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SECTION 3 — NUCLEAR STRUCTURE

3.1. Solving the many-body problem with linear programing

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3.2. Quadrupole moments of the even-even vibrational nuclei

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Recent measurements of the quadrupole moments in the even-even vibrational nuclei have provided a good test for different nuclear models. The purpose of the present report is to extract the main effects responsible for the existence of the quadrupole moment and to compare the mechanisms provided by different types of approach.

In analogy with the polarization effect observed in the odd-proton nuclei, such as Sb, In, etc., where the large ground-state quadrupole moment was explained by coupling one particle to the harmonic vibrator

$$Q(j)=Q_{s.p.}(j)\left[e^{p}_{\mathrm{eff}}+\frac{5}{4\pi}Ze\frac{\langle k\rangle}{C}\right],$$

one expects a similar process for two or more particles coupled to the vibrator[1,2,3]. However, in going from the one- to the two-particle case, a qualitatively new feature, i. e., the short-range residual force between the two particles, is encountered.

The quadrupole moment of a two-particle 2^+ state is given as

$$eQ(2^+_1)=Q[(j)^2 2]\cdot\left[e^{p}_{\mathrm{eff}}+\frac{5}{4\pi}Ze\frac{\langle k\rangle}{C}\right], \tag{1}$$

exhibiting the same polarization effect as in the one-particle case. However, in a vibration nucleus the 2^+_1 state is of the one-phonon rather than of the two-particle type due to the large energy gap Δ separating the paired from the broken pair states. In this case both the zeroth and first order contributions vanish. The lowest nonzero particle and vibrational contributions are of second and third order, giving the result

$$eQ(2^+_1)=Q_{s.p.}[(j)^2 2]\left[e^{p}_{\mathrm{eff}}+\frac{5}{4\pi}Ze\frac{\langle k\rangle}{C}\right]\cdot x^2(j),$$

which is identical to (1) except for the last factor including the energy denominators. The corresponding diagrams are shown in Fig. 1.

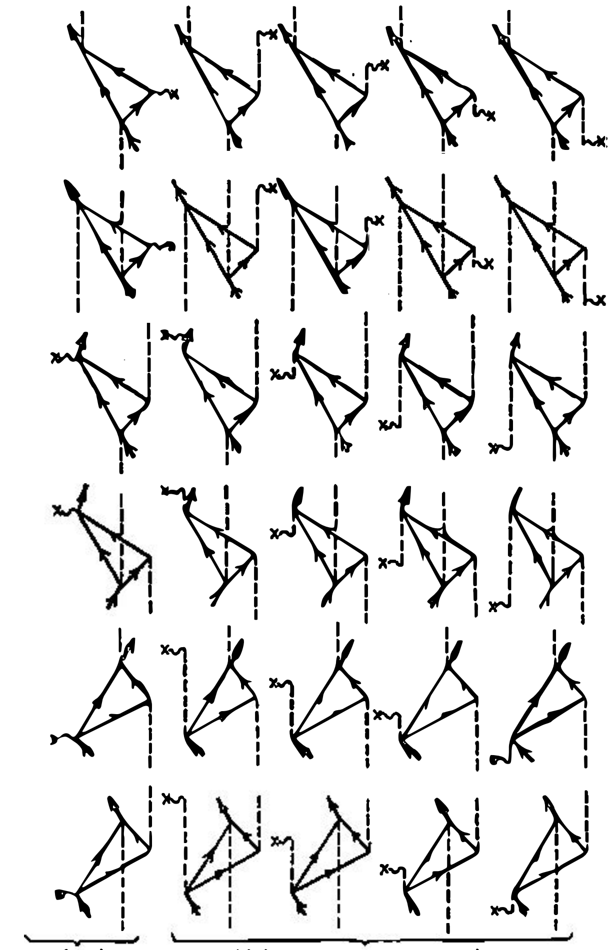

Fig. 1.

If several configurations are available, the effect is the same, but the sum has to be taken over all intermediate particle states.

The semimicroscopic result can be connected with that obtained in the phenomenological anharmonic model[2,4]. If the third-order terms of the type $\sim A_1 b^+ b^+ b$ are added to the harmonic Hamiltonian, the quadrupole moment becomes, in the first approximation,

$$eQ(2_1^+) = \sqrt{\frac{16\pi}{5}} \begin{pmatrix} 2 & 2 & 2 \\ -2 & 0 & 2 \end{pmatrix} \left(\frac{3}{4\pi} Ze \sqrt{\frac{\hbar\omega}{2C}} R_0^2 \right) (-4 A_1). \tag{2}$$

The nature of the phenomenological constant A_1 can be understood by searching for the quasi-particle excitations building the phonon. The diagrams corresponding to (2) are shown in Fig. 2. Comparison with Fig. 1 shows that the semimicroscopic model is more complete, because it includes processes with the explicit presence of particles.

Both the semimicroscopic and the anharmonic model quadrupole moment can be written in the following form:

$$eQ(2_1^+) = K e_{\mathrm{eff}}^{M} \sum_{jj'j''} \langle j'' \| Y_2 \| j \rangle \langle j \| Y \| j' \rangle \langle j' \| Y \| j'' \rangle \begin{Bmatrix} 2 & 2 & 2 \\ j & j' & j'' \end{Bmatrix} D^M(jj'j'')\, U^M(jj'j'') \quad (3)$$

where M denotes the model-dependent quatities e_{eff}^{M}, D^M and U^M which characterize the effective charge, the energy denominators and the occupation probabilities of the particle orbits, respectively. Essentially the same form (3) is obtained in the pairing + quadrupole study of the vibrational nuclei[5].

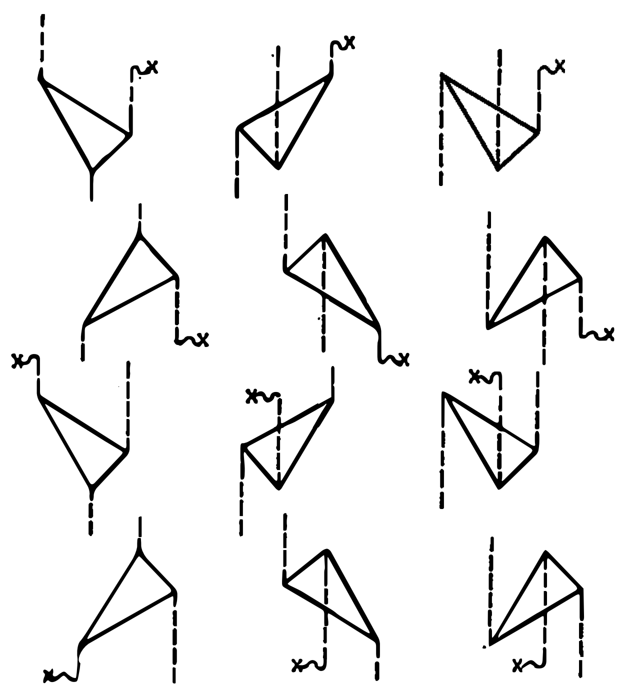

Fig. 2.

Expression (3) shows that the structure of the 2_1^+ state is strongly affected by the particle properties in spite of the apparently vibration-like character of this state. The competition of different configurations will determine the sign of the total quadrupole moment. Generally, terms with nondiagonal matrix elements have a sign opposite to the sign of terms containing diagonal elements. For two particles coupled to the vibrator the diagonal terms are positive and the nondiagonal

TABLE

Nucleus	Available proton configurations	Dominant matrix element		Quadrupole moment		
		Diagonal	Nondiagonal	A	Exper.	Theory (semimicroscopic)
$_{26}Fe$	$f^{-1}_{7/2}$	$<f^{-1}_{7/2} \| Y_2 \| f^{-1}_{7/2}>$	—	56	—0.22	—0.21
$_{80}Hg$	$s^{-1}_{1/2}, d^{-1}_{3/2}, h^{-1}_{11/2}, d^{-1}_{5/2}$	—	$<s^{-1}_{1/2} \| Y_2 \| d^{-1}_{3/2}>$	198	—	0.83
$_{52}Te$	$g_{7/2}, d_{5/2}, s_{1/2}, d_{3/2}, h_{11/2}$	$<g_{7/2} \| Y_2 \| g_{7/2}>$	$<g_{7/2} \| Y_2 \| d_{3/2}>$	122 124 126 128	—0.50 —0.08 —0.40 +0.07 —0.27 —0.01	—0.37 —0.18
$_{48}Cd$	$g^{-1}_{9/2}, p^{-1}_{1/2}, p^{-1}_{3/2}, (f^{-1}_{5/2})$	$<g^{-1}_{9/2} \| Y_2 \| g^{-1}_{9/2}>$	$(<p^{-1}_{1/2} \| Y_2 \| f^{-1}_{5/2}>)$	114	—0.38	—0.33

negative. The opposite holds for holes. Whatever the result of this shell effect is, it is always enhanced several times due to the coherence of the particle and vibrational contributions.

This basic mechanism is valid in both models. The discrepancy in the results should be attributed to the model-dependent quantities and to the choice of the subspace over which the sum in (3) is taken.

For illustration the Table gives the dominant diagonal and nondiagonal proton matrix elements as well as some characteristic experimental and theoretical values for some isotopes with two protons or two proton holes in the closed shell. In such cases the two proton cluster imposes its motion to the whole nucleus and is almost entirely responsible for the existence of the nonzero quadrupole moment. More details of the calculations as well as references on experiments are given in Ref. 6.

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3.3. Application of the »bootstrap« on the states and processes around the doubly closed shell nuclei. The Pb case

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A unified picture is developed to describe the situation around a closed shell nuclear system[1-8]. This approach is based on the diagrammatic method[1-3], accounting for the dominant processes, in particular for those states which are strongly excited in the (t, p), (p, t) reactions and/or inelastic scattering processes. In the shell-model language these are the 2p, 2h and 1p—1h states (pairing and surface vibrational states), and those generated by coupling these states among themselves (two-phonon states).

The bootstrap study was carried out in three successive steps:

i) the calculation of the elementary modes of excitation, i. e. the low-lying states, of each spin λ and parity $(-)^{\lambda}$. The residual forces which are diagonalized are multipole particle-hole and pairing interactions[2,5-7]. The only free parameters