PRESSURE DEPENDENCE AND TRIPLET CORRELATION EFFECTS IN SPECTRUM OF SINGLE EXCITATIONS IN SUPERFLUID ^{4}He

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Abstract

Pressure dependence of single excitations spectrum and influence of triplet correlation were examined using Brillouin-Wigner perturbation theory. We found that theoretical results were improved about 20 – 25% if triplet correlations were included. We also showed that Family's structure factors didn't reproduce pressure dependence quite well when compared with experimental results.

1. INTRODUCTION

Precise neutron scattering experiments of Cowley and Woods[1)] showed two distinct branches in the excitation spectrum of superfluid ^{4}He (Fig.)[2)]. The lower branch corresponds to the excitation of a single quasiparticle from the condensate and it had been known before. The upper branch may be thought of as corresponding to the excitations of two or more quasiparticles from the condensate.

In this paper we investigated the pressure dependence of the spectrum of single excitations in superfluid ^{4}He and the influence of triplet correlations under these conditions. Some parts of these investigations were published in Refs. 3 and 4, further denoted by I and II. In this paper we kept the same notations as in II.

For the energy of single excitation we used the relation

$$\epsilon = \epsilon_0 + \epsilon_2/(1 - \epsilon_3/\epsilon_2), \tag{1}$$

where all quantities have the same meaning and form as in II. The realtion (1) contains quantities S_k, S_3, and S_4 which are two-, three- and four-body structure factors. In the convolution approximation (CA) the last two have forms[3, 5, 6)]

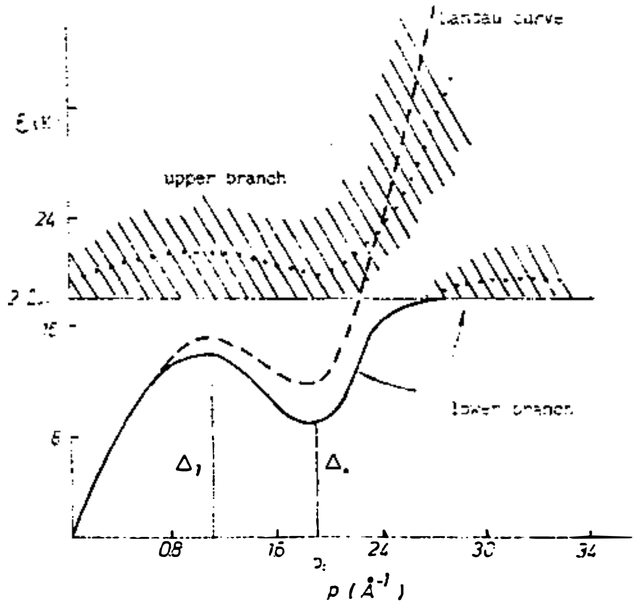

Figure 1. Schematic depiction of excitation spectrum of superfluid ^{4}He

$$S_3^{\,c}(-k, l, k-l) = S_k S_l S_{k-l} \tag{2}$$

$$S_4^{\,c} = S_{k-l} S_{k-l'} S_l S_{l'} \cdot (-2 + S_k + S_{l-l'} + S_{k-l-l'}). \tag{3}$$

Although there are several more approximations for the function S_3, which are available[7,8], only the Feenberg form (FA) of this function was used

$$S_3^{\,F} = \left\{k^2/S_k + l^2/S_l + |\vec{k} - \vec{l}|^2/S_{k-l}\right\}^{-1} \cdot \left\{0.5\,(k^2 + l^2 + |k-l|^2) \cdot S_3^{\,c}\right.$$

$$\left. + 0.5\,(|\vec{k} - \vec{l}|^2 - k^2 - l^2)(S_l + S_k - S_{k-l}) + l^2 S_k + k^2 S_l\right\}. \tag{4}$$

It is worth to notice that relation (4) is transformed into (2) if triplet correlations are neglected in the ground state wave function[7].

2. NUMERICAL COMPUTATIONS AND DISCUSSION

Relation (1) is an integral equation for ϵ which includes two-dimensional and six-dimensional integrals. The same numerical algorithm as in I and II was used. 32 point Gauss integration was applied in calculation of 2-dimensional integrals. The quantity $_3$ which contains six-dimensional integrals was calculated using the number theoretic method.

Table 1. The correspondence between pressure and density in superfluid ^{4}He

p (atm)	T (K)	m (kg m^{-3} 10^3)	(m^{-3} 10^{30})
SVP	0.6	0.145005	2.1816
	1.5	0.145045	2.1822
1	1.5	0.1469	2.21
5	1.5	0.1533	2.3
10	1.5	0.1595	2.399
14	1.5	0.1639	2.466
16	1.5	0.1660	2.497
20	1.5	0.1690	2.543
24	1.5	0.1735	2.610

The energy of elementary excitations, as it was shown in I, is a function of density of particles and two-body structure factor (at least after the above stated approximation) which of course depends on density as well. Two body structure factors and the corresponding pressure are available in literature.

In order to obtain the pressure dependence of spectrum we used the two body structure factors from Family's paper[9]. Two body structure functions, in this paper, are plotted for: SVP, 5, 10, 15, 20 and 24 atm at zero temperature. Our relation (1) includes the density of particles which is the unknown function of the pressure. For this correlation we employed experimental results (Table 1).

First calculation was done for $\epsilon_3 = 0$, i.e. $\epsilon = \epsilon_0 + \epsilon_2$. The results for maxon and roton part of spectrum are presented in Table 2.

The calculation of energy with complete expression (1) was performed as well. For entire spectrum and one value of pressure it took 10 — 11 hours at Univac 1100 computer. We found the numbers very close to those in Table 2.

Table 2. The values of single quasiparticle energy in maxon and roton part of spectrum in convolution (Feenberg) approxmation

p (atm) / k (10^{10} m^{-1})	5	10	15	20	24
1.0	19.414 (24.346)	19.806 (25.397)	20.250 (26.414)	19.646 (26.413)	19.927 (27.344)
1.1	19.397 (24.441)	19.815 (25.574)	19.934 (26.094)	19.172 (25.835)	19.525 (26.946)
1.2	19.705 (24.791)	20.153 (26.002)	20.036 (26.160)	19.163 (25.716)	19.572 (26.968)
1.9	17.837 (16.946)	17.247 (16.343)	18.027 (17.389)	17.014 (16.259)	16.766 (16.041)
2.0	18.413 (17.066)	17.721 (16.290)	17.726 (16.354)	16.842 (15.352)	16.979 (15.560)
2.1	19.495 (17.750)	18.927 (17.107)	18.406 (16.541)	17.639 (15.653)	17.586 (15.609)

We notice a great discrepancy between the results of CA and FA. Comparing these numbers with experimental results[1, 10], we see that the Feenberg approximation is better. It means that triplet correlations play an important role in single excitation spectrum. This effect is about 20 – 25%. Also we see that our results in maxon region are lower and in roton region higher than the corresponding experimental points. It is a well known experimental fact that energy difference of roton minimum Δ_0 and maxon maximum Δ_1, becomes higher and higher with the increase in pressure. Beyond about 24 atm $\Delta_1 > 2\Delta_0$. Although we notice a slight qualitative change in this sence, our results are rather far from the experimental picture. We did a preliminary calculation using another form of S_k[8] and produced quite good agreement with experiments. From this we could conclude that Family's functions S are not good or that our supstitution of his curves with numbers (what is less unlikely) was bad.

This kind of theory is not adopted to describe multi-excitations (upper branch). We believe it could be done if one starts with the wave function having at least two quasiparticles out of the condensate.

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