the anomalous energy spread at both the injection and pre-injection foci and also to estimate the effect due to the cathode region. Extrapolating our experimental data to zero current we have been able to obtain the energy spread due to the cathode temperature alone.

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1.15 Field lens application in electron-atom collision experiments

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1.16 The effective geometrical factor $(Ld\Omega)_{eff}$ in differential cross section measurements

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1. Angular scattering of electrons

An electron-atom collision with the angular analysis of scattered electron including the most important experimental parameters, such as the electron beam divergence angle $2\,\alpha$, the analyser view angle $2\,\beta$, gun (r) and detector (R) distances from the scattering center, in the horizonatal plane, is shown schematicaly in Fig. 1.

In the formula for differential cross section calculation from experimentally measured data

$$\frac{d^2\sigma}{dE\,d\Omega}=\frac{I_s}{I_0}\cdot\frac{1}{n\cdot L\cdot d\Omega\cdot dE} \tag{1}$$

a geometrical factor ($Ld\Omega$) appeares which takes into account the different scattering geometry at various scattering angles θ. It is necessary either to normalize all measurements to the same scattering geometry, or to determine exactly the value of the geometrical factor for each scattering angle and introduce it into the cross section formula. Most experimentators use an approximative normalization by simply multiplying the scattered electron current I_s by $\sin\theta$. According to Trajmer et al.[1] and Kuyatt[2] this procedure yields proper normalization only if the electron beam entering scattering region and the detected scattered electron beam have small angular divergence. A more sofisticated treatment was used by Trajmar et al.[1], but the procedure of that calculation was not given in the paper.

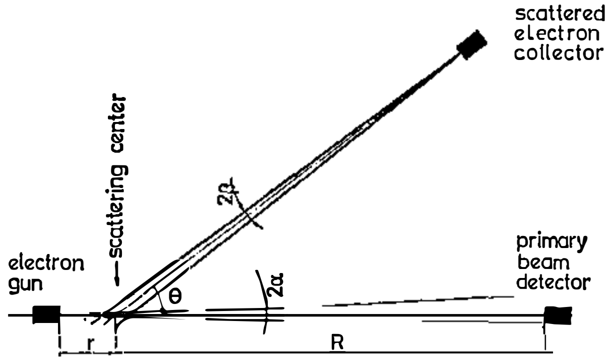

Fig. 1. Schematic representation of an electron-atom angular scattering experiment.

For the differential scattering experiment in progress in our laboratory, we needed to know the exact values of the geometrical factor in the cross section formula. At present we use a tape shaped electron beam without monoenergetization, but a spherical monochromator with cylindrical beam geometry will substitute it very soon. For both these beam profiles we calculated the effective geometrical factor $(L\mathrm{d}\Omega)_{\mathrm{eff}}$.

2. Tape-shaped electron beams

In the case of the tape-shaped electron beam the interaction volume, as seen by the detector, was determined by the intersection of two prisms, one representing the incident beam and the other the view cone of the detector. This volume was calculated using the formula

$$V = \int_{z_j}^{z_k} P(z_i) \cdot \mathrm{d}z_i, \qquad (2)$$

the lower and upper limits of integration being determined by the disappearance of the two prisms intersection. The magnitude $P(z_i)$ is the intersection area at a z coordinate of z_i, defined as

$$P(z_i) = P(\alpha, \beta, r, R, \theta), \qquad (3)$$

where α and β are the incident beam and view cone semiangles, r and R are distances from the beam gun and/or the scattered electron detector to the scattering volume center, respectively. The volume V was calculated on an IBM 360/44 computer, and results for two different values of the angle α were normalized to the value at 90°. The obtained curves are shown in Fig. 2.

The value of the effective geometrical factor for any angle θ can be obtained from

$$(L \cdot \mathrm{d}\,\Omega)_{\mathrm{eff}\,(\theta^\circ)} = (L \cdot \mathrm{d}\,\Omega)_{\mathrm{eff}\,(90^\circ)} \cdot \left[\frac{V(\theta^\circ)}{V(90^\circ)}\right]. \qquad (4)$$

The percentual difference between the exactly calculated values for the corection factor and the approximative correction by $\sin\theta$ multiplication is given in Fig. 3.

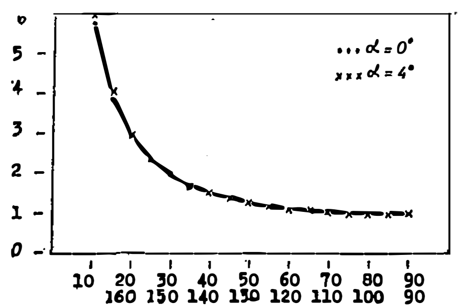

Fig. 2. Ratio $V(\theta^\circ)/V(90^\circ)$ for tape-shaped electron beam and electron collector view angle.

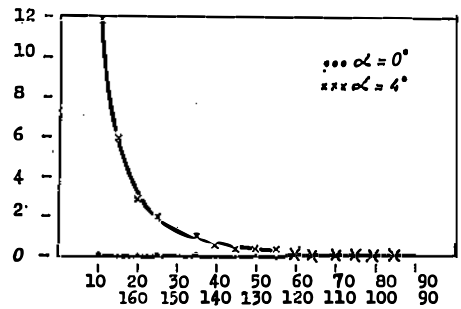

Fig. 3. Percentual difference between the calculated correction factor for the tape-shaped beam and the $\sin\theta$ correction.

3. Cylindrical electron beams

In the case of cylindrical electron beam the cross section of the primary beam and the detector view cone with planes parallel to the XY plane are hyperbolas. Tangents of these hyperbolas are directrices of cones in the horizontal cross section. They are in fact straight lines with an angle α, or β in respect to the cone axis, respectively. Starting from these geometrical parameters a mathematical model for the interaction valume calculation at any scattering angle θ was derived. This model is most general and it is valid for beams which can be divergent, paralel or convergent inside the scattering region.

If the ratio $V(\theta^\circ)/V(90^\circ)$ versus scattering angle θ is known, the geometrical factor $(Ld\Omega)_{\mathrm{eff}}$ for any angle θ can be simply determined by calculating the interaction volume at 90°.

The formula for the area common to two hyperbolas

$$P = \int_{X(5)}^{X(9)} \mathrm{HIP}\,3 \cdot \mathrm{d}X + \int_{X(9)}^{X(3)} \mathrm{HIP}\,1 \cdot \mathrm{d}X + \int_{X(3)}^{-X(5)} \mathrm{HIP}\,3 \cdot \mathrm{d}X +$$

$$+ \int_{X(8)}^{X(5)} \mathrm{HIP}\,3 \cdot \mathrm{d}X + \int_{X(8)}^{X(4)} \mathrm{HIP}\,1 \cdot \mathrm{d}X + \int_{-X(5)}^{X(4)} \mathrm{HIP}\,3 \cdot \mathrm{d}X,$$

for $X(5) > X(7)$,

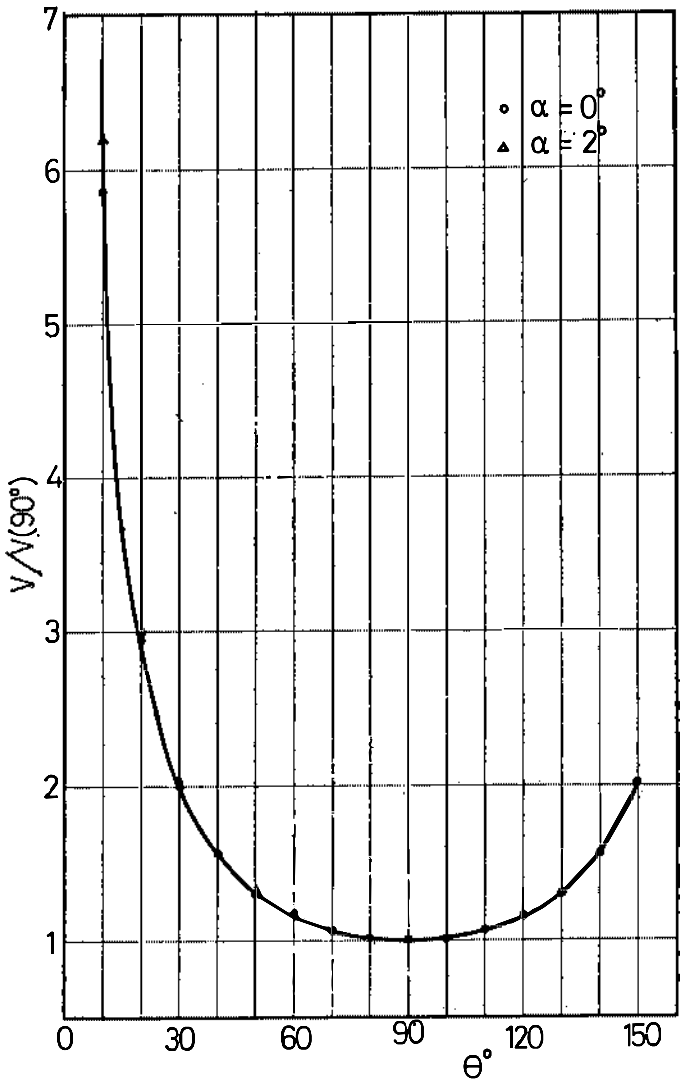

Fig. 4. Ratio $V(\theta°)/V(90°)$ for cylindrical electron beam and electron collector view angle.

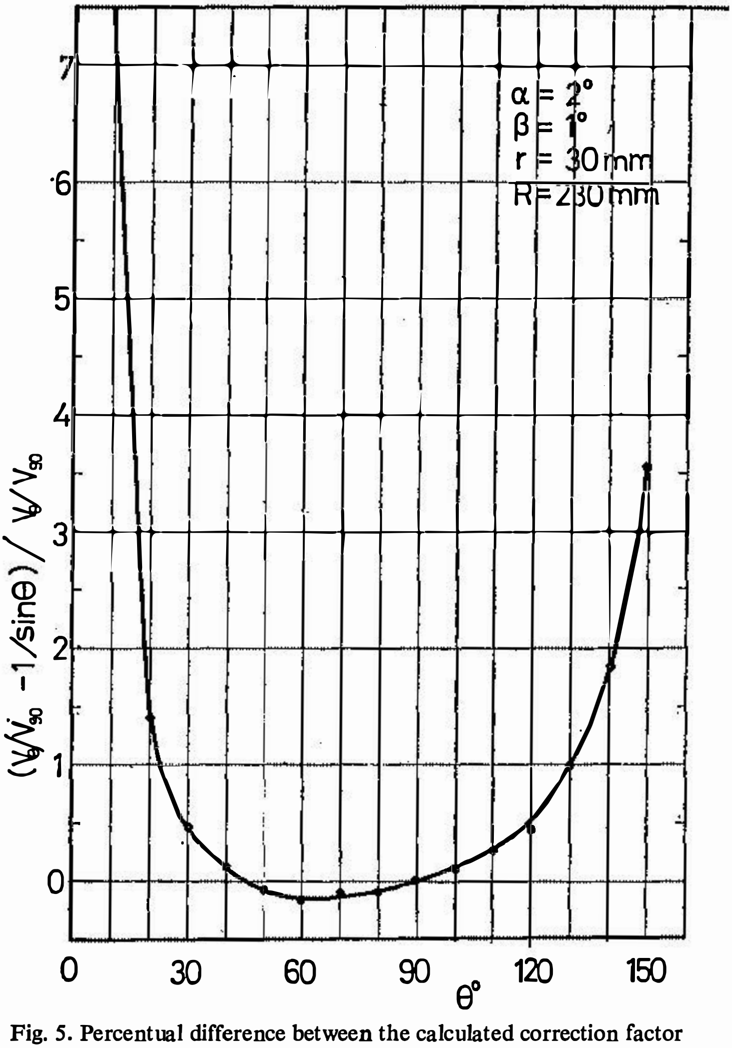

Fig. 5. Percentual difference between the calculated correction factor for the cylindrical beam and the $\sin\theta$ correction.

and (5)

$$P = \int_{X(7)}^{X(3)} \mathrm{HIP}\,1 \cdot \mathrm{d}X + \int_{X(3)}^{-X(5)} \mathrm{HIP}\,3 \cdot \mathrm{d}X + $$
$$+ \left| \int_{X(7)}^{X(4)} \mathrm{HIP}\,1 \cdot \mathrm{d}X + \int_{-X(5)}^{X(4)} \mathrm{HIP}\,3 \cdot \mathrm{d}X \right|,$$
$$\text{for } X(5) < X(7) < -X(5).$$

The scattering volume was calculated from

$$V = 2 \cdot \Delta b \cdot \sum_{b=0}^{b_n} P(b), \tag{6}$$

the limit being defined by $P(b_n) = 0$.

In equations (5) HIP1 and HIP3 are expressions for the hyperbolas

$$\mathrm{HIP}\,1 = \mathrm{tg}\,\alpha \cdot \sqrt{X + 2\,rX + C_1},$$
$$\mathrm{HIP}\,3 = -R + \frac{1}{2}\sqrt{4\,R - 4\,(C_3\,X^2 + C_{15})}, \tag{7}$$

and the limits of integration are defined by the two hyperbola intersection points.

The upper relations have been used for the $V(\theta°)/V(90°)$ ratio calculation, done on a computer, and the results are presented in Fig. 4. The percentual difference between the calculated correction factor and the $\sin\theta$ multiplication correction is given in Fig. 5.

4. Conclusion

As one can see for both the tape-shaped and cylindrical electron beams, the simple $1/\sin\theta$ correction for the differential cross section calculation from experimental data gives an error less than 1% for scattering angles between 40° and 140°, and/or 25° and 130°, in these two cases respectively. For smaller and larger angles the error rises steaply, showing that the $1/\sin\theta$ correction for these values of the scattering angle θ is a very rough approximation and that it should not be used for differential cross section determinations.

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