A. Primakoff: Phys. Rev. 58, 1098 (1940)