SOLITARY WAVES IN THE CLASSICAL ONE-DIMENSIONAL ISING MODEL IN A TRANSVERSE FIELD

B.Žekš
Institute of Biophysics, Medical Faculty and J.Stefan Institute, Edvard Kardelj University, Ljubljana, Yugoslavia

M.A. de Moura and C. Cordeiro
Departamento de Física, Universidade Federal de Pernambuco, Recife, Brasil

In recent years there has been a considerable interest in solitons and their relevance to low-temperature thermodynamic and dynamic properties of one-dimensional systems. Sine-Gordon system, one-dimensional ϕ^4 model and some spin systems (isotropic Heisenberg model, XY model) have been shown[1-7] to admit solitary wave excitations which usually exhibit remarkable stability and particle properties and essentially contribute to some low-temperature macroscopic properties of these systems. We study in this paper the solitary wave excitations in a classical Ising model in a transverse field, the quantum spin-$\frac{1}{2}$ version of which can be used to describe some properties of quasi one--dimensional hydrogen bonded ferroelectrics[8,9].

The system we consider is an infinite linear chain of spins $\vec{S}_i = (S_i^x, S_i^y, S_i^z)$ which are classical vectors with $|\vec{S}_i| = 1$ and satisfy the Poisson bracket relations

$$[S_i^\alpha , S_j^\beta] = \epsilon_{\alpha\beta\gamma} S_i^\gamma \delta_{ij} \qquad \alpha,\beta,\gamma = 1,2,3 \quad (1)$$

$\epsilon_{\alpha\beta\gamma}$ being the completely antisymmetric tensor of the third rank. The Hamiltonian of the system is given by

$$H = -J \sum_i S_i^z S_{i+1}^z - \Omega \sum_i S_i^x \quad (2)$$

where J represents the nearest neighbour interaction constant and Ω corresponds to the transverse field. From the equation of motion for spin components

$$\frac{dS_i^\alpha}{dt} = [S_i^\alpha , H] \tag{3}$$

one gets

$$\frac{dS_i^x}{dt} = J\, S_i^y\, (S_{i+1}^z + S_{i-1}^z) \tag{4a}$$

$$\frac{dS_i^y}{dt} = -\, J\, S_i^x\, (S_{i+1}^z + S_{i-1}^z) + \Omega\, S_i^z \tag{4b}$$

$$\frac{dS_i^z}{dt} = -\, \Omega\, S_i^y \tag{4c}$$

In the continuum approximation, i.e. assuming that $S_i^\alpha(t) = S^\alpha(ai,t) = S^\alpha(x,t)$ is a slowly varying function of x, one obtains for the energy density h(x) the expression

$$h(x) = \frac{1}{a}\, \{-J(S^z)^2 - \Omega\, S^x + \frac{1}{2}\, Ja^2 (\frac{\partial S^z}{\partial x})^2\} \; , \tag{5}$$

a being the lattice constant. The equations (4) can be rewritten as

$$\frac{\partial S^x}{\partial t} = J\, S^y\, (2\, S^z + a^2\, \frac{\partial^2 S^z}{\partial x^2}) \tag{6a}$$

$$\frac{\partial S^y}{\partial t} = -J\, S^x\, (2\, S^z + a^2\, \frac{\partial^2 S^z}{\partial x^2}) + \Omega\, S^z \tag{6b}$$

$$\frac{\partial S^z}{\partial t} = -\, \Omega\, S^y \tag{6c}$$

Minimizing the energy (eq. 5) one obtains for $\Omega<2J$ the ground state

$$S_o^x = \frac{\Omega}{2J}\ , \quad S_o^y = 0\ , \quad S_o^z = \pm\{1 - (\frac{\Omega}{2J})^2\}^{1/2} \tag{7}$$

which is polar ($S_o^z \neq 0$) and degenerated (two domains).

We are looking for the solitary wave excitations, i.e. for the solutions of eqs. (6) which have a travelling wave form

$$\vec{S}\ (x,t) = \vec{S}(\xi)\ , \qquad \xi = \frac{x}{a} - vt \tag{8}$$

(v being the solitary wave velocity in units of lattice constants per time) and which at both infinities equal to the ground state solution

$$\xi \to \pm\infty\ , \quad \vec{S} \to \vec{S}_o\ , \quad \frac{d\vec{S}}{d\xi} \to 0 \tag{9}$$

The system of equations (6) can be solved analytically. For the solitary wave shape one gets

$$\sqrt{2}\xi = \arcsin\frac{\sqrt{2}z}{\sqrt{A+1}} + \frac{\sqrt{A-1}}{\sqrt{2}}\ln\{\frac{\sqrt{A+1-2z^2} + z\sqrt{A-1}}{(1-z)\ \sqrt{A+1}}\} \tag{10}$$

and its energy equals to

$$\Delta E = J(S_o^z)^2\ \{\frac{1}{\sqrt{2}}\ (A+1)\ \arcsin\sqrt{\frac{2}{A+1}} - \sqrt{A-1}\} \tag{11}$$

Here z is the order parameter normalized to its ground state value ($z = S^z/S_o^z$) and A is a function of the parameters of the system and of the solitary wave velocity v

$$A = \frac{1}{(S_o^z)^2}\ \{[2-(S_o^z)^2] - 4[1-(S_o^z)^2]\ (\frac{v}{\Omega})^2\} \tag{12}$$

To have real solutions A has to be bigger than one, which determines the upper limit of the solitary wave velocity $v_{max}^2 = \Omega^2/2$.

The obtained solitary wave has a form of kink (moving domain wall) and its width depends only on the parameter A. For $A \gg 1$, which corresponds to $\Omega \lesssim 2J$ and $v \ll v_{max}$, the equations (10) and (11) reduce to

$$z = \tanh \frac{\xi}{\sqrt{A}} \quad , \qquad \Delta E = \frac{4}{3\sqrt{A}} J(S_o^z)^2 \tag{13}$$

These expressions are characteristic for ϕ^4 solitary waves[1] with the domain wall width being equal to $2a\sqrt{A}$. For $A \to 1$ $(v \to v_{max})$ the width equals to $\pi a/\sqrt{2}$ and the continuum approximation is not justified in this case. For static domain walls (v=0) our expression is similar to the one obtained by Bishop et al.[10] for quantum spin-1/2 system by using a variation approximation. Recently, Takeno[11] treated the quantum case by using the coherent-state representation and obtained the solitary wave excitations of the same form as eq. (10).

It is easy to see that the solitary wave energies are in general lower than the lowest energy of the magnon band $\omega_{min} = 2JS_o^z$. For example, for $\Omega = J$ and $0 < v < v_{max}$ one obtains $0.50 \leq \Delta E/\omega_{min} \leq 0.97$.

For low temperature thermodynamic properties the low velocity solitary waves are the most important ones. So one is allowed to use the simplified solitary wave expression for $A \gg 1$ (eq. 13) when calculating low-temperature thermodynamic and dynamic properties. In the ideal solitary wave gas approximation[1,12] one obtains for the equal time correlation function

$$\langle S^z(x)\, S^z(0)\rangle = (S_o^z)^2\, e^{-\frac{x}{\lambda}} \tag{14}$$

where the correlation length

$$\lambda = 2a \frac{\{2-(S_o^z)^2\}^{1/2}}{S_o^z} \exp\left[\frac{4J}{3kT} \frac{(S_o^z)^3}{\{2-(S_o^z)^2\}}\right] \tag{15}$$

exponentially diverges as $T \to 0$. For the dynamic correlation function one gets

$$\langle S^z(t)\ S^z(0)\rangle = (S_o^z)^2\ e^{-\frac{t}{\tau}} \tag{16}$$

where the relaxation frequency $\frac{1}{\tau}$ is given by

$$\frac{1}{\tau} = \frac{\Omega\ S_o^z}{2\sqrt{2}\sqrt{2-(S_o^z)^2}}\ \exp\left[\frac{4J}{3kT}\ \frac{(S_o^z)^3}{\{2-(S_o^z)^2\}}\right] \tag{17}$$

and shows the exponential temperature dependence and a strong dependence on the transverse field Ω .

References:

(1) J.A.Krumhansl and J.R. Schrieffer, Phys.Rev. B11 3535 (1975)
(2) H.J. Mikeska, J.Phys. C11, L29 (1978)
(3) T.Schneider and E.Stoll, Ferroelectrics 24 67 (1980)
(4) G.F. Mazenho and P.S. Sahni, Phys.Rev. B18, 6139 (1978)
(5) E. Stoll, T.Schneider and A.R. Bishop, Phys. Rev. Lett. 42, 937 (1979)
(6) T. Schneider and E. Stoll, Phys. Rev. (to be published)
(7) J. Tjon and J. Wright, Phys. Rev. B 15, 3470 (1977)
(8) A.V. de Carvalho and S.R. Salinas, J.Phys.Soc.Japan 44, 238 (1978)
(9) R.Blinc, B.Žekš, A.Levstik, C. Filipič, J. Slak, M. Burgar, I.Zupančič and L.A. Shuvalov, Phys. Rev. Lett. 43, 231 (1979)
(10) A.R. Bishop, E. Domany and J.A.Krumhansl, Ferroelectrics 16, 183 (1977)
(11) S.Takeno, J. Phys.Soc.Japan 48, 1075 (1980)
(12) J.F.Currie, J.A.Krumhansl, A.R.Bishop and S.E.Trullinger, Phys.Rev. B (to be published)