EFFECTIVE PINNING POTENTIAL IN PERIODIC STRUCTURES

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In recent years, there has been great interest in the problem of commensurability of two interpenetrating periodic structures. First theoretical works concerning the epithaxial growth of a monolayer on a crystal substrate(1) and distorsions of two interpenetrating ionic sublattices(2) have been done in the rigid external potential approximation(2). In the latter case some self-consistent calculations of the pinning potential as the function of configuration have been done too(3). The problem is much more complex when one deals with statics and dynamics of flux lines (FL) in a periodically modulated superconductor,(4)(5) In this case, for each value of the external magnetic field H the effective pinning potential acting on FL has to be calculated in a self-consistent manner as the function of the equilibrium FL configuration, which also depends on H. It turns out that the pinning potential has not always a simple structure of spatial modulation of the specimen. To illustrate this point, we consider an inhomogeneous superconductor with the periodically modulated penetration depth λ in a given direction,

$$\lambda^2(\vec{r}) = \bar{\lambda}^2\cdot(1 + \delta\cos\vec{q}\vec{r}) \qquad (1)$$

with $q = 2\pi/L$.

The external magnetic field is assumed to be parallel to tne planes λ=const. For this case we solved the generalized London equation numerically. The results show that a good approximation for the field distribution $h(\vec{r},\vec{r}_i)$ around the FL situated at $\vec{r}_i$ is given by the formula

$$h(\vec{r},\vec{r}_i) = \frac{\phi_o}{2\pi\lambda(\vec{r})\lambda(\vec{r}_i)} \cdot K_o\left(\frac{|\vec{r}-\vec{r}_i|}{\bar{\lambda}}\right) \qquad (2)$$

The error is $\lesssim 10\%$ for $\delta \le 0.5$ and at the distances $|\vec{r}-\vec{r}_i| \ge \bar{\xi}$, where $\bar{\xi}$ is the (temperature dependent) mean coherence length. Therefore, eq. (2) can be used to calculate the interaction energy between the FL in low and intermediate fields (London limit):

$$F_{int} = \frac{\phi_o^{\,2}}{16\pi^2\bar{\lambda}^2} \sum_{i,j=1}^{N} \frac{K_o\left(\frac{|\vec{r}_i-\vec{r}_j|}{\bar{\lambda}}\right)}{(1+\delta\cdot\cos\frac{2\pi}{L}x_i)^{1/2}(1+\delta\cdot\cos\frac{2\pi}{L}x_j)^{1/2}} \qquad (3)$$

The self energy of the FL is derived, in a local approximation, from the field at the core

$$h|_{core} = \frac{\phi_o}{2\pi\lambda^2(\vec{r}_i)} K_o\left(\frac{1}{K(\vec{r}_i)}\right) \qquad (4)$$

This gives

$$F_{self} = \frac{\phi_o H_{C1}}{4\pi} \sum_{i=1}^{N} \frac{1+\delta}{1+\delta\cdot\cos\frac{2\pi}{L}x_i} \qquad (5)$$

where $H_{C1} = H_{C1}(\lambda_{max})$. The total free energy (per unit length) is $F = F_{int} + F_{self}$.

Note that the positions $\{\vec{r}_i\}$ of FL are different from their positions $\{\vec{r}_i^{\,o}\}$ in the absence of modulation. Using the Fourrier expansion of the function

$$\frac{1}{1+\delta\cdot\cos\vec{q}\cdot\vec{r}_i} = \frac{1}{1+\delta\cos\frac{2\pi}{L}x_i}$$

we write F in the form

$$F=\overline{F}(r_{ji}/\overline{\lambda})-\frac{\phi_o H_{C1}}{4\pi}\sum_{i=1}^{N} f(\delta,x_i)-\frac{\phi_o^{\ 2}}{16\pi^2\,\overline{\lambda}^2}\sum_{\substack{i,j=1\\(i\neq j)}}^{N} K_o(r_{ji}/\overline{\lambda})\{1-[1-f(\delta,x_i)]^{1/2}$$

$$[1-f(\delta,x_j)]^{1/2}\} \qquad (6)$$

where $r_{ji} \equiv |\vec{r}_j-\vec{r}_i|$ and

$$f(\delta,x_i) \doteq \frac{\delta}{1+\delta} + 2\,\frac{\frac{1}{\sqrt{1-\delta^2}}-1}{1-\sqrt{1-\delta^2}}\sum_{n=1}^{\infty}\left(-\frac{1-\sqrt{1-\delta^2}}{\delta}\right)^n\left(1-\cos\frac{2\pi\,x_i}{L}\right) \qquad (7)$$

In (6) we separated

$$\overline{F}(r_{ji}/\overline{\lambda})' = \frac{\phi_o^{\ 2}}{16\pi^2\,\overline{\lambda}^2}\sum_{i,j=1}^{N} K_o(r_{je}/\overline{\lambda})$$

from the terms coming directly from the interaction of FL with the inhomogeneities. In the general case, this interaction is a complicated function of $\vec{r}_i$, although the spatial distribution of inhomogeneitis leading to (1) is given by a single cosine function. The free energy can be represented as a sum of the hexagonal lattice energy $\overline{F}\,(r^o_{ji}/\overline{\lambda})$, of the elastic energy $\Delta\overline{F}_{el}(r^o_{ji}/\overline{\lambda})$, and of an effective pinning energy in the weak pinning limit only. In this limit, $\delta\to 0$ and the configuration of FL is not very different from the one in the absence of modulation. Keeping the terms of the 1st order in δ in the pinning energy, we obtain

$$F \doteq \overline{F}(r^o_{ji}/\overline{\lambda}) + \Delta\overline{F}_{el}(r^o_{ji}/\overline{\lambda}) - \varepsilon\sum_{i=1}^{N}\cos\frac{2\pi}{L}x_i \qquad (8)$$

where

$$\varepsilon = \delta\,\frac{\overline{F}(r^o_{ji}/\overline{\lambda})}{N} \qquad (9)$$

Eq. (9) shows that 2 is not constant, but increases with H, since

$$\overline{F} = \frac{\phi_o\,\overline{H}_{C1}}{4\pi}\cdot N + \overline{F}_{int}(H)\ .$$

In the opposite limit of strong pinning, the equilibrium configuration of FL is imposed by modulation. The ground state configuration in a large domain of field is a lattice simply commensurate with the modulation period (C_1 FL lattice) and it consists of isocelle triangles[6][7]. In this case the amplitude of the effective pinning potential can be found considering displacements $\Delta\vec{r}_i = \vec{r}_i - \vec{r}_i^{\,c}$ of the FL from their positions $\vec{r}_i^{\,c}$ in the C_1 lattice. We start with eq. (3) and (4) and assume a slow variation of displacements $\Delta\vec{r}_i$ with position,

$$|\Delta\vec{r}_i - \Delta\vec{r}_j| \ll \bar{\lambda} ,$$

and also $\delta \left|1 - \cos \frac{2\pi}{L} \Delta x_i\right| \ll 1.$

Keeping again only the 1^{st} order terms in the pinning energy, we obtain

$$F \simeq F^c + \Delta F^c_{el} + \varepsilon \sum_{i=1}^{N} \left(1 - \cos \frac{2\pi}{L} \Delta x_i\right) \qquad (10)$$

i.e. the free energy can be expressed as a sum of the energy F^c of C_1, its elastic deformation energy ΔF^c_{el}, and of the variation of the pinning potential relative to C_1 (in C_1 configuration the last term in (10) is zero). The pinning potential amplitude ε is proportional to the free energy (per FL and per unit length) of the C_1 configuration,

$$\varepsilon = \frac{\delta}{1+\delta} \cdot \frac{F^c}{N} = \frac{\delta}{1+\delta} \left\{ \frac{\phi_o H_{C1}}{4\pi} + \frac{\phi_o^{\,2}}{16\pi^2 \bar{\lambda}^2 (1+\delta)} \cdot \sum_{i=1}^{N} K_o\left(\frac{\vec{r}_i^{\,c}}{\bar{\lambda}}\right) \right\} \qquad (11)$$

and it increases with H in the same way as F^c.

Comparing the results (9) and (11), obtained in the weak pinning limit and in the strong pinning limit, respectively, we see that the effective pinning potential amplitude ε depends strongly on the configuration of FL. On the other hand in both cases ε becomes H independent only if one neglects the change in the interaction of FL due to the presence of the modulation. It appears that this is a good approximation in the case of a thin superconducting film with modulated thickness in a perpendicular field[5].

REFERENCES

(1) J. Villain, Lectures at the NATO Adv. Study Institute, Geilo, April, 1979.

(2) G. Theodorou and T.M. Rice, Phys, Rev. B, 18, p. 2840 (1978)

(3) H.U. Beyeler et al, in Physics of Superionic Conductors, ed. by M.B. Salamon, Spriniger Verlag (1979).

(4) H. Raffy, Ph.D. Thesis, Orsay-Paris (1980)

(5) P. Martinoli Phys.Rev. B, 17, 1175 (1978)

(6) L. Dobrosavljević, J.Phys. (Paris) 37, 23 (1976)

(7) Z.Radović and L. Dobrosavljević, Proceedings of the 1980 Annual Conference CMDEPS, Antwerpen (1980)