SUPERFLUIDITY OF THE HEISENBERG FERROMAGNET

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The Hamiltonian of the isotropic Heisenberg ferromagnet [1] with spin 1/2 (from [2], and in approximation: $S^{+} = P = B$, $S^{-}S^{+} = P^{+} P = B^{+} B - B^{+2} B^{2}$):

$$H = h \sum_{\vec{n}} B^{+}_{\vec{n}} B_{\vec{n}} - \frac{1}{2} \sum_{\vec{n},\vec{m}} I_{\vec{n}\vec{m}} B^{+}_{\vec{n}} B_{\vec{m}} - h \sum_{\vec{n}} B^{+2}_{\vec{n}} B^{2}_{\vec{n}} - \frac{1}{2} \sum_{\vec{n},\vec{m}} I_{\vec{n}\vec{m}} B^{+}_{\vec{n}} B_{\vec{n}} B^{+}_{\vec{m}} B_{\vec{m}} \tag{1}$$

is similar to the excitonic Hamiltonian, with respect to kinematics of the operators featuring in it.

The results of the previous paper [3] dedicated to the problem of BEC (Bose--Einstein condensation) and SF (superfluidity) in the system of Frenkel excitons indicated that kinematical interaction of excitons can induce their BEC and consequently their SF transfer through the crystal. It is natural to ask whether the formal similarity of spin waves with Frenkel excitons can lead to some similar properties.

In order to make use of the complete analogy of magnon and exciton Hamiltonian, it is important to examine whether the last term in (1) can be treated by the perturbation theory (δ — potential scattering). The excitation energy of the isolated molecule (Δ) is about 100 times larger than the energy of dipole-dipole interaction (W) of molecules, the same type of condition would imply:

$$h \equiv \mu \mathcal{H} + I \geqslant 10^{2}\, I \Rightarrow \mu \mathcal{H} >> I. \tag{2}$$

The condition, which would permit us to exploit the complete analogy between exciton and magnon system, isverydiffliculdfor practical realization, because it demands fields of order 1000 T, at least.

Let us suppose that the condition (2) is fulfilled. We can estimate the relaxation time (time necessary for attaining the thermal equilibrium with phonons) in the same way as in [4]. The result is (for details see [5]): $\tau_r \sim 10^{-12}$ s. This time is shorter than the magnon lifetime ($\tau_l \sim 10^{-8}$ s) [6, 7], which means that the condition for the quasi-particle BEC [8] is satisfied, and BEC can be formed with all its properties. In the case that (2) is not fulfilled, i.e. when $\mu \mathcal{H} \leqslant I$, the relaxation time is much longer ($\tau_r \sim 10^{-7}$ s), so the condition $\tau_l > \tau_r$ [8] is not satisfied and the condensate cannot be formed.

In the previouse calculations of the magnon time relaxation we use standard phonon Hamiltonian [9]:

$$H_{ph} = \sum_{\vec{k}} \epsilon_{ph}(\vec{k})\, b^+_{\vec{k}}\, b^-_{\vec{k}}\ ; \qquad \epsilon_{ph}(\vec{k}) = \hbar v_s k, \tag{3}$$

and Hamiltonian of magnon- (longitudinal) phonon interaction [10]:

$$H_{int} = \sum_{\vec{k},\vec{q}} F(\vec{q})\, B^+_{\vec{k}}\, B_{\vec{k}-\vec{q}}\, (b_{\vec{q}} + b^+_{-\vec{q}});$$

$$F(\vec{q}) = i\sqrt{\frac{2\hbar I^2}{MN v_s q}}\, (\vec{q}\, \vec{l}_{\vec{q}}). \tag{4}$$

Magnon Hamiltonian in momentum space is of the form (in absence of an external magnetic field):

$$H_m = \sum_{\vec{k}} \epsilon_m(\vec{k})\, B^+_{\vec{k}}\, B_{\vec{k}}\ ; \qquad \epsilon_m(\vec{k}) = I(1 - \cos ak). \tag{5}$$

With those Hamiltonians we was able to examine the existention coupled states in magnon system. Works are presented in papers [11 to 14]. Now, we are interested in the consequences of existence of such condensate that is could such BEC — magnon drops, have a SF „motion".

A new method of solving the gap-equation for 1d structures was recently proposed [15]. This method was used for an analysis of the exciton drops spectrum in the molecular chains. Our intention is to test the magnon drop spectrum by the use of this method.

The virtual exchange of phonons leads to effective interaction of spin waves. One of the possible consequences of this interaction is the creation of magnon drops which arise when two spin waves with opposite momenta collide. The formed drop is unstable and desintegrates into two new (with respect to the initial) elementary excitations. The spectrum of these excitations will be the object of our investigations. Using the procedure similar to one used in BCS theory of superconductivity (Fröhlich and Bogolyubov's transformations, etc, methologicaly exposed in [3]) we get dispersion law of elementary excitations (see in [14], too):

$$E(\vec{k}) = [\epsilon_m^2(\vec{k}) - |\phi(\vec{k})|^2]^{1/2}, \tag{6}$$

through equations of motion for operators 8:

$$i\hbar\dot{B}_{\vec{k}} = [B_{\vec{k}}, H]\,; \qquad H = H_m + H_{ph} + H_{int}. \tag{7}$$

Where, in eq. (6), $\phi(\vec{k})$ are the transformational functions, defining the gap elementary excitations and satisfying the gap-equation (singular difference equation). The result of applikation of the new method of solving the gap-equation, we can represented graphically of fig. 1 and 2.

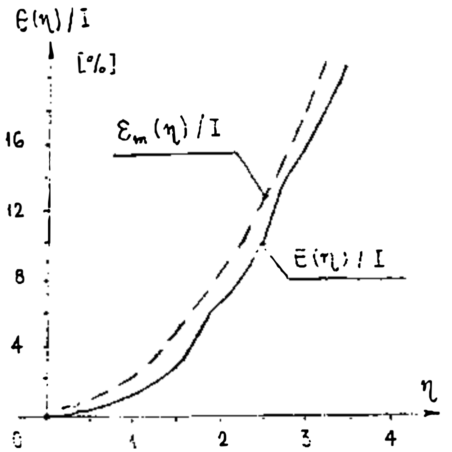

Fig. 1

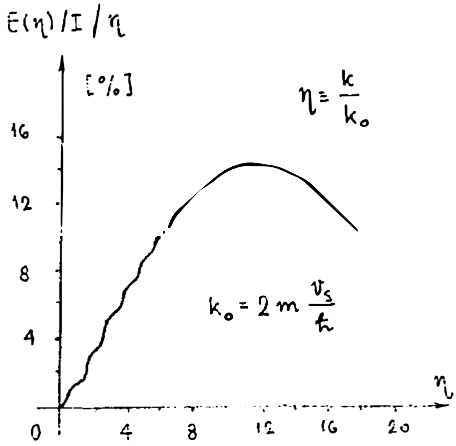

Fig. 2

It is seen from Fig. 1 that the energies E of desintegrated drops are lower than the energies ϵ of spin waves. This is compatible with result of papers [11, 12]. The function E $(\vec{k})$ /k (Fig. 2) was also numerically calculated and it was shown that this function monotonously increases for all k. Consequently, these elementary excitations do not possess SF properties.

Comparing results of this analysis to the results of [15] (the excitons possess a large gap) one can conclude than SF effects could eventually be expected in a ferromagnet placed in a very strong external magnetic field $\mathcal{H}$, such than $\mu\,\mathcal{H} >> 1$.

Finally, we seek the existence of solitary mechanism of SF. If there in 1d ferromagnetic structures magnon-phonon interaction, such that quasi-particles with velocity (v) smaller than the longitudinal sound velocity (v_s) are created, they will behave as the solitary waves. The probability amplitudes A (x, t) of single particle solitary states satisfies the following solitary equation (the formation of such, cubic Schrödinger equation can be found in [16]):

$$i\hbar\,\frac{\partial A}{\partial t} - (\hbar\Omega + E_p)\,A + \frac{1}{2}\,a^2 I\,\frac{\partial^2 A}{\partial x^2} + \frac{4a^2\,\mathcal{J}^2\,\mathcal{H}}{M\,(v^2 - v_s^2)}\,|A|^2\,A = 0. \tag{8}$$

When the quasi-particles are created in the system with velocities $v > v_s$, this equation, however, changes its meaning, since the last term becomes negative. In this case, it gets the form like the phenomenological equation of hydrodynamics than describes SF liquid flow [17]. Buth, we have to mentioned that the last term on the left side eq. (8) proportional to the intensity of the external magnetic field: $J_{\mathcal{H}} = \mu \mathcal{H} \sum_k k \sin ak$, and the equation (8) have meaning only in the presence of external magnetic field.

In general, we conclude that in ferromagnetic system we can expect SF effects only if they are in the strong external magnetic field.

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