THE HYBRID EXCITATIONS IN THE MAGNON-PHONON SYSTEM

S.D.Stojanović, M.J.Škrinjar and J.P.Šetrajčić

Institute of Physics, Faculty of Sciences, Novi Sad

In this paper we analysed the hybrid magneto-elastic excitations due to the magnon-phonon interaction in the anisotropic ferromagnets. We have also calculated the magneto-mechanical tensor of the system.

This paper treats the problem of the magnon-phonon interaction from the standpoint of the possibility of the existence of collective modes (hybrid excitations) in the spin-phonon system. Those hybrid excitations represent in fact the quanta of the magnetoelastic waves in ferromagnetics, whose phenomenological theory was elaborated in [1] . A problem similar to ours was discussed for the case of antiferromagnetic [2] .

The Hamiltonian of the magnon-phonon system of anisotropic ferromagnetic with one easy axis (chosen to be z-axis) and one magnetic ion per unit cell can be written in the following form:

$$H = H_0 + H_S + H_P + H_{SP} \tag{1}$$

$$H_0 = -\frac{1+C}{2} S^2 J_0 N - \mu_B \mathcal{H} S N \ , \quad H_P = \sum_{\vec{k}j} \hbar\omega_{\vec{k}j}\left(\frac{1}{2} + b^+_{\vec{k}j} b_{\vec{k}j}\right)$$

$$H_S = \sum_{\vec{k}} \mathcal{E}_S(\vec{k}) B^+_{\vec{k}} B_{\vec{k}} \ , \quad H_{SP} = \sum_{\vec{k}\vec{q}j} \Phi_j(\vec{k},\vec{q}) \left(b_{\vec{q}j} + b^+_{-\vec{q}j}\right) B^+_{\vec{k}} B_{\vec{k}-\vec{q}}$$

$$\Phi_j(\vec{k},\vec{q}) = i\left[\frac{\hbar}{2MN\omega_{\vec{q}j}}\right]^{1/2} \left\{\vec{q}\vec{e}_{\vec{q}j}\left(\Delta - SJ_{\vec{k}-\vec{q}}\right) + \vec{k}\vec{e}_{\vec{q}j} S\left(J_{\vec{k}-\vec{q}} - J_{\vec{k}}\right)\right\}$$

$$\mathcal{E}_S(\vec{k}) = \Delta - SJ_{\vec{k}} \ , \quad \Delta = S(1+C)J_0 + \mu_B \mathcal{H}$$

C - is the anisotropy constant.

The Hamiltonian (1) defined in terms of "standard" magnon-phonon interaction cannot lead to the hybridization in the system, because it contains the terms of the type $B^+_{\vec{k}} B_{\vec{k}-\vec{q}} (b_{\vec{q}} + b^+_{-\vec{q}})$. One can calculate the influence of the magnon-phonon interaction on the magnon lifetime and phase transition temperature, on the basis of this Hamiltonian (see [3] and the references cited therein).

In order to discuss the hybrid modes in this system, we shall perform an unitary transformation of the Hamiltonian (1), which introduce a bilinear terms of the type $B^+b^+,\ B^+b, \dots$

$$H_{el} = e^{-\hat{S}} H e^{\hat{S}} \cong H - [\hat{S}, H] + \frac{1}{2}[\hat{S}, [\hat{S}, H]] \tag{2}$$

where

$$\hat{S} = \hat{S}_1 - \hat{S}_1^+ \ , \quad \hat{S}_1 = \sum_{\vec{\nu}} X_{\vec{\nu}} B_{\vec{\nu}} + \sum_{\vec{\nu}j} Y_{\vec{\nu}j} B_{\vec{\nu}} b_{-\vec{\nu}j} \tag{3}$$

Assuming the interaction with longitudinal phonon branch only (with the accuracu up to the third order terms), we obtain

$$H_{el} = H_{el}^{(0)} + H_{el}^{(1)} + H_{el}^{(2)} \tag{4}$$

where

$$H_{el}^{(0)} = H_0 + \frac{1}{2}\sum_{\vec{k}j}\omega_{\vec{k}j} + \sum_{\vec{k}}|Y_{\vec{k}}|^2[\xi_S(\vec{k}) + \omega_{\vec{k}l}] + \frac{1}{2}\Big\{|X_0|^2\xi_S(0) + \sum_{\vec{k}} X_0 \Phi_l^*(\vec{k}) Y_{\vec{k}}^* +$$
$$+ \sum_{\vec{k}} X_0 Y_{\vec{k}}^* \Phi_l(\vec{k}) + C.C.\Big\} \tag{5}$$

$$H_{el}^{(1)} = \xi_S(0) Y_0 X_0^* b_{0l} - \Big[\xi_S(0) X_0 + \sum_{\vec{k}} Y_{\vec{k}} \Phi_l(\vec{k})\Big] B_0 + C.C. \tag{6}$$

$$H_{el}^{(2)} = \sum_{\vec{k}}\big[\xi_S(\vec{k}) + |Y_{\vec{k}}|^2(\xi_S(\vec{k}) + \omega_{\vec{k}l})\big] B_{\vec{k}}^+ B_{\vec{k}} + \sum_{\vec{k}j}\omega_{\vec{k}j} b_{\vec{k}j}^+ b_{\vec{k}j} + \Big\{\sum_{\vec{k}} X_0 Y_{\vec{k}} \Phi_l(\vec{k}) B_{\vec{k}} B_{-\vec{k}} +$$
$$+ \frac{1}{2}\sum_{\vec{k}}|Y_{\vec{k}}|^2[\xi_S(\vec{k}) + \omega_{\vec{k}l}] b_{-\vec{k}l}^+ b_{\vec{k}} - \sum_{\vec{k}} X_0 \Phi_l(\vec{k}) B_{\vec{k}} (b_{\vec{k}l}^+ + b_{-\vec{k}l}) -$$
$$- \sum_{\vec{k}}[\xi_S(\vec{k}) + \omega_{\vec{k}l}] Y_{\vec{k}} B_{\vec{k}} b_{-\vec{k}l} + C.C.\Big\} \tag{7}$$

The function X and Y are determined from the conditions

$$\begin{aligned} &\tfrac{1}{2}S^2 c J_0 + \mu_B S H = |Y_{\vec{k}}|^2[\xi_S(\vec{k}) + \omega_{\vec{k}l}] \\ &\xi_S(0) X_0 + \sum_{\vec{k}} Y_{\vec{k}} \Phi_l(\vec{k}) = 0 \end{aligned} \tag{8}$$

In the resonance region ($\xi_S(\vec{k}) = \omega_{\vec{k}l}$), the system (8) gives

$$X_0 \cong i\,\frac{\xi_S(\vec{k})}{\xi_S(0)}\sqrt{\frac{\Delta_0 N}{4Mv^2}} \ , \quad |Y_{\vec{k}}| \cong \sqrt{\frac{\Delta_0}{6\hbar\omega_D}} \ , \ |X_0| \sim 10^{7}, \ |Y_{\vec{k}}| \sim 10^{-1} \div 10^{-2} \tag{9}$$

while the expression (4) turns into

$$H_{el} = H_{el}^{(0)} + H_{el}^{(2)} \tag{10}$$

where

$$H_{el}^{(0)} = H_0 + \frac{1}{2}\sum_{\vec{k}j}\omega_{\vec{k}j} + \frac{1}{2}\sum_{\vec{k}_1}(S^2 c J_0 + \mu_B S H)\Big(1 + \frac{\xi_S(0)}{12Mv^2}\Big) \tag{11}$$

$$H_{el}^{(2)} = \sum_{\vec{k}} H_{\vec{k}} \ , \quad H_{\vec{k}} = \sum_{\alpha\beta=1}^{4} S_{\alpha\beta} C_\alpha^+ C_\beta + \sum_{\alpha\beta=1}^{4}(R_{\alpha\beta} C_\alpha^+ C_\beta^+ + R_{\alpha\beta}^* C_\alpha C_\beta) \tag{12}$$

309

$$C_1 = B_{\vec{k}}\ ,\ C_2 = b_{\vec{k}l}\ ,\ C_3 = b_{\vec{k}t_1}\ ,\ C_4 = b_{\vec{k}t_2}\ ,\quad S_{11} = \mathcal{E}_S(\vec{k}) + \Delta_0\ ,\ S_{33} = \omega_{\vec{k}t_1}$$

$$S_{22} = \omega_{\vec{k}l} + \Delta_0\ ,\ S_{44} = \omega_{\vec{k}t_2}\ ,\ R_{12} \approx S_{12} = -\chi_0 \Phi_l(\vec{k}),\ R_{11} \approx 0\ ,\ \Delta_0 = \frac{1}{2} S \bar{c}^2 J_0 + S\mu_0 \mathcal{H}$$

v- is the velocity of longitudinal waves and ω_D- is the Debye frequency. One should not be confused by the large value for χ_0, because it figures in the expansion as $\chi_0 \Phi_l(\vec{k}) Y_{\vec{k}}$ which ensures the convergence of the expansion.

The Hamiltonian (10) is diagonalized by the canonical transformation

$$C_\alpha = \sum_{\nu=1}^{4} \left(U_{\alpha\nu} \xi_\nu + V_{\alpha\nu} \xi_\nu^+ \right)\ ;\quad \alpha = 1,2,3,4$$

giving

$$H = \sum_{\vec{k}\,\mu=1}^{4} E_\mu(\vec{k})\, \xi_{\mu\vec{k}}^+ \xi_{\mu\vec{k}} \tag{13}$$

$$E_{1,2}^2(\vec{k}) = \frac{1}{2}(S_{11}^2 + S_{22}^2) \pm \frac{1}{2}\sqrt{(S_{11}^2 - S_{22}^2)^2 + 16 S_{11} S_{22} S_{12}^2} \tag{14}$$

$$E_3 = \hbar\omega_{\vec{k}t_1}\ ,\quad E_4 = \hbar\omega_{\vec{k}t_2} \tag{15}$$

Expressions (14) represent the energies of hybridized magnon-phonon excitations and their dispersion law is given at the Fig. 1.

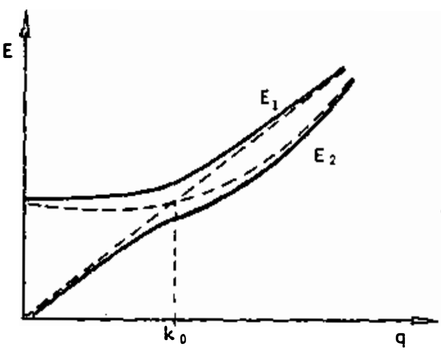

Fig. 1

Magnon-phonon hybridization enables us to connect magnetic and mechanical characteristics of the crystal, because it is possible by the action of the high frequency magnetic field, as an external perturbation, to induce the change of the mean atomic displacement in the crystal and vice-verse, by applying ultrasound as an external perturbation to induce additional magnetic moment in the crystal.

The interaction of the crystal with the external magnetic field $\vec{h}_{\vec{n}}(t)$ will be taken in the form

$$H_{int}(t) = -2\mu_B \sum_{\vec{n}\beta} S^{\beta}_{\vec{n}}(t)\, h^{\beta}_{\vec{n}}(t) \tag{16}$$

Applying the linear response theory, we have calculated the change of the average value of atomic displacement

$$\langle \vec{r}(\vec{k},\omega)\rangle_{neq} - \langle \vec{r}(\vec{k},\omega)\rangle_0 = \hat{R}(\vec{k},\omega)\, \vec{h}(\vec{k},\omega) \tag{17}$$

where $\langle \vec{r}(\vec{k},\omega)\rangle_{neq}$ and $\langle \vec{r}(\vec{k},\omega)\rangle_0$ are nonequilibrium and equilibrium mean values respectively. The quantity $\hat{R}(\vec{k},\omega)$ is the tensor with the elements in the resonance region given by

$$R_{\alpha\beta}(\vec{k},\omega) = 4\pi i \mu_B \sqrt{\frac{1}{2M\hbar\omega_{\vec{k}t}}}\; e^{\alpha}_{\vec{k}}\, \Gamma_{\beta}(\vec{k},\omega) \tag{18}$$

$$\Gamma_x(\vec{k},\omega) = \sqrt{\frac{S}{2}}\left\{U_{11}U_{21}\left[G_1(\vec{k},\omega) - G_1^{+}(\vec{k},\omega)\right] + U_{22}^2\left[G_2(\vec{k},\omega) - G_2^{+}(\vec{k},\omega)\right]\right\}$$

$$\Gamma_y(\vec{k},\omega) = i\sqrt{\frac{S}{2}}\left\{U_{11}U_{21}\left[G_1(\vec{k},\omega) - G_1^{+}(\vec{k},\omega)\right] + U_{22}U_{21}\left[G_2(\vec{k},\omega) - G_2^{+}(\vec{k},\omega)\right]\right\}$$

$$\Gamma_z(\vec{k},\omega) = 0 \quad , \quad G_{\rho}(\vec{k},\omega) = \langle\langle \xi_{\rho\vec{k}} \,|\, \xi^{+}_{\rho\vec{k}} \rangle\rangle_{\omega} \quad , \quad G^{+}_{\rho}(\vec{k},\omega) = \langle\langle \xi^{+}_{\rho\vec{k}} \,|\, \xi_{\rho\vec{k}} \rangle\rangle_{\omega}$$

From $R_{\alpha z} = 0$ it follows that sound waves are not being excited when the external field is directed along the easy axis.

In an analogous way we can calculate high-frequency magnetic susceptibility, i.e. the change of the magnetic moment of the crystal due to the input of mechanical energy. In this way we obtain

$$\langle \vec{m}(\vec{k},\omega)\rangle_{neq} - \langle \vec{m}(\vec{k},\omega)\rangle_0 = \hat{\chi}(\vec{k},\omega)\, \vec{h}(\vec{k},\omega) \tag{19}$$

where $\hat{\chi}(\vec{k},\omega)$ is the magnetic susceptibility tensor with the elements

$$\chi_{\alpha\beta}(\vec{k},\omega) = \frac{8\pi i \mu_B^2}{\hbar\, \Omega_0}\, D_{\alpha\beta}(\vec{k},\omega) \tag{20}$$

$$D_{xx}(\vec{k},\omega) = D_{yy}(\vec{k},\omega) = \frac{S}{2}\left\{U_{11}^2\left[G_1(\vec{k},\omega) + G_1^{+}(\vec{k},\omega)\right] + U_{12}^2\left[G_2(\vec{k},\omega) + G_2^{+}(\vec{k},\omega)\right]\right\}$$

$$D_{xy}(\vec{k},\omega) = -D_{yx}(\vec{k},\omega) = -\frac{S}{2i}\left\{U_{11}^2\left[G_1(\vec{k},\omega) - G_1^{+}(\vec{k},\omega)\right] + U_{12}^2\left[G_2(\vec{k},\omega) - G_2^{+}(\vec{k},\omega)\right]\right\}$$

$$D_{zz} = \chi_0 \quad , \quad D_{xz} = D_{yz} = D_{zx} = D_{xz} = 0$$

χ_0 equilibrium susceptibility and Ω_0 is unit cell volume.

If the external magnetic field acts paralel to the easy-axis there is no mechanical energy transfer, a situation completely opposite to the one in antiferromagnetics [2] .

References:

[1] A.I.Akhiezer, V.G.Bariakhtar, S.V.Peletminskij, Spin Waves, Nauka, Moscow 1967 (pp. 156-164), (in Russian).

[2] V.G.Bariakhtar, M.A.Savčenko, V.V.Tarasenko, Zh. eksper. teor. Fis. 49 (1965), 944.

[3] P.Gluch and O.Wohlman-Entin, Phys. Stat. Sol. (b) 52, (1972), 323.