

EM FIELD DISTRIBUTION INSIDE LASER RESONATORS

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Abstract

We have developed a numerical procedure for the analysis of the spatial properties of a fast pulse from a Q-switched laser. We present the numerical method and show some results of the application.

INTRODUCTION

The problem of steady state modes of a laser has been extensively treated in the literature [1], [2], [3], [4], [5], [6], [7], [8], for the cases of stable and unstable resonators with active medium. However, the analysis of the spatial properties of a fast pulse from a Q-switched laser has not been done yet. Such lasers usually have output mirrors with rather low reflectivity. The resonators are also often unstable or nearly unstable. In such a situation the concept of a stationary resonator mode is no longer appropriate for the description of a transient giant pulse. We have developed a numerical procedure to calculate the time evolution of the electric field in a laser resonator. The initial EM field is due to spontaneous emission. This initial field is then propagated according to the paraxial wave equation. At each propagation step the field was corrected for gain and additional noise produced by spontaneous emission.

CALCULATION TECHNIQUE

The model used in our calculations is shown in figure 1. The space between the mirrors was divided in M equidistant segments. The distance between segments was Δz , $\Delta z = D/M$, where D is the distance between the mirrors. In x and y directions the segments were divided into J. L. calls. Between these segments the active medium is located in thin sheets. In the resonator cavity we have the wave equation

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right) \vec{U}(x, y, z, t) = 0 \quad (1)$$

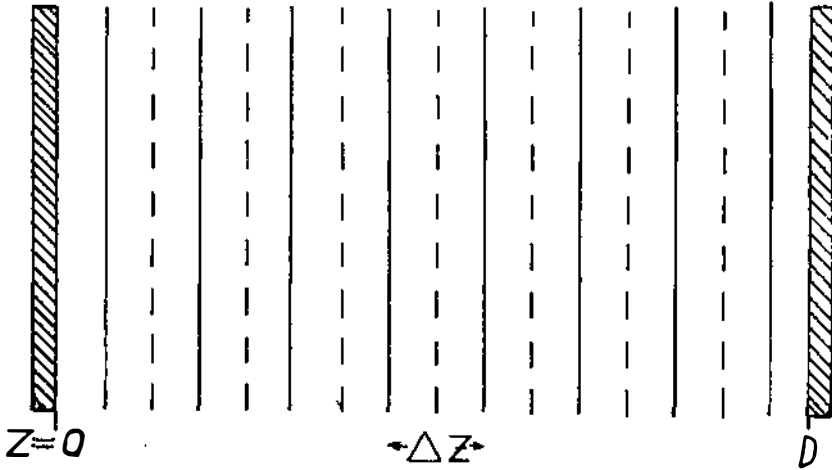


Figure 1. Model of laser resonator used in the calculations. The EM field is calculated at planes indicated with solid lines, and the gain medium is located in thin sheets between them.

Electric field strength vector \vec{U} within the resonator can be represented by plane polarized monochromatic travelling wave,

$$\vec{U}(x, y, z, t) = \vec{e} \operatorname{Re} [U(x, y, z) U_p(-i\omega t)] \quad (2)$$

where \vec{e} is a unit polarization vector, ω is the optical frequency ($\omega = c \cdot k$), k is the free space wave vector, $U(x, y, z)$ is the complex scalar electric field depending on the position coordinates. With $U(x, y, z)$ the wave equation becomes

$$(\nabla^2 + k^2 n^2) U(x, y, z) = Q \quad (3)$$

where n is the complex index of refraction. Equation (3) can be converted into an integral equation by means of the Green's function

$$U(x, y, z + \Delta z) \approx U_0(x, y, z + \Delta z) U_p \left(\frac{g(x, y)}{2} - ik(n-1)\Delta z \right) \quad (4)$$

where U_0 is free propagated field for distance Δz , the exponential term corrects the free propagated field for gain and refractive index inhomogeneities. The calculation was carried out in the following steps. First we generated the random electric field coming from spontaneous emission on all segments. Then we propagated all segments for one step to the right and to the left. The resulting free propagated field was then amplified by the amount of stimulated emission and a random field, corresponding to spontaneous emission, was added. Gain was calculated in the two level atom approximation. The spontaneous emission field was taken to be a two dimensional uniformly distributed random variable with the average value corresponding to the spontaneous emission rate. In the end we computed the changes in population inversion due to gain and spontaneous emission. Fields were corrected for mirror curvature, shape and reflectivity in the case when the left and right travelling waves were reflected from the mirrors. This procedure was repeated until the laser pulse is formed.

PROPAGATION ALGORITHM

When we insert left and right travelling waves with slowly varying amplitude $E_{L,R}(x, y, z)$ in equation (3)

$$U_{L,R}(x, y, z) = E_{L,R}^L(x, y, z) \exp(\pm i k z) \quad (5)$$

we can neglect second derivatives in z and we get paraxial wave equations for left and right travelling waves

$$\begin{aligned} 2ik \frac{\partial E_R}{\partial z} + \nabla_T^2 E_R &= 0 \\ -2ik \frac{\partial E_L}{\partial z} + \nabla_T^2 E_L &= 0 \end{aligned} \quad (6)$$

where $\Delta_T^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, E_L, E_R are slowly varying amplitudes of left and right travelling waves. These free space wave equations were solved by finite difference method of Dufort and Frankel [10]

$$\begin{aligned} \frac{\partial E}{\partial z} &= \frac{1}{2\Delta z} (E_{j,l}^{m+1} - E_{j,l}^{m-1}) \\ \frac{\partial^2 E}{\partial x^2} &= \frac{1}{\Delta x^2} (E_{j+1,l}^m - E_{j,l}^{m+1} - E_{j,l}^{m-1} + E_{j-1,l}^m) \\ \frac{\partial^2 E}{\partial y^2} &= \frac{1}{\Delta y^2} (E_{j,l+1}^m - E_{j,l}^{m+1} - E_{j,l}^{m-1} + E_{j,l-1}^m) \end{aligned} \quad (7)$$

These difference equation scheme is explicit and allows the calculation of left and right free propagated fields from the fields on the two preceding layers. However at the reflection at the mirrors we know the fields only on one preceding layer. So we need another propagation algorithm for the layers next to the mirrors

$$\begin{aligned} \frac{\partial E}{\partial z} &= \frac{1}{\Delta z} (E_{j,l}^{m+1} - E_{j,l}^m) \\ \frac{\partial^2 E}{\partial x^2} &= \frac{1}{\Delta x^2} (E_{j+1,l}^m - 2E_{j,l}^m + E_{j-1,l}^m) \\ \frac{\partial^2 E}{\partial y^2} &= \frac{1}{\Delta y^2} (E_{j,l+1}^m - 2E_{j,l}^m + E_{j,l-1}^m) \end{aligned} \quad (8)$$

The medium was described by a two level atomic system with lower laser level empty all the time. The incident wave intensity impinging upon the active medium is changed by

$$\Delta j = \frac{\sigma N_2 j \Delta z}{\Delta V} + j_{\text{noise}} \quad (9)$$

where σ is the stimulated emission cross section, j is the incoming intensity, N_2 is inverse population in given segment and ΔV is segment volume. The first term describes the change in intensity caused by amplification by stimulated emission and the second j_{noise} is the intensity corresponding to spontaneously emitted photons within the given frequency range. This frequency range is chosen to be the mode width of the resonator and not the full gain line width.

$$j_{\text{noise}} = \frac{\epsilon \epsilon_0 |E_{\text{noise}}|^2}{2} \cdot \frac{\Delta V_L}{\Delta V_{\text{gain}}} \quad (10)$$

where E_{noise} is a two dimensional random variable with an amplitude corresponding to the full gain linewidth, $\Delta\mu_2$ is mode width, and ΔV_{gain} is gain linewidth. In the last step we computed the change in population inversion in each segment

$$\Delta N_2 = -N_{\text{sp}} - \frac{\sigma N_2 j \Delta t}{\hbar \omega} \quad (11)$$

where N_{sp} is the number of spontaneously emitted photons calculated from j_{noise} .

RESULTS

We applied the procedure to the case of two and three dimensional plane parallel laser resonators with active medium Nd : YAG. We show the results of the two dimensional calculations of the unidirectional ring laser. The resonator was divided into 23 segments along z axis with $\Delta z = 3$ mm and 31 segments along x axis with $\Delta x = 0.2$ mm. Two dimensional calculation uses the propagation algorithm (7), (8) for z and x coordinates only. For calculating the gain from (9) the segment volume is needed so we set the thickness of the two dimensional active material to Δx . We assumed that the EM field was coherent across this second transverse dimension. This could be achieved by suitable refractive index modulation on the boundaries.

The stimulated emission cross section was taken $\sigma = 3.3 \text{ E} - 23 \text{ m}^2$ and spontaneous emission lifetime of the upper laser level with $1/A_{21} = 230 \mu\text{s}$. Mirrors were smaller in size than the laser rod and had tapered edges because the propagation algorithm cannot sharp edges. The reflectivity of the mirrors was

$$R = \begin{cases} R_0 & r < r_0 \\ R_0 \exp\left(-\frac{(r-r_0)^2}{p^2}\right) & r \geq r_0 \end{cases} \quad (12)$$

In our calculation we set $r_0 = 1.4 \text{ mm}$, $\rho = 0.3 \text{ mm}$. The initial inversion was $8 \times 10^{23} \text{ m}^{-3}$. Untapered output mirror reflectivity was 0.64, so 36% of the intensity was fed back into the laser. We have simulated the Q-switching of the laser by linearly increasing the output mirror reflectivity

$$R_{\text{out}}(t) = \begin{cases} 0 & , t < t_0 \\ \frac{R_0(t - t_0)}{t_2 - t_0} & , t_0 \leq t < t_q \\ R_0 & , t \geq t_q \end{cases} \quad (13)$$

We chose $t_0 = 10 \text{ ns}$ to allow the laser to establish the quasi equilibrium state of the spontaneous emission and $t_q = 40 \text{ ns}$ for opening the Q-switch. In the instantaneous power plot (figure 2) of the laser output we see that laser pulse is formed 5 ns after the Q-switch was completely opened. The pulse was 1.6 ns long. The pulse shape is modu-

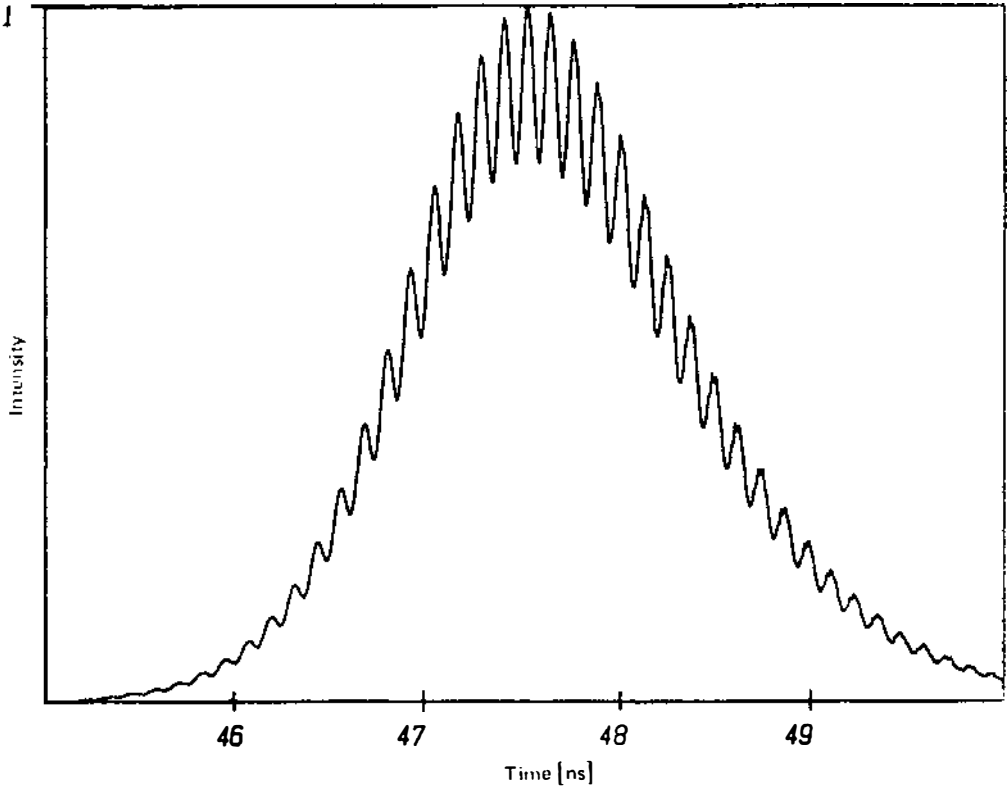


Figure 2. Instantaneous power plot of the laser pulse

lated periodically with period corresponding to one half of the resonator length. We believe these oscillations to be the amplified remnants of the step discontinuity arising from the initial conditions. The depth of this modulation depends strongly on the detuning of the laser frequency from the empty resonator mode frequency and on the rate of change of the resonator quality. The results shown in figure 2 and 3 were obtained with zero detuning. The observed intensity profiles (figure 3) of the beam at the pulse peak ($t = 47.5$ ns) show several spikes so that the divergence of the laser output is 2.5 mrad. The phase profiles show almost constant phase across the beam. The spatial coherence of the laser beam varies as the pulse decays. At the end of the pulse the phase profile is very smooth with very small variations across the beam.

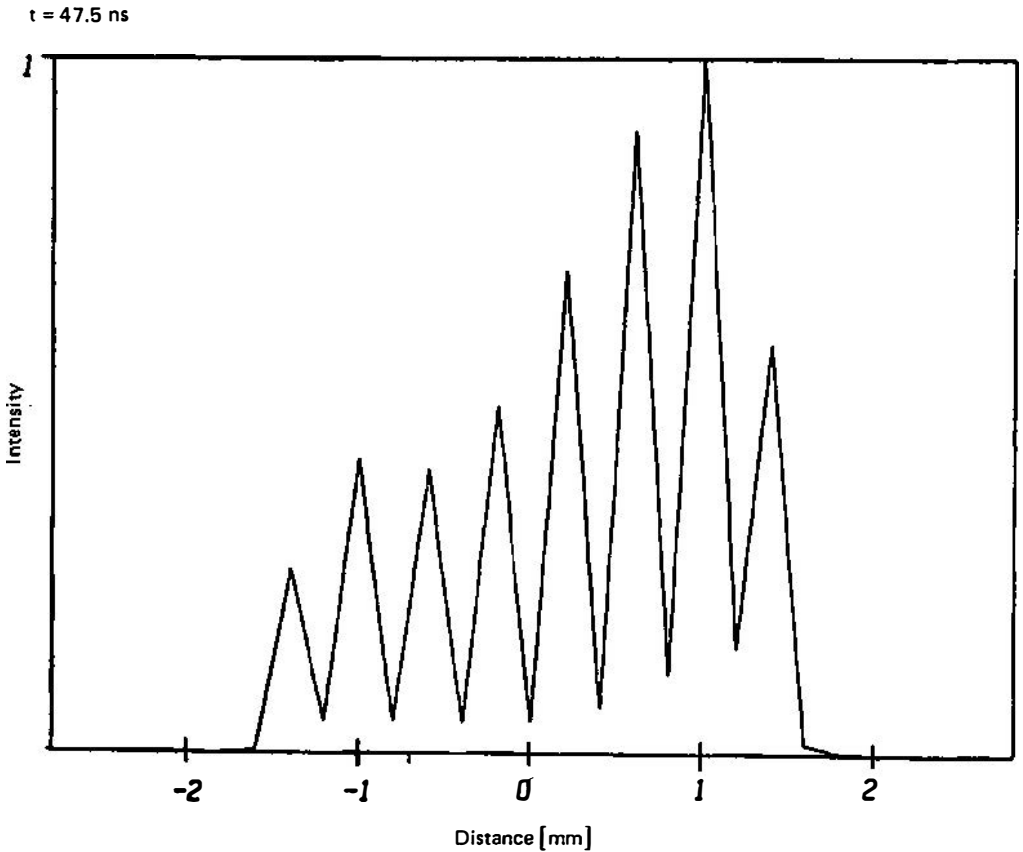
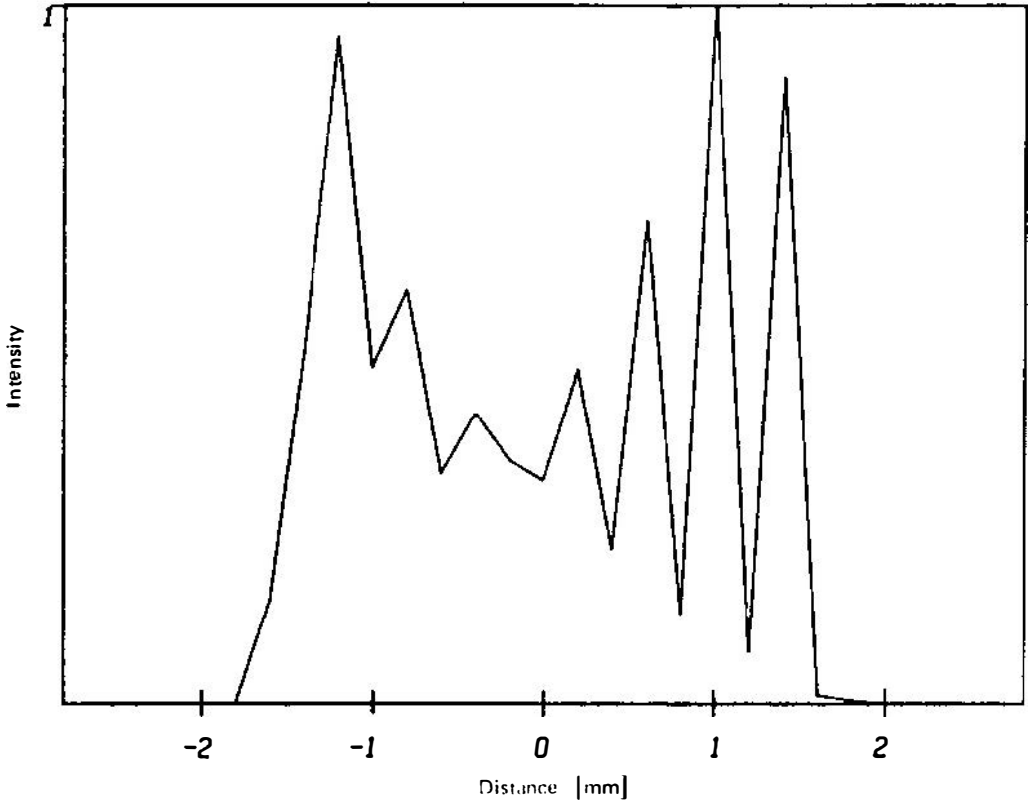
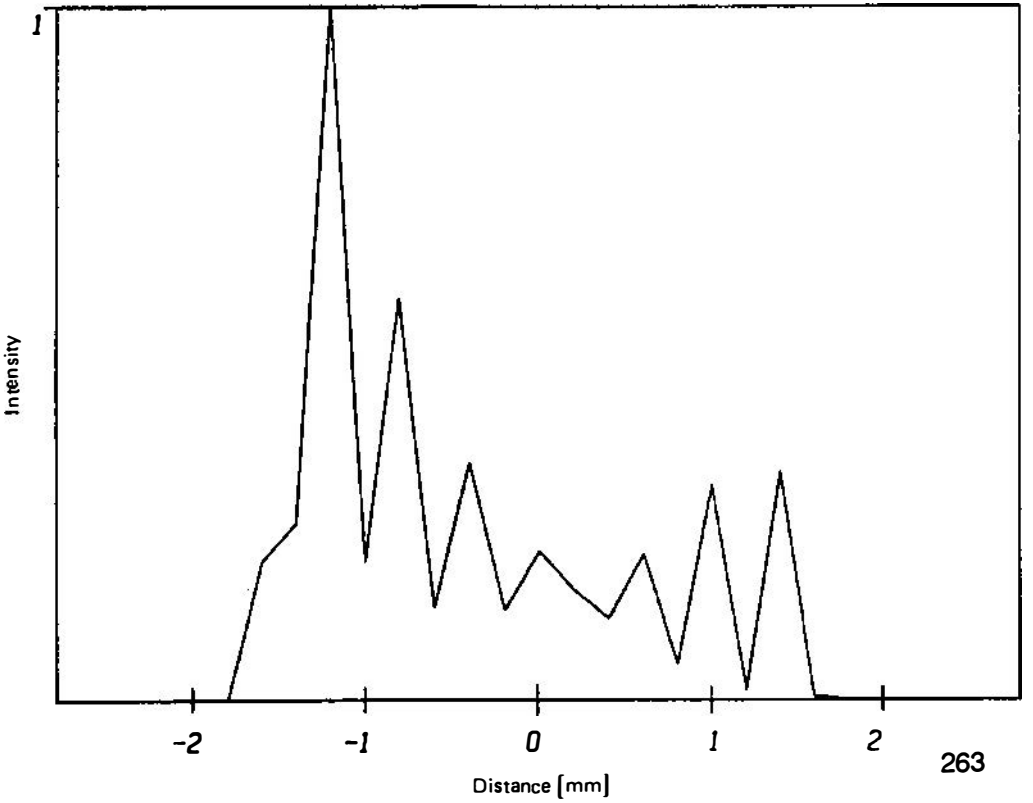


Figure 3. Phase and intensity profiles of the laser beam at different times ($t = 47.5$ ns, $t = 49$ ns, $t = 50$ ns)

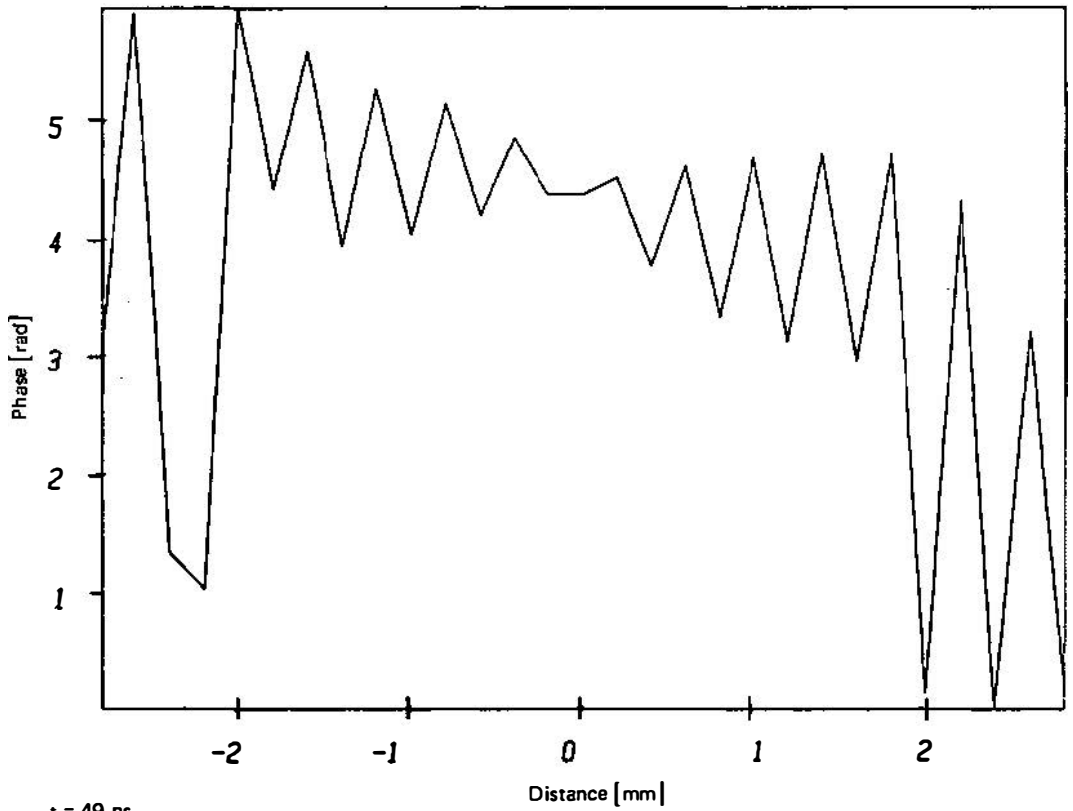
t = 49 ns



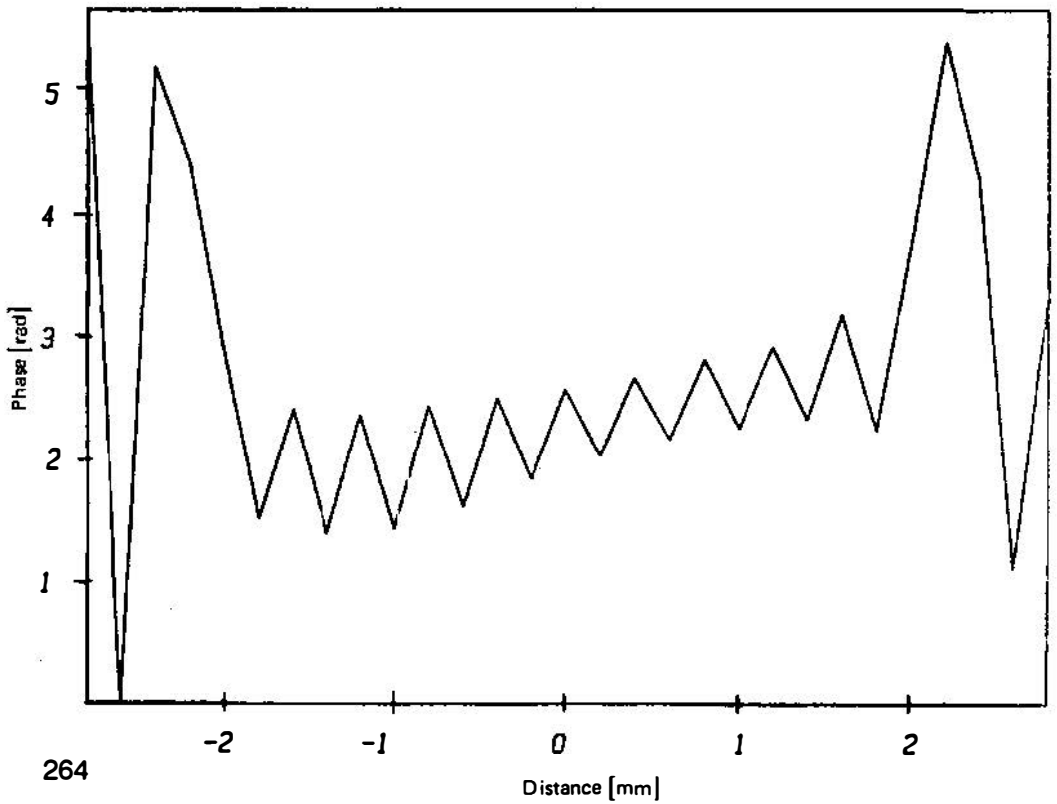
t = 50 ns



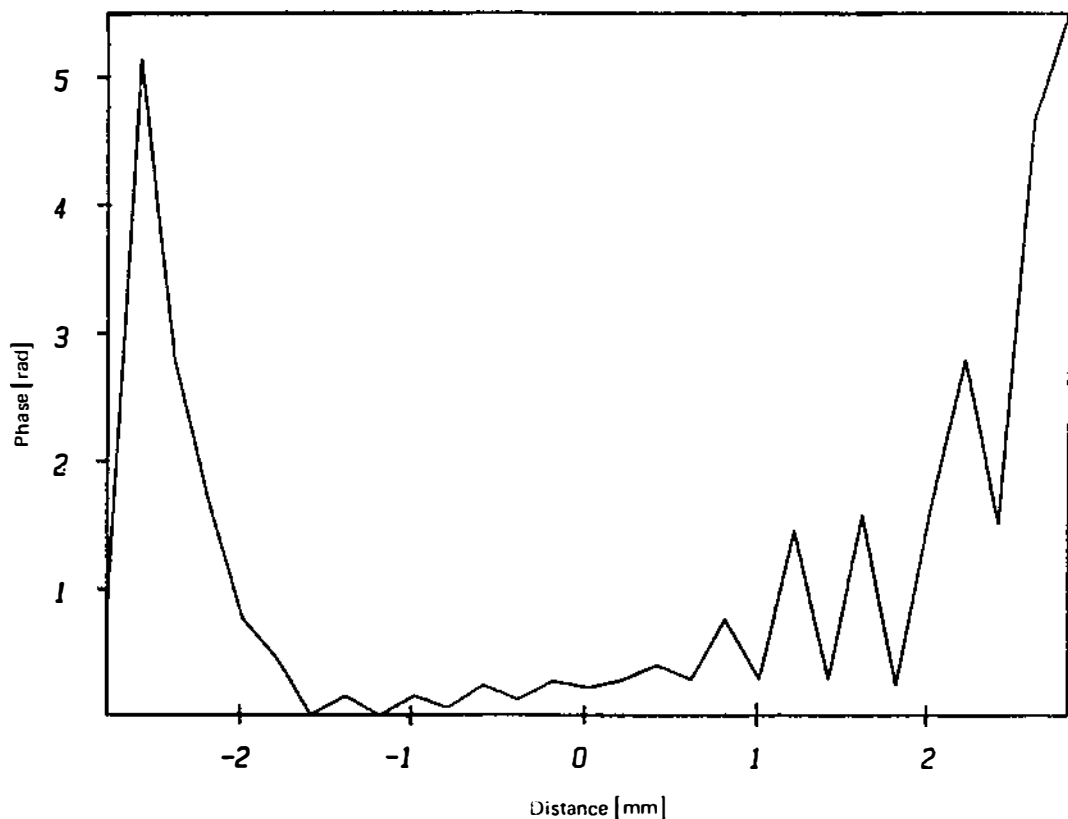
$t = 47.5 \text{ ns}$



$t = 49 \text{ ns}$



$t = 50 \text{ ns}$



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