SOLITONS IN 1-d FERROMAGNET WITH PLANAR ANISOTROPY IN THE PRESENCE OF SPIN-PHONON INTERACTION

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We shall study compressible one-dimensional Heisenberg ferromagnetic model with planar anisotropy, where effects of cubic lattice anharmonicity are included. (Similar problem for the fourth order anharmonicity was studied recently [1].)

The Hamiltonian of the system is given by:

$$H = -I\sum_i \vec{S}_i\vec{S}_{i+1} + A\sum_i (S_i^z)^2 - 2g\mu_B h^x \sum_i S_i^x - \alpha\sum_i (U_{i+1}-U_i)\,\vec{S}_i\,\vec{S}_{i+1} +$$

$$+ \sum_i \frac{p_i^2}{2m} + \frac{\mathcal{H}}{2}\sum_i (U_{i+1}-U_i)^2 + \beta'\sum_i (U_{i+1}-U_i)^3 \quad (1)$$

Here $\vec{S}_i$ is the spin of the paramagnetic ion on the i-th site, U_i is the displacement of this ion, p_i – corresponding momentum. m is the ion mass, I – exchange integral, A – anisotropy parameter, h^x – magnetic field, α – spin-phonon coupling constant, $\mathcal{H}$ – „spring constant" ($\mathcal{H}a^2 = m V_a^2$ where a is the lattice constant and V_a the sound velocity) and β' is the anharmonicity parameter.

Following [1] we reat $\vec{S}_i$ as the classical vectors and in the continuum approximation ($\vec{S}_i(t) \rightarrow \vec{S}(x,t)$) we obtain the following Hamiltonian density:

$$\mathcal{H} = \frac{Ia^2}{2}\left(\frac{\partial\vec{S}}{\partial x}\right)^2 + A\,(S^z)^2 - 2g\,\mu_B h^x S^x + \frac{\alpha a^3}{2}\frac{\partial U}{\partial x}\left(\frac{\partial\vec{S}}{\partial x}\right)^2 + \frac{p^2}{2m} + \frac{\mathcal{H}a^3}{2}\left(\frac{\partial U}{\partial x}\right)^2 +$$

$$+ \beta' a^3 \left(\frac{\partial U}{\partial x}\right)^3 - \frac{\mathcal{H}a^4}{24}\left(\frac{\partial^2 U}{\partial x^2}\right)^2 - \alpha\, a\, S^2 \left(\frac{\partial U}{\partial x}\right) \quad (2)$$

Now we can formulate the equations of motion and study two substantially different cases, depending on the relation between the sound velocity V_a and the velocities of magnetic soliton V_m and elastic kink velocity V_u.

a) $V_m, V_u << V_a$

In this case, one can neglect the cubic anharmonism ($\beta' = 0$) and the term $\partial^4 U/\partial_x^4$ in the equation of motion. The equation of motion for U becomes

$$\frac{\partial^2 U}{\partial t^2} - V_a^2 \frac{\partial^2 U}{\partial x^2} = \frac{\alpha a^3}{2m} \left(\frac{\partial \vec{S}}{\partial x}\right)^2 \tag{3}$$

This equation can lead to soliton solution under the assumption [2] that $V_m = V_u = V$ and that there exists so called stationary solution of the form $U = U(x - vt)$ and $\frac{\partial S}{\partial x} = \frac{\partial \vec{S}}{\partial x}(x - vt)$.

Performing a canonical transformation, one obtains [3]:

$$U_{stat} = - \frac{\alpha a}{2\mathcal{H}\left[1 - \left(\frac{V}{V_a}\right)^2\right]} \int_{-\infty}^{x - vt} \left(\frac{\partial \vec{S}}{\partial z}\right)^2 dz \tag{4}$$

giving rise to an effective Hamiltonian density

$$\mathcal{H}_{eff} = \frac{I a^2}{2} \left[1 + \frac{\alpha^2 S^2}{\mathcal{H} I \left(1 - \frac{V^2}{V_a^2}\right)}\right] \left(\frac{\partial \vec{S}}{\partial x}\right)^2 + A (S^z)^2 - 2g\mu_B h^x S^x + O(a^4) \tag{5}$$

This Hamiltonian describes, within the framework of the sine-Gordon approximation ($S^z \approx 0$) a typical sine-Gordon system, but with the renormalized exchange interaction

$$I \rightarrow \tilde{I} = I \left[1 + \frac{\alpha^2 S^2}{\mathcal{H} I \left(1 - \frac{V^2}{V_a^2}\right)}\right] \tag{6}$$

The solutions of the problem are well-known [4], although the properties of the solutions are, of course, influenced by the velocity dependence of the renormalized exchange interaction.

b) $V_m, V_u >> V_a$

In this case one studies the interaction of the magnetic SG-kink and elastic kink whose coupling is described by the system of equations (expressed in terms of polar coordinates of the spin vector):

$$\frac{\partial^2 \varphi}{\partial t^2} - C_o^2 \frac{\partial^2 \varphi}{\partial x^2} + \omega_o{}^2 \sin\varphi = \frac{\alpha a}{I} C_o^2 \frac{\partial}{\partial x}\left(\frac{\partial \varphi}{\partial x} \frac{\partial U}{\partial x}\right) \tag{7a}$$

$$\frac{\partial^2 U}{\partial t^2} - V_a^2 \frac{\partial^2 U}{\partial x^2} - \frac{V_a^2 a^2}{12} \frac{\partial^4 U}{\partial x^4} - \frac{6\beta' a^3}{m} \frac{\partial U}{\partial x} \frac{\partial^2 U}{\partial x^2} = \frac{\alpha a^3 S^2}{2m} \frac{\partial}{\partial x} [\sin^2\theta \left(\frac{\partial \varphi}{\partial x}\right)^2 +$$

$$+ \left(\frac{\partial \theta}{\partial x}\right)^2] ; \quad C_o{}^2 = 2A\tilde{I} S^2 a^2, \; \omega_o{}^2 = 2AS \mu_B h^x g \tag{7b}$$

For $\alpha = 0$, the solutions are SG-kink for φ and elastic kink for U:

$$\varphi_{SG}(x, t) = 4 \operatorname{arctg} \exp \left\{ \pm \frac{\frac{\omega_o}{C_o} x - \omega_o \frac{V_m}{C_o} t}{\sqrt{1 - (V_m/C_o)^2}} \right\} \tag{8a}$$

$$U(x, t) = U_o \operatorname{th} \frac{x - V_u t}{2 l} \; ; \qquad l = \frac{a^2 V_a^2}{12 (V_u^2 - V_a^2)} \tag{8b}$$

We look for the solution of (7) in the approximation linear in α:

$$\varphi = \varphi_{SG} + \varphi_1, \qquad \varphi_1 \sim \alpha \tag{9}$$

We obtain the linearized equation for φ_1 expressed in terms of dimensionaless parameters $\delta = \omega_o t$, $y = \frac{\omega_o}{C_o} x$

$$\frac{\partial^2 \varphi_1}{\partial \delta^2} - \frac{\partial^2 \varphi_1}{\partial y^2} + (\cos \varphi_{SG}) \varphi_1 = \frac{a\alpha}{I} \frac{\omega_o}{C_o} \frac{\partial}{\partial y}\left(\frac{\partial \varphi_{SG}}{\partial y} \frac{\partial U}{\partial y}\right) \tag{10}$$

We wish to present here an explicit solution for the particular case $V_m = V_u$. The expression derived in [5] is not valid in this case so one must solve (10) directly. Applying the condition proposed in [1] $\frac{\partial \varphi_{SG}}{\partial y} \frac{\partial^2 U}{\partial y^2} << \frac{\partial^2 \varphi_{SG}}{\partial y^2} \frac{\partial U}{\partial y}$, one arrives to the following results: for $V_m = V_u$ the Fourier-transform $\varphi_1(y, \omega)$ of $\varphi_1(y, \delta)$ is proportional to $\delta(\omega)$, so $\varphi_1(y, \delta)$ is time independent and it is given by the expression

$$\varphi_1(y, \delta) = \varphi_1(y) = \frac{1}{48\bar{u}} \frac{U_o}{l} \frac{\alpha a}{I} \frac{1}{\mathrm{ch}\, y} (7\, \mathrm{th}\, y + 4\, y) \tag{11}$$

for $y > 0$, when the magnetic field can be chosen in such a way that the condition
$\frac{2l}{\sqrt{1 - \frac{V^2}{C_o^2}}} \approx 1$ is fulfilled.

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