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PRELIMINARY OIL FILM ANALYSIS OF EHL BASED ON CUTTER TILTED VH-CATT CYLINDRICAL GEAR

Summary

This study investigates the factors influencing oil film thickness on the tooth surface of variable hyperbolic circular-arc-tooth-trace (VH-CATT) cylindrical gears, aiming to optimise lubrication performance and mitigate poor oil film formation near meshing points. A mathematical model of the VH-CATT gear tooth surface is developed based on large cutter tilt milling principles, forming the basis for tooth contact analysis (TCA) and elastohydrodynamic lubrication (EHL) models. These models predict key lubrication parameters, such as minimum and central oil film thicknesses, under various gear design and operational conditions. The analysis shows that increasing the cutter tilt angle enhances both central and minimum oil film thicknesses during meshing. Similarly, higher modulus and pressure angles significantly improve lubrication performance, while a slight increase in the tooth line radius also contributes to thicker oil films. The findings provide theoretical insights into the interaction between gear design parameters and lubrication behaviour, offering practical guidance for improving lubrication performance, reducing friction and wear, and enhancing the efficiency of VH-CATT cylindrical gear systems. These results have significant implications for the design and operation of advanced gear transmission systems in engineering applications.

Key words: *VH-CATT cylindrical gears; the forming principle of large cutter tilt processing; central oil film thickness; minimum oil film thickness; preliminary analysis of oil film*

1. Introduction

The VH-CATT cylindrical gear is a novel type of mechanical transmission component [1]. The tooth profile in the middle section of the gear follows an involute profile, while the other sections exhibit changing hyperbola profiles, with the tooth line forming an arc segment. Currently, research on VH-CATT cylindrical gears primarily focuses on meshing principles, accurate modelling techniques, tooth surface contact characteristics, tooth surface modification methods, and machine tool processing. For instance, Ma et al. [2] deduced a mathematical model for the tooth surface based on rotary cutter milling principles and successfully

reconstructed the tooth surface of VH-CATT cylindrical gears. Additionally, Ma et al. [3] presented a design approach for tooth surface modification and achieved a numerical software-based reconstruction of the tooth surface. Sun et al. [4] employed CREO software to parametrically model circular cylindrical gears, providing an accurate theoretical foundation for investigating deep contact characteristics in VH-CATT cylindrical gears. In terms of studying tooth surface contact characteristics in VH-CATT cylindrical gear pairs, Wei et al. [5], Wu et al. [6], Ma et al. [7], Zhang et al. [8], Ma et al. [9], along with numerous other scholars, have conducted extensive research and proposed optimisation algorithms aimed at enhancing accuracy and reliability in studying contact characteristics. Regarding machine tool processing, many scholars [10,11,12,13,14,15,16] have carried out a series of studies on the circular arc tooth line cylindrical gear machine tool, and analysed the effect of error on the processing gear, and promoted the processing application of VH-CATT cylindrical gear. While there is a wealth of research on VH-CATT cylindrical gears under optimal processing conditions, tool installation errors are unavoidable in real-world processing, and there is still a lack of research in this area. The current understanding of VH-CATT cylindrical gears lays a theoretical foundation for studying these gears while processing with a significant cutter head tilt.

Many researchers have studied the calculation of gear EHL film thickness. For example, Pu et al. [17] examined the impact of entrainment velocity in various directions on oil film thickness in elliptical contacts, taking surface roughness into account. Liu et al. [18] proposed a starved thermoelastic EHL model to evaluate the tribological properties of helical gears. Zhang et al. [19] proposed a fast prediction method to numerically predict the Stribeck curve. Wassim [20,21] presented a method for quantitatively predicting the minimum film thickness of EHL elliptical contact, then an analytical formula for predicting the maximum inlet pressure and shear stress. Abd [22] analysed the effects of acceleration and final entrainment velocity on the onset of ultra-thin film motion. Luo et al. [23] established a TEHL model of elliptical contact of VH-CAT cylindrical gear transmission. Wei et al. [24] proposed a comprehensive wear prediction model based on hybrid EHL. Farrenkopf et al. [25] suggested a transient EHL contact modelling method for gear meshing taking into consideration mixed lubrication and thermal effects. Cao et al. [26] examined the boundary slip at any angle in elliptical contact based on the modified Reynolds equation, and carried out numerical simulation. Yue et al. [27], based on hybrid EHL, proposed an improved calculation method of gear meshing power loss. Using the fluid dynamics model, Mo et al. [28] analysed the lubrication characteristics of the arc tooth line cylindrical gear by the fuel injection mode and the fuel injection condition. Gu et al. [29] considered a time-varying mesh stiffness (TVMS) model of hybrid EHL gears. Nevertheless, there is still a lack of initial research regarding the oil film thickness in EHL of VH-CATT cylindrical gears when subjected to large cutter tilt machining. However, the current studies on EHL offer a theoretical foundation for investigating the oil film thickness on the meshing surfaces of VH-CAT cylindrical gear transmissions that utilise large cutterhead tilt machining.

While numerous scholars have examined EHL oil film thickness, the current theoretical foundations are insufficient to substantiate the universal applicability of EHL oil film thickness in VH-CATT cylindrical gears. To gain a deeper understanding of the changes in EHL oil film thickness of these gears during large cutter tilt processing, additional research on the EHL oil film thickness of the gear is required. In this study, a TCA (Transmission Contact Analysis) model is developed to analyse the meshing process of VH-CATT cylindrical gears subjected to large cutter head tilt. Utilising the Downson-Higginson elastohydrodynamic lubrication theory, a model is designed for the central oil film and the minimum oil film thickness in the meshing process of EHL. The study further investigates and analyses the impact of gear design parameters on the EHL central oil film thickness and minimum oil film thickness during the meshing process of VH-CATT cylindrical gears across various cutter head inclination angles.

2. Forming principle and tooth surface equation

This research examines the use of the large cutter tilting double-edged milling technique for the fabrication of VH-CATT cylindrical gears. The methodology entails affixing a double-edged large cutter head processing tool onto the rotating spindle of the machining apparatus. The spindle operates at a defined angular speed, with numerous double-edged milling tools positioned along the circumference of the large cutter head. The processing of the gear blank workpiece involves both the rotation of the gear blank around the mandrel and the horizontal feed motion to facilitate movement during the machining process.

The detailed machining motion is as follows: when the large cutterhead completes a periodic rotation motion, the double-edged milling tool completes the milling of the gear blank, and uses the internal and external cutting edges to form two cones of the same size but in the opposite cone direction, thereby forming the concave and convex tooth surface of the gear. When the concave and convex tooth surface of a tooth groove is completely milled, the gear blank is rotated to the next tooth groove to be processed, and the processing is continued until the entire gear blank is completely cut.

During the inclined double-edged milling process of the VH-CATT cylindrical gear with a large cutter head, the positioning of the cutter head at an inclined angle enables the gear to be adjusted along the tooth line direction. This adjustment results in an expansion of the contact area and a reduction in contact stress between gear pairs. Furthermore, shaping the drum tooth in the tooth line direction serves to mitigate installation inaccuracies and promotes the development of a dynamic pressure oil film, thereby improving lubrication effectiveness [30].

Fig.1 shows the schematic diagram of the VH-CAT cylindrical gear tilted by rotating the double-edged cutter, where $O_{do}X_{do}Y_{do}Z_{do}$ is the tool static coordinate system, $O_{df}X_{df}Y_{df}Z_{df}$ is the cutter dynamic coordinate system, $O_dX_dY_dZ_d$ is the static coordinate system of the cutterhead, $O_0X_0Y_0Z_0$ is the auxiliary coordinate system, $O_fX_fY_fZ_f$ is the static coordinate system of the gear blank, and $O_1X_1Y_1Z_1$ is the dynamic coordinate system of the gear blank.

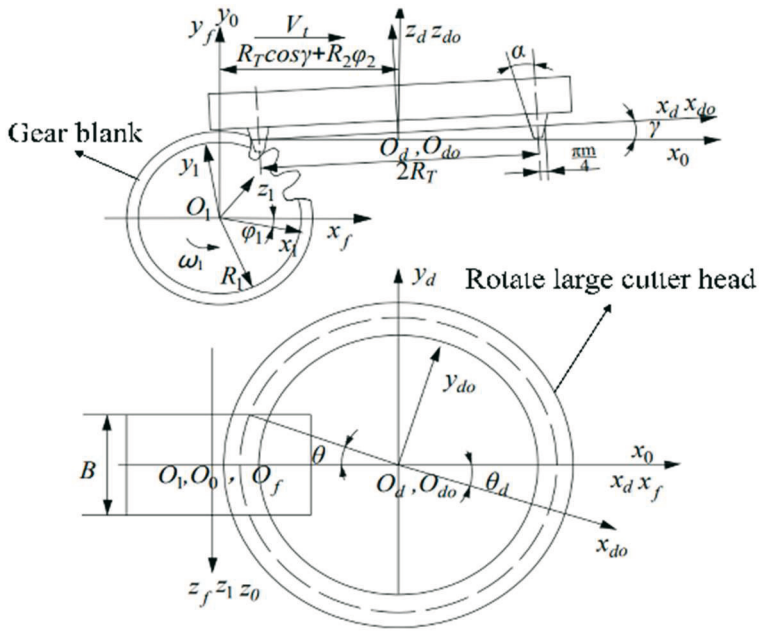


Fig. 1 VH-CATT cylindrical gear large cutter head tilting forming diagram

Combined with Fig. 1 and literature [3], the tooth surface equation of the VH-CATT under $O_1X_1Y_1Z_1$ can be obtained as follows:

$$\begin{cases} x_1 = \left[\mp\dfrac{\pi m}{4}\cos\gamma + u\sin(\gamma \mp \alpha) - R_T\right]\cos\theta\cos(\gamma+\varphi_1) \\ +\left[\mp\dfrac{\pi m}{4}\sin\gamma - u\cos(\gamma\mp\alpha)\right]\sin(\gamma+\varphi_1) + R_T\cos(\gamma+\varphi_1) + R_1\varphi_1\cos\varphi_1 - R_1\sin\varphi_1 \\ y_1 = \left[\mp\dfrac{\pi m}{4}\cos\gamma + u\sin(\gamma \mp \alpha) - R_T\right]\cos\theta\sin(\gamma+\varphi_1) \\ -\left[\mp\dfrac{\pi m}{4}\sin\gamma - u\cos(\gamma\mp\alpha)\right]\cos(\gamma+\varphi_1) + R_T\sin(\gamma+\varphi_1) + R_1\varphi_1\sin\varphi_1 + R_1\cos\varphi_1 \\ z_1 = \left[\mp\dfrac{\pi m}{4}\cos\gamma + u\sin(\gamma \mp \alpha) - R_T\right]\sin\theta \\ u = \dfrac{1}{\cos\theta}\left[R_1\varphi_1\sin\gamma\cos\theta\cos(\gamma\mp\alpha) - \dfrac{\pi m}{4}\sin\alpha\cos\theta - (R_1\varphi_1\cos\gamma + R_T - R_T\cos\theta)\sin(\gamma\mp\alpha)\right] \end{cases} \tag{1}$$

In the formula, the concave surface of the tooth surface in '$\mp$' takes the upper part of the symbol, and the convex surface of the tooth surface takes the lower part of the symbol, γ is the inclination angle of the cutterhead, φ_1 is the gear blank spread angle, θ is the expansion angle of the cutterhead, ω_1 is the rotation speed of gear blank, B is tooth width, and R_T is the tooth line radius.

3. TCA analysis model

The spatial meshing coordinate system of the VH-CATT cylindrical gear pair is shown in Fig. 2. Here, $O_f X_f Y_f Z_f$ is the fixed coordinate system of the driving wheel, $O_p X_p Y_p Z_p$ and $O_g X_g Y_g Z_g$ are the follow-up coordinate systems of the driving wheel and the driven wheel respectively, $O_m X_m Y_m Z_m$ is the auxiliary coordinate system, E is the standard centre distance, ϕ_p is the rotation angle of the driving wheel, and ϕ_g is the rotation angle of the driven wheel.

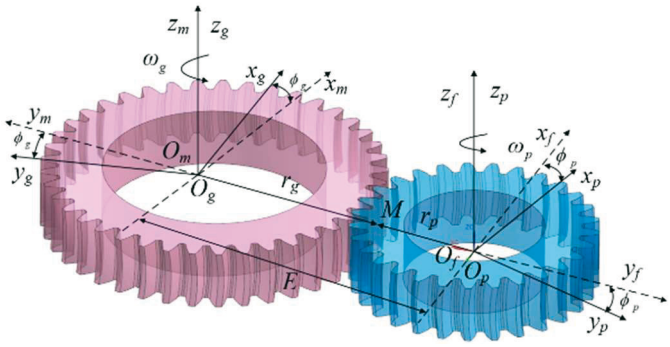


Fig. 2 Space meshing coordinate system

In this paper, the driving wheel of the VH-CATT cylindrical gear pair is not processed by the cutter inclination angle ($\gamma = 0^\circ$), and the concave tooth surface is selected. The driven wheel is tilted by the cutter head (that is, $\gamma = t^\circ$, t is any value within the critical value range, and the tooth surface contact is excluded as 'bridge contact'), and the convex tooth surface is chosen as the object of analysis.

According to the tooth surface equation (1), the expression $\boldsymbol{r}_{fp}$ and $\boldsymbol{r}_{fg}$ of the tooth surface equation of the concave tooth surface of the driving gear and the convex tooth surface of the driven gear in the fixed coordinate system $O_f X_f Y_f Z_f$ can be obtained as follows:

$$\begin{cases} \boldsymbol{r}_{\boldsymbol{fp}}(\theta_p,\varphi_p,\phi_p) = x_{fp}(\theta_p,\varphi_p,\phi_p)\boldsymbol{i} + y_{fp}(\theta_p,\varphi_p,\phi_p)\boldsymbol{j} + z_{fp}(\theta_p,\varphi_p,\phi_p)\boldsymbol{k} \\ \boldsymbol{r}_{\boldsymbol{fg}}(\theta_g,\varphi_g,\phi_g) = x_{fg}(\theta_g,\varphi_g,\phi_g)\boldsymbol{i} + y_{fg}(\theta_g,\varphi_g,\phi_g)\boldsymbol{j} + z_{fg}(\theta_g,\varphi_g,\phi_g)\boldsymbol{k} \end{cases} \tag{2}$$

The TCA analysis model for the VH-CATT cylindrical gear pair during gear transmission is established based on the continuous tangent condition of the two tooth surfaces. The expression for this model is as follows:

$$\begin{cases} \boldsymbol{r}_{\boldsymbol{fp}}(\theta_p,\varphi_p,\phi_p) - \boldsymbol{r}_{\boldsymbol{fg}}(\theta_g,\varphi_g,\phi_g) = 0 \\ \boldsymbol{n}_{\boldsymbol{fp}}(\theta_p,\varphi_p,\phi_p) - \boldsymbol{n}_{\boldsymbol{fg}}(\theta_g,\varphi_g,\phi_g) = 0 \end{cases}. \tag{3}$$

Here, ϕ_p is a known parameter, and the specific range is determined by [31]. Five independent equations containing all unknown parameters θ_p, φ_p, θ_g, φ_g and ϕ_g of the two tooth surfaces are obtained by the above equation.

4. Establishing the EHL model

4.1 Principal curvature analysis of surface contact points

The geometric characteristics of the meshing tooth surface of the VH-CATT cylindrical gear pair play a crucial role in determining the thickness of the oil film present. Enhancing the assessment of oil film thickness in elastohydrodynamic lubrication (EHL) involves an initial determination of the primary curvature of the gear pair based on gear meshing principles and spatial geometry theory. Following the approach detailed in Litvin [32], mathematical expressions are formulated to determine the principal curvature of the concave surface of the driving wheel and the convex surface of the driven wheel in both the tooth line and tooth profile orientations. Subsequently, the overall curvature radius along the tooth line and the overall curvature radius of the tooth profile are calculated using the methodologies proposed by Wei et al. [34], as outlined in Equations (7) and (8) respectively.

$$K_{iG} = \frac{L_{iK}N_{iK} - M_{iK}^2}{E_{iK}G_{iK} - F_{iK}^2}(i = p,g) \tag{4}$$

$$K_{iA} = \frac{L_{iK}G_{iK} - 2M_{iK}F_{iK} + N_{iK}E_{iK}}{2(E_{iK}G_{iK} - F_{iK}^2)}(i = p,g) \tag{5}$$

$$\begin{cases} K_i^a = K_{iA} + \sqrt{K_{iA}^2 - K_{iG}} \\ K_i^b = K_{iA} - \sqrt{K_{iA}^2 - K_{iG}} \end{cases}(i = p,g) \tag{6}$$

In the formula, $E_{iK} = \boldsymbol{r}_{\iota\theta}^2$, $F_{iK} = \boldsymbol{r}_{\iota\theta} \cdot \boldsymbol{r}_{i\varphi}$, $G_{iK} = \boldsymbol{r}_{i\varphi}^2$, $L_{iK} = \boldsymbol{r}_{\iota\theta\theta} \cdot \boldsymbol{n}$, $M_{iK} = \boldsymbol{r}_{\iota\theta\varphi} \cdot \boldsymbol{n}$, and $N_{iK} = \boldsymbol{r}_{\iota\varphi\varphi} \cdot \boldsymbol{n}$, where E, F, G are the first basic quantities of VH-CATT cylindrical gears; L, M, N are the second basic quantities of the VH-CATT cylindrical gear; K_{iG} and K_{iA} are the Gaussian curvature and mean curvature of the tooth surface, respectively; the principal curvature K_i^a and K_i^b are expressed as the principal curvature of the VH-CATT cylindrical gear in the tooth line direction and the tooth profile direction, respectively.

$$R_x = R_{px}R_{gx} / (R_{px} + R_{gx}) \tag{7}$$

$$R_y = R_{py}R_{gy} / (R_{py} + R_{gy}) \tag{8}$$

The equation concerns the radii of curvature in the tooth profile orientation of the driving and driven wheels, labelled as R_{px} and R_{gx} respectively, as well as in the tooth line orientation, denoted as R_{py} and R_{py} for the driving and driven wheels, respectively.

4.2 Contact tooth surface velocity analysis

The assessment of EHL oil film thickness requires an analysis of the key variables that significantly impact both the thickness of the oil film and the pressure within it. Among these crucial factors, speed parameters play a vital role, particularly the entrainment speed during meshing. Hence, the tooth surface speed parameter throughout a meshing cycle is a pivotal dynamic component essential for investigating EHL oil film thickness in VH-CATT cylindrical gear transmissions.

Wei et al. [33] employ equations (9), (10), and (11) to analyse the dynamic properties of velocity as shown in Fig. 3, including entrainment velocity, sliding relative velocity, and slip-roll ratio, for specific meshing points on the tangent plane during the gear pair meshing process.

$$v_r = v_p^T - v_g^T \tag{9}$$

$$u_e = (v_p^T + v_g^T)/2 \tag{10}$$

$$S = \frac{2|v_r|}{|u_e|} = \frac{2|v_p^T - v_g^T|}{|v_p^T + v_g^T|} \tag{11}$$

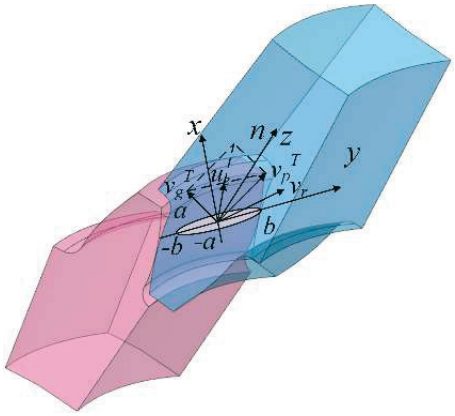


Fig. 3 Speed diagram at the meshing point

4.3 Theoretical analysis of EHL minimum oil film and central oil film thickness

When the large cutterhead is used for milling, the meshing of the VH-CATT cylindrical gear pair is in the form of an elliptical point contact [34]. By applying the empirical formula for determining the minimum oil film and central oil film thickness in Downson-Higginson EHL [35], the formula is applicable to the elliptic point contact form. The dimensionless calculation equations can be expressed as follows:

$$H_{\min} = 3.63U^{0.68}G^{0.49}W^{-0.073}(1 - e^{-0.68k}) \tag{12}$$

$$H_{cen} = 2.69U^{0.67}G^{0.53}W^{-0.067}(1 - 0.61e^{-0.73k}) \tag{13}$$

where U is the velocity parameter, W is the load parameter, G is the material parameter, e is the natural constant, and k is the elliptic parameter.

$$\begin{cases} U = \eta_0 u_e / E' R_x ,\ G = \alpha_0 E' ,\ W = F / E' R_x^2 \\ k = 1.03\left(R_y / R_x \right)^{0.64} ,\ E' = \left[\dfrac{1}{2}\left(\dfrac{1-\mu_p^2}{E_p} + \dfrac{1-\mu_g^2}{E_g} \right) \right]^{-1} \end{cases} \tag{14}$$

In the formula, E' is the comprehensive elastic modulus, E_p and E_g are the elastic modulus of the main and driven wheels, respectively, and the value is 2.07×10^{11} Pa ; η_0 is the initial viscosity of lubricating oil, where $\eta_0 = 0.08$ Pa·s ; μ_p and μ_g are the Poisson's ratios of the main and driven wheels, respectively, and both values are 0.3; α_0 is the pressure viscosity coefficient, where $\alpha_0 = 2.3\times10^{-8}$ Pa^{-1}; F is the contact force of the driven wheel, and the value is $F = T_g / R_g$.

The dimensional formulas for the minimum and central EHL film thicknesses are given as follows:

$$h_{cen} = H_{cen} R_x \tag{15}$$

$$h_{\min} = H_{\min} R_x \tag{16}$$

5. Analysis of EHL minimum oil film and central oil film thickness

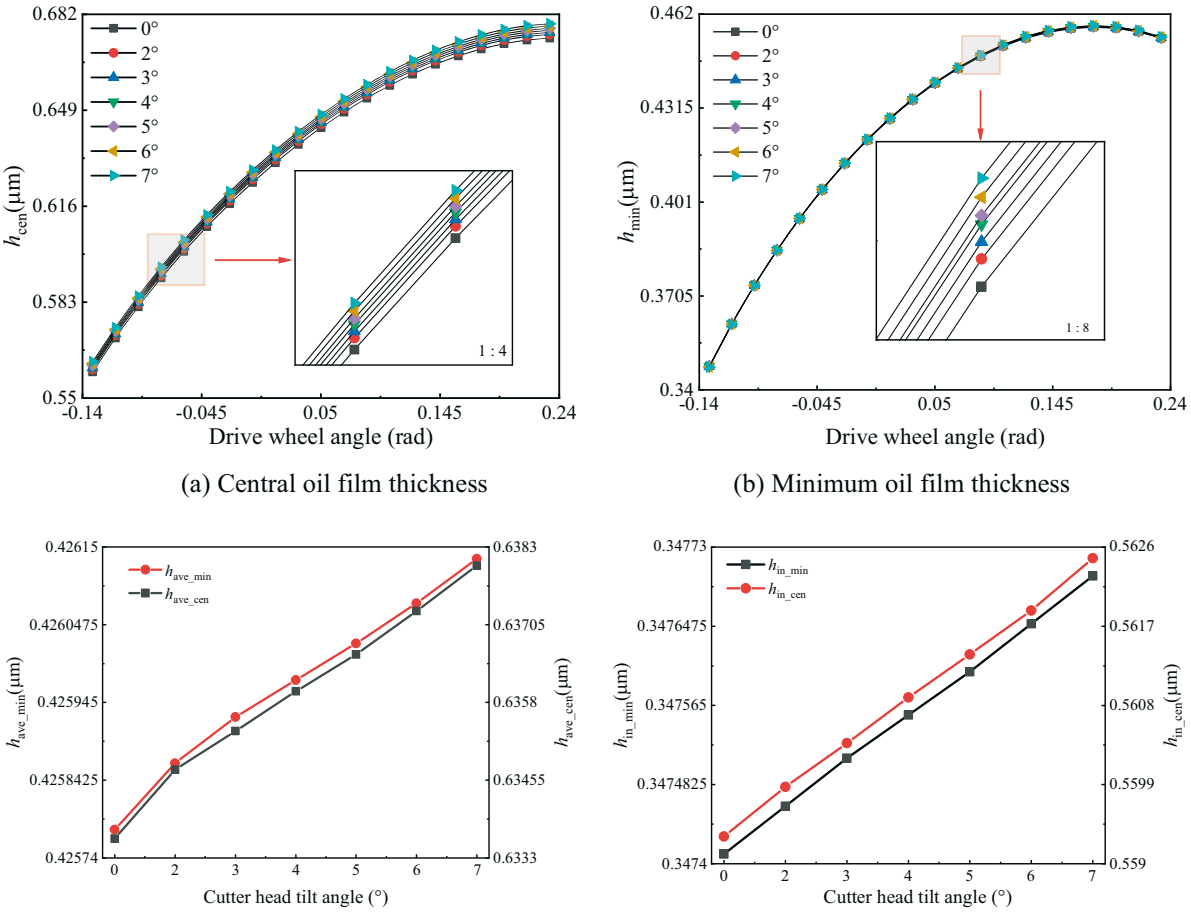
To investigate the factors that affect the EHL oil film thickness near the meshing area, the relevant gear design parameters are shown in Table 1. The study delves into the impact of gear design parameters (modulus, pressure angle, tooth line radius) on the EHL oil film thickness under the cutter inclination angle during the meshing process of the VH-CATT cylindrical gear pair. In order to further quantify the influence of various parameters on the VH-CAT cylindrical gear transmission, the minimum EHL and the central oil film thickness h at the meshing point of each gear pair are defined as the EHL meshing point oil film thickness (h_{in}). At the same time, the average EHL minimum and central oil film thickness of each design parameter group in a meshing period are calculated respectively, which is defined as the average EHL oil film thickness (h_{ave}).

Table 1 Design parameters

parametric variable(unit)	numerical value
Cutter inclination angle γ (°)	0, 2, 3, 4, 5, 6, 7
Tooth number z_p / z_g	29/41
Input speed of the driving gear np (r/min)	1000
Input load of the driven gear T_g (N·m)	1000
Modulus m (mm)	4, 6, 8
Pressure angle α (°)	18, 20, 22
Tooth line radius R_{T} (mm)	200, 300, 400
Δγ (°)	0, 3, 5, 7

5.1 The influence of the modulus parameters on the minimum oil film thickness and the central oil film thickness

According to Table 1, if $\alpha = 20^{\circ}$ and $R_T = 200$ mm, the remaining parameters are unchanged. Firstly, the influence of different cutter head inclination angles on the EHL oil film thickness under the same modulus is analysed. The variation of $h_{\min}$ and h_{cen} of the tooth surface under the cutter head inclination angle parameters during the meshing process of the VH-CATT cylindrical gear pair is obtained as shown in Figs. 4 to 6.



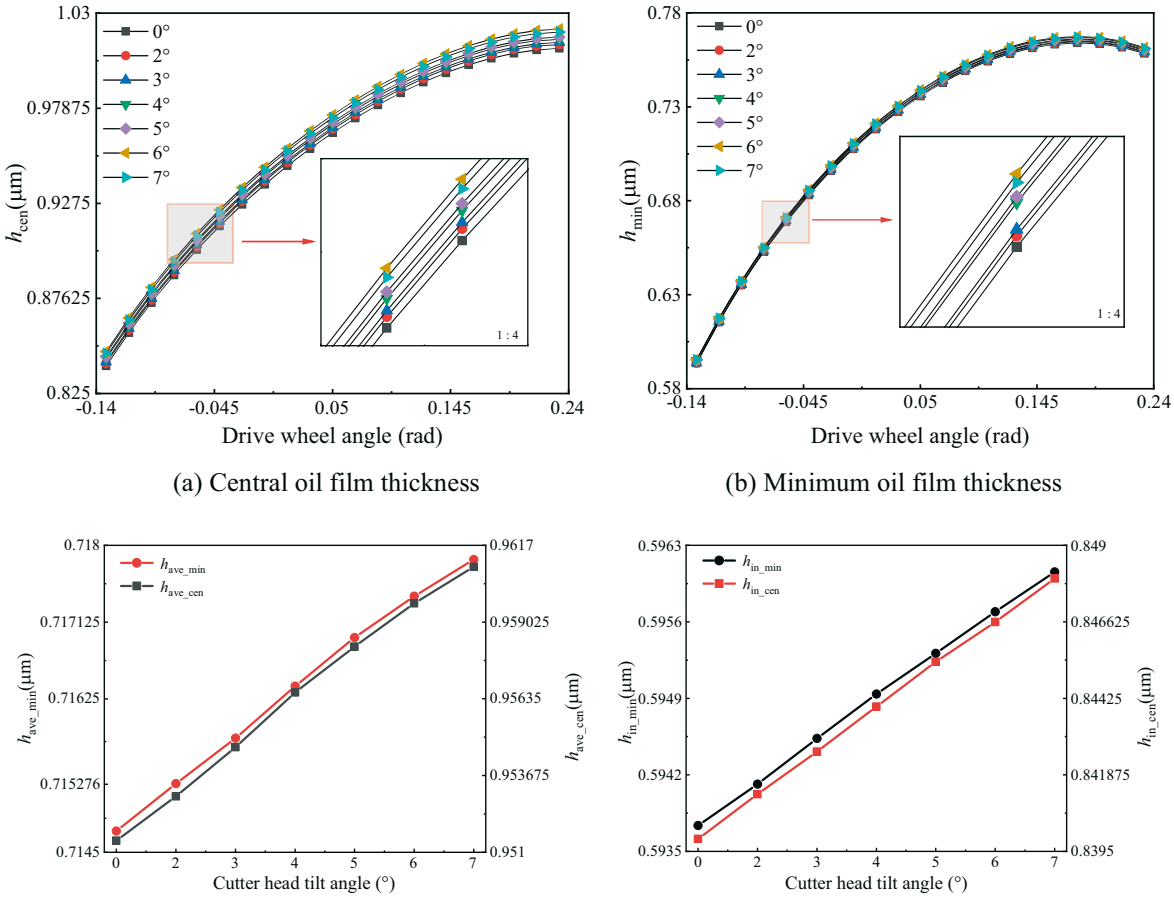
(a) Central oil film thickness

(b) Minimum oil film thickness

(c) Average oil film thickness

(d) Oil film thickness at entry point

Fig. 4 $m = 4$ mm. Changes of central oil film thickness and minimum oil film thickness under the action of cutter head tilt angle



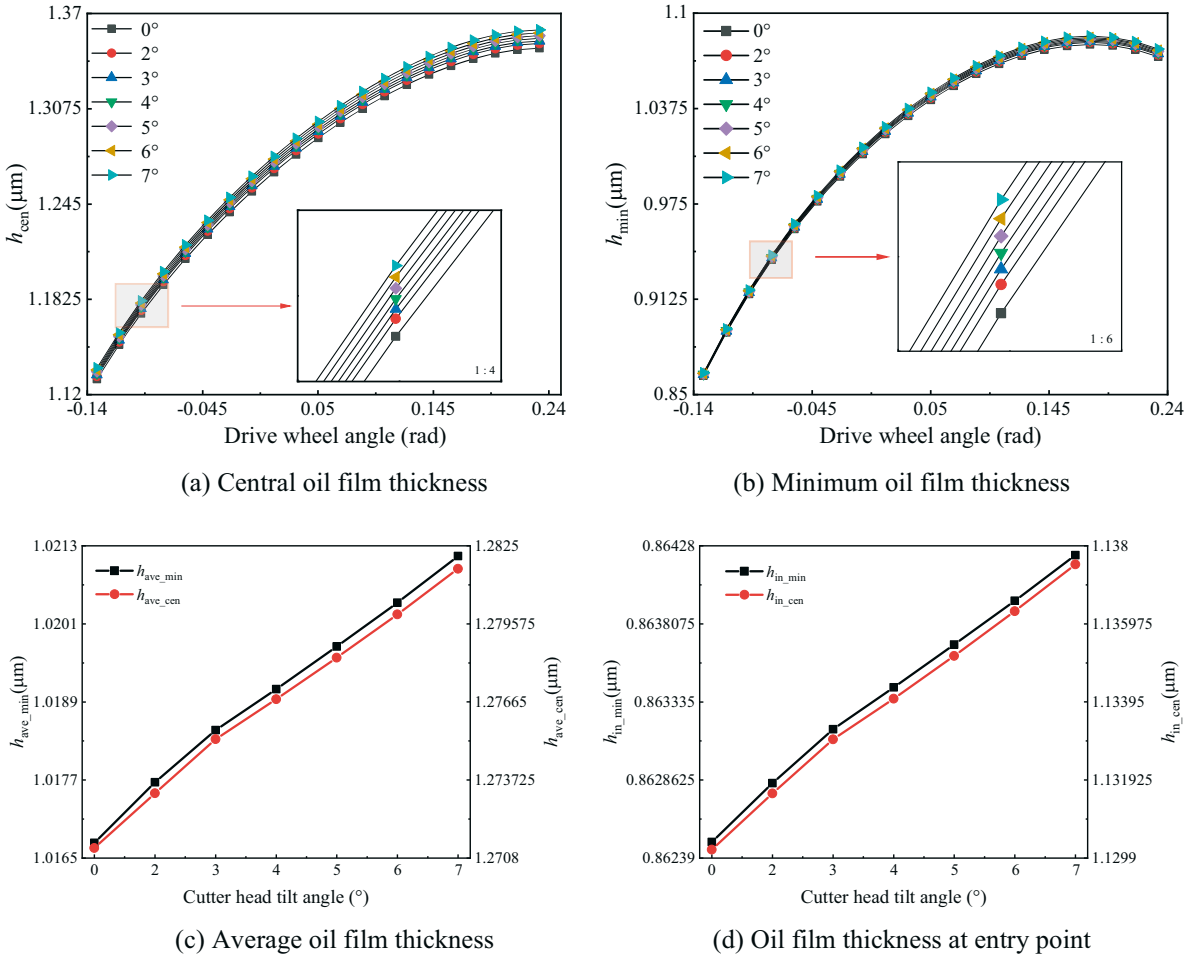
(a) Central oil film thickness

(b) Minimum oil film thickness

(c) Average oil film thickness

(d) Oil film thickness at entry point

Fig. 5 m = 6 mm. Changes of central oil film thickness and minimum oil film thickness under the action of cutter head tilt angle



(a) Central oil film thickness

(b) Minimum oil film thickness

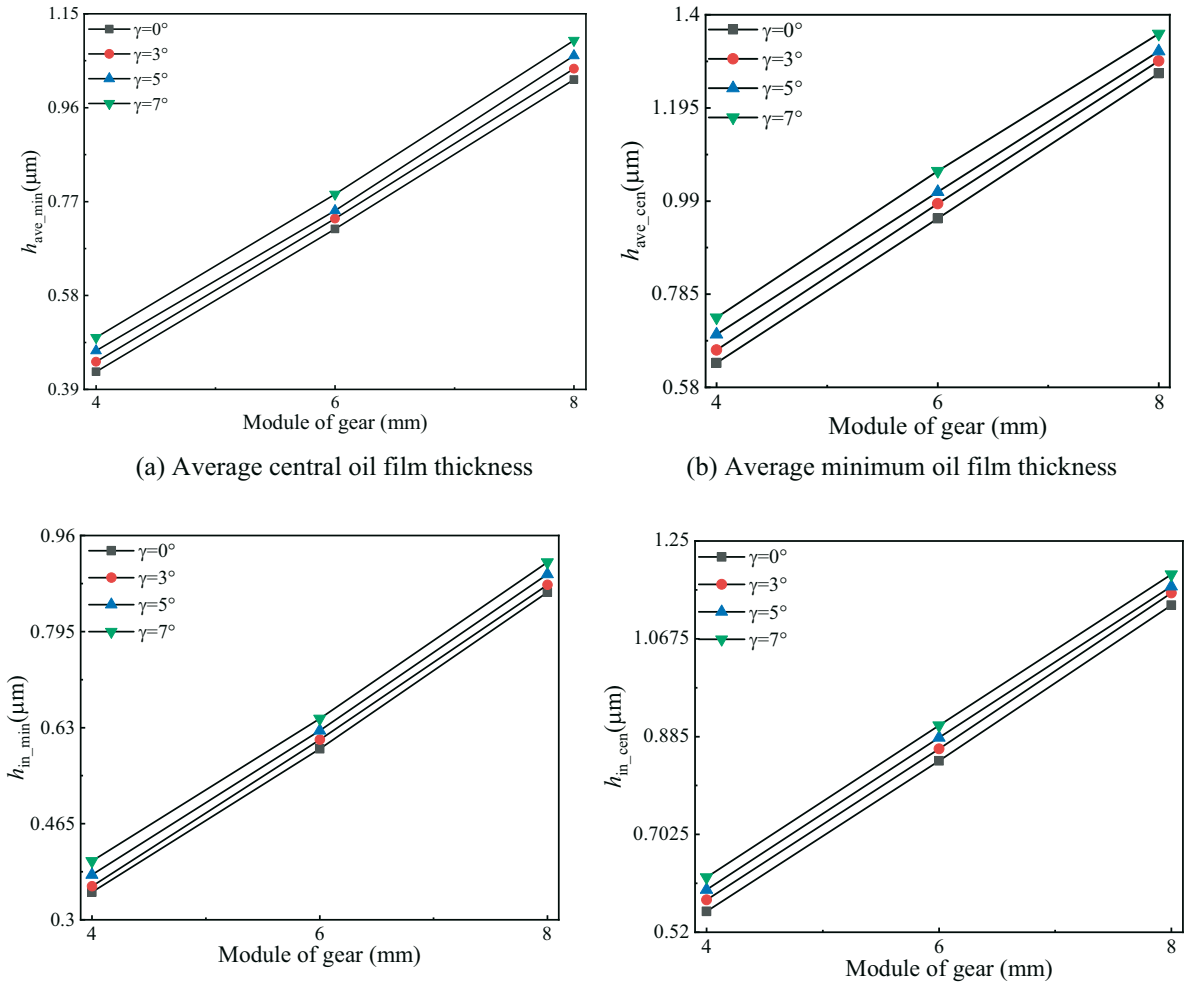
(c) Average oil film thickness

(d) Oil film thickness at entry point

Fig. 6 m = 8 mm. Changes of central oil film thickness and minimum oil film thickness under the action of cutter head tilt angle

Figures 4 to 6 show that when the modulus is set to 4 mm, 6 mm, and 8 mm, both the EHL h_{cen} and $h_{\min}$ of the VH-CATT cylindrical gear pair increase as the cutterhead inclination angle rises during a meshing cycle. On average, the central and minimum oil film thicknesses increase by 0.78% and 0.30%, respectively. The cutterhead inclination angle significantly impacts the oil film thickness. For modulus values of 4 mm, 6 mm, and 8 mm, the average central oil film thickness (h_{ave_cen}) increases by 0.69%, 0.99%, and 0.82%, while the meshing point centre thickness (h_{in_cen}) rises by 0.62%, 0.93%, and 0.65%, respectively. Increased inclination enhances lubrication by improving pressure distribution and hydrodynamic effects, resulting in thicker and more stable oil films.

Additionally, the impact of the VH-CATT cylindrical gear modulus on the oil film thickness of EHL is examined by considering the cutterhead inclination angles as presented in Table 1($\Delta\gamma$). The changes in the average oil film thickness (h_{ave}) on the tooth surface and the oil film thickness (h_{in}) at the meshing point during the meshing process of the VH-CATT cylindrical gear pair under different modulus parameters are illustrated in Fig. 7.



(a) Average central oil film thickness

(b) Average minimum oil film thickness

(c) Central oil film thickness at entry point

(d) Minimum oil film thickness at entry point

Fig. 7 Changes of average oil film thickness and oil film thickness at the point of entry under different modulus parameters

Figs 4(a), 5(a), and 6(a) illustrate that as the modulus increases, the EHL central oil film thickness rises. Similarly, Figs 4(b), 5(b), and 6(b) show that the minimum oil film thickness also increases with the modulus. When the cutterhead inclination angles are $\gamma = 0^{\circ}$, $\gamma = 3^{\circ}$, $\gamma = 5^{\circ}$, $\gamma = 7^{\circ}$, the average minimum oil film thickness increases by 138%, 139.12%, 139.38%, and 139.64%, respectively, while the average central oil film thickness rises by 100.62%, 100.72%, 100.81%, and 100.88%. The minimum oil film thickness at the meshing point grows by 148%, 148.31%, 148.57%, and 148.75%, with the central oil film thickness increasing by 102.16%, 102.18%, 102.23%, and 102.23%, respectively. Notably, the increase in the minimum oil film thickness is greater when the modulus changes from 6 to 8 compared to from 4 to 6. Under the same cutterhead angles, the average minimum oil film thickness increases by approximately 71%, and the entry point thickness rises by about 77%.

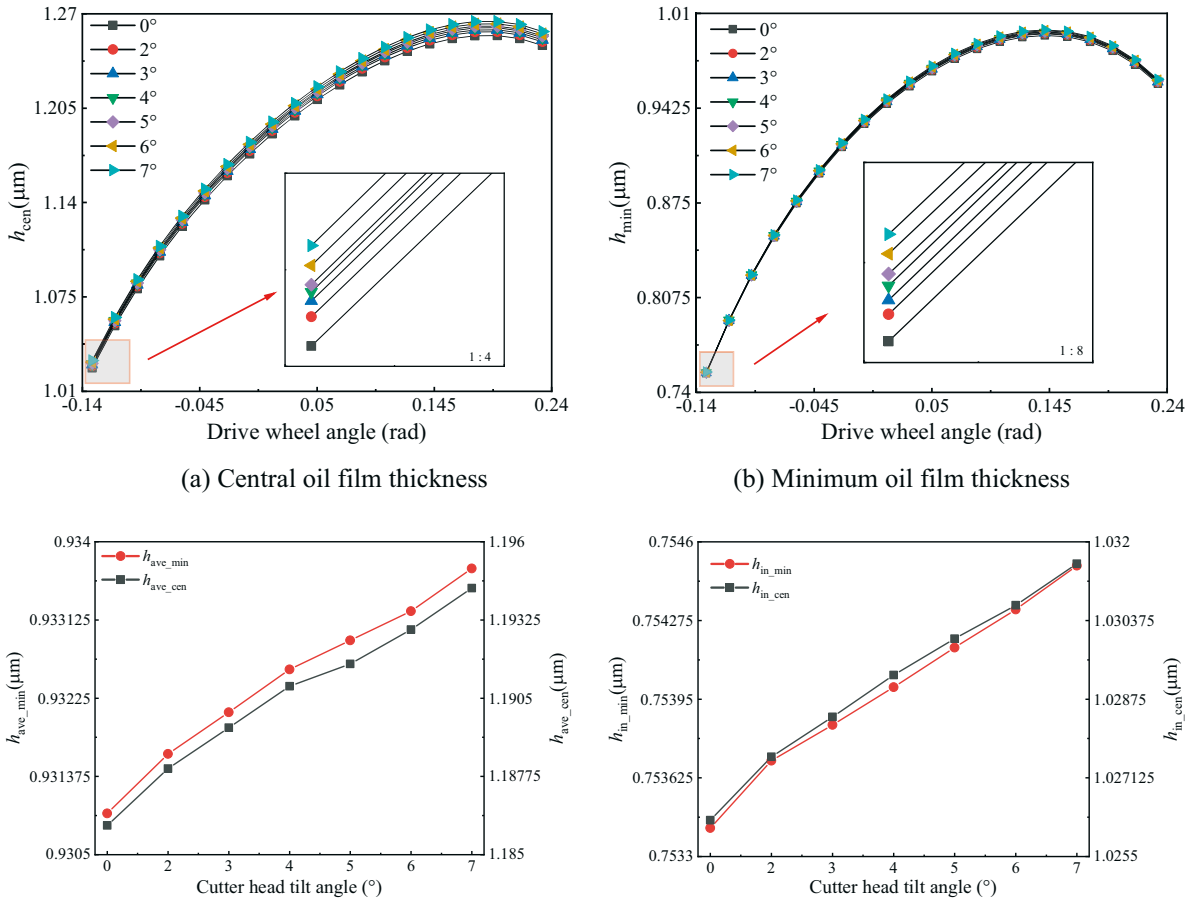
These findings indicate that larger modulus values lead to thicker EHL oil films due to increased pitch, which expands the tooth contact area and enhances the central oil film thickness. However, the localised minimum oil film thickness between the gear teeth becomes thinner as the pitch increases, slightly offsetting the gain.

When the gear modulus increases, h_{ave_cen} and h_{ave_min} of EHL also increase. This is because a higher modulus enlarges the tooth surface size of the VH-CATT cylindrical gear, resulting in a greater tooth pitch and a broader contact area between the tooth tip and root. These changes improve the load distribution across the tooth surface, reducing local contact stress.

By lowering the contact stress, increasing the gear modulus minimises the risk of surface damage and wear. Additionally, the enhanced load distribution improves lubrication near the meshing point of the gear pair, facilitating the formation of a more robust oil film between the tooth surfaces. This not only improves lubrication efficiency but also extends the gear's operational lifespan by reducing friction and wear, ensuring more reliable and durable performance under load.

5.2 The influence of pressure angle parameters on the minimum oil film thickness and the central oil film thickness

Based on the data presented in Table 1, the parameters $m = 8$ mm and $R_T = 200$ mm were taken into consideration, while the other parameters remained constant. The study first determined the variation in h_{min} and h_{cen} of the tooth surface during the meshing process of the VH-CATT cylindrical gear pair under the cutterhead inclination angle. This information is illustrated in Figs. 8–10.



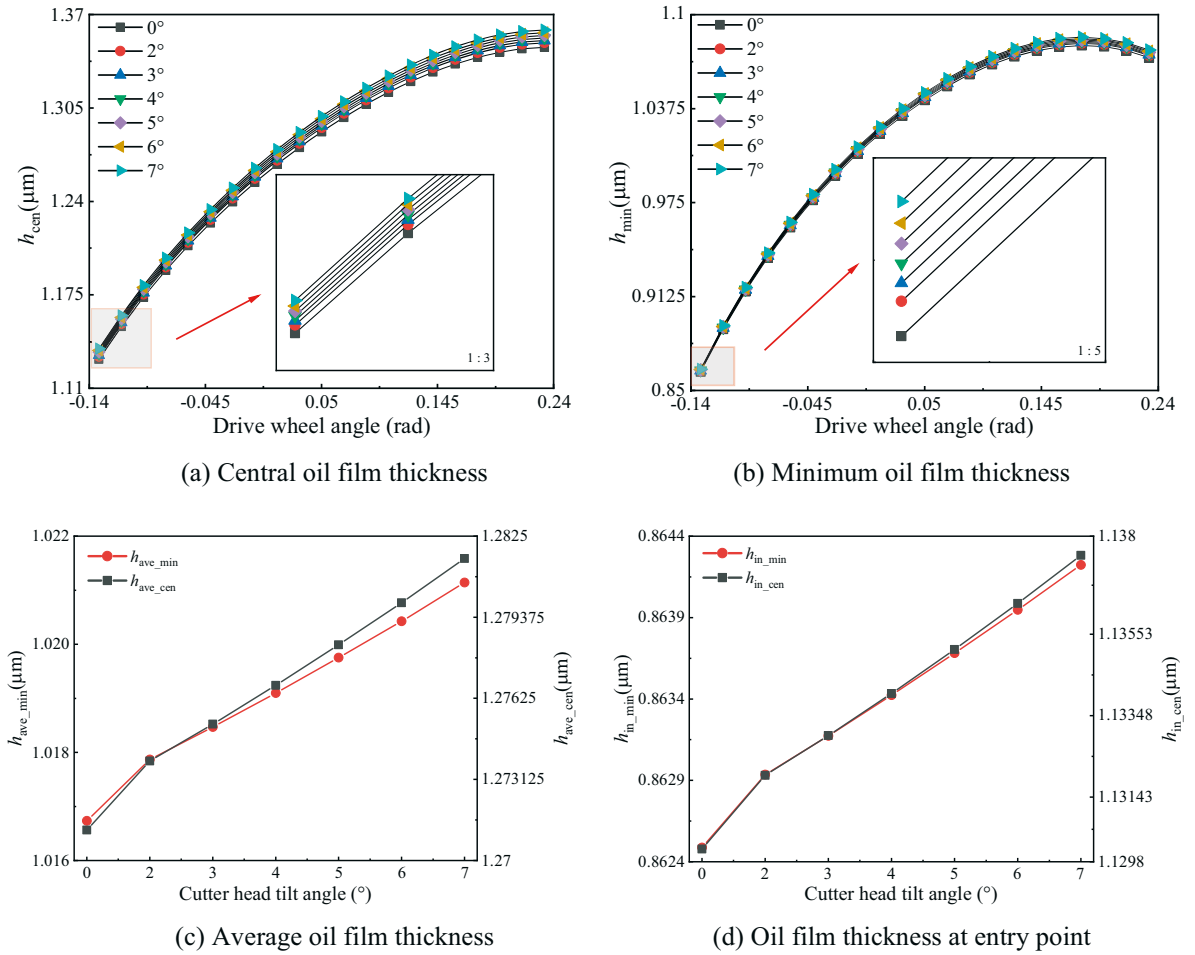
(a) Central oil film thickness

(b) Minimum oil film thickness

(c) Average oil film thickness

(d) Oil film thickness at entry point

Fig. 8 $\alpha = 18^{\circ}$. Changes of central oil film thickness and minimum oil film thickness under the action of the cutter head tilt angle



(a) Central oil film thickness

(b) Minimum oil film thickness

(c) Average oil film thickness

(d) Oil film thickness at entry point

Fig. 9 $\alpha = 20^{\circ}$. Changes of central oil film thickness and minimum oil film thickness under the action of the cutter head tilt angle

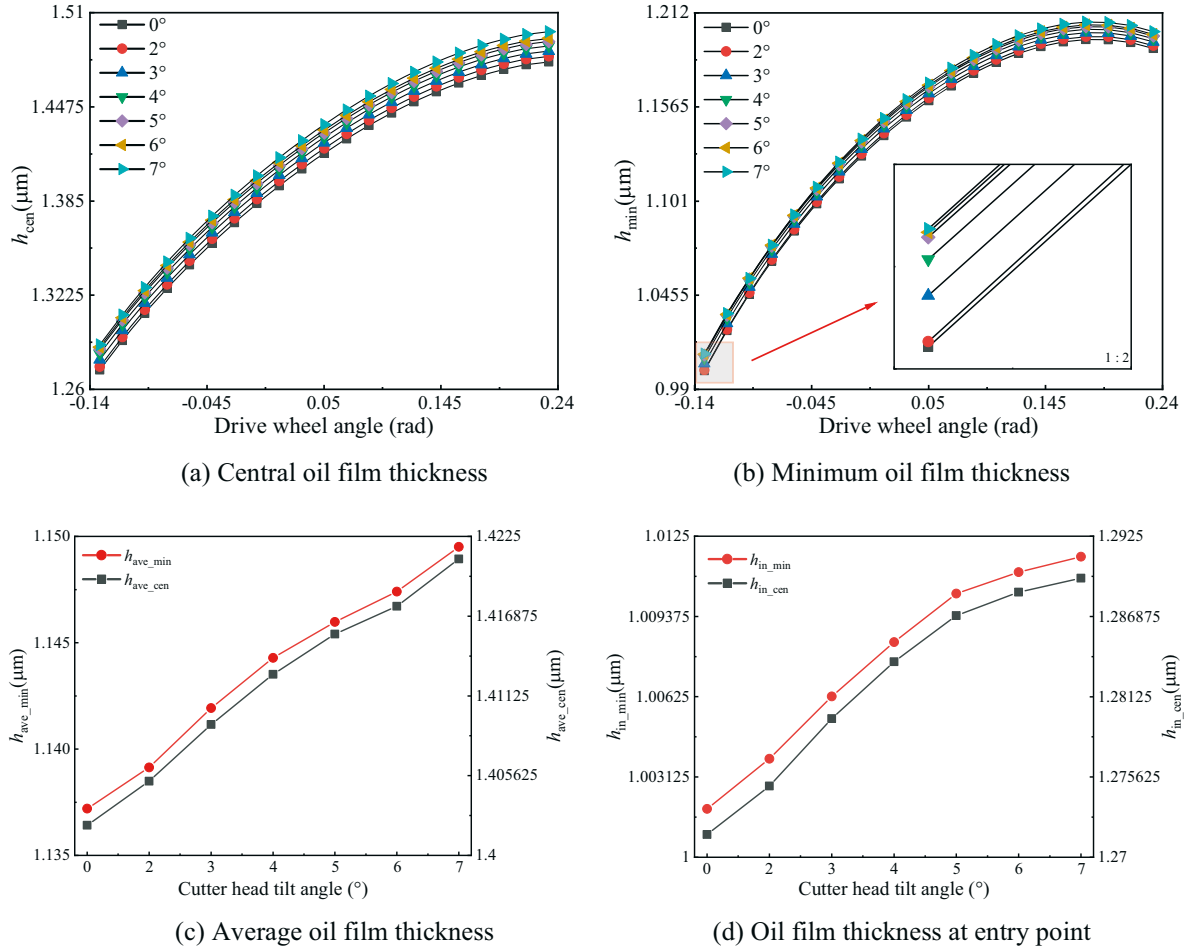
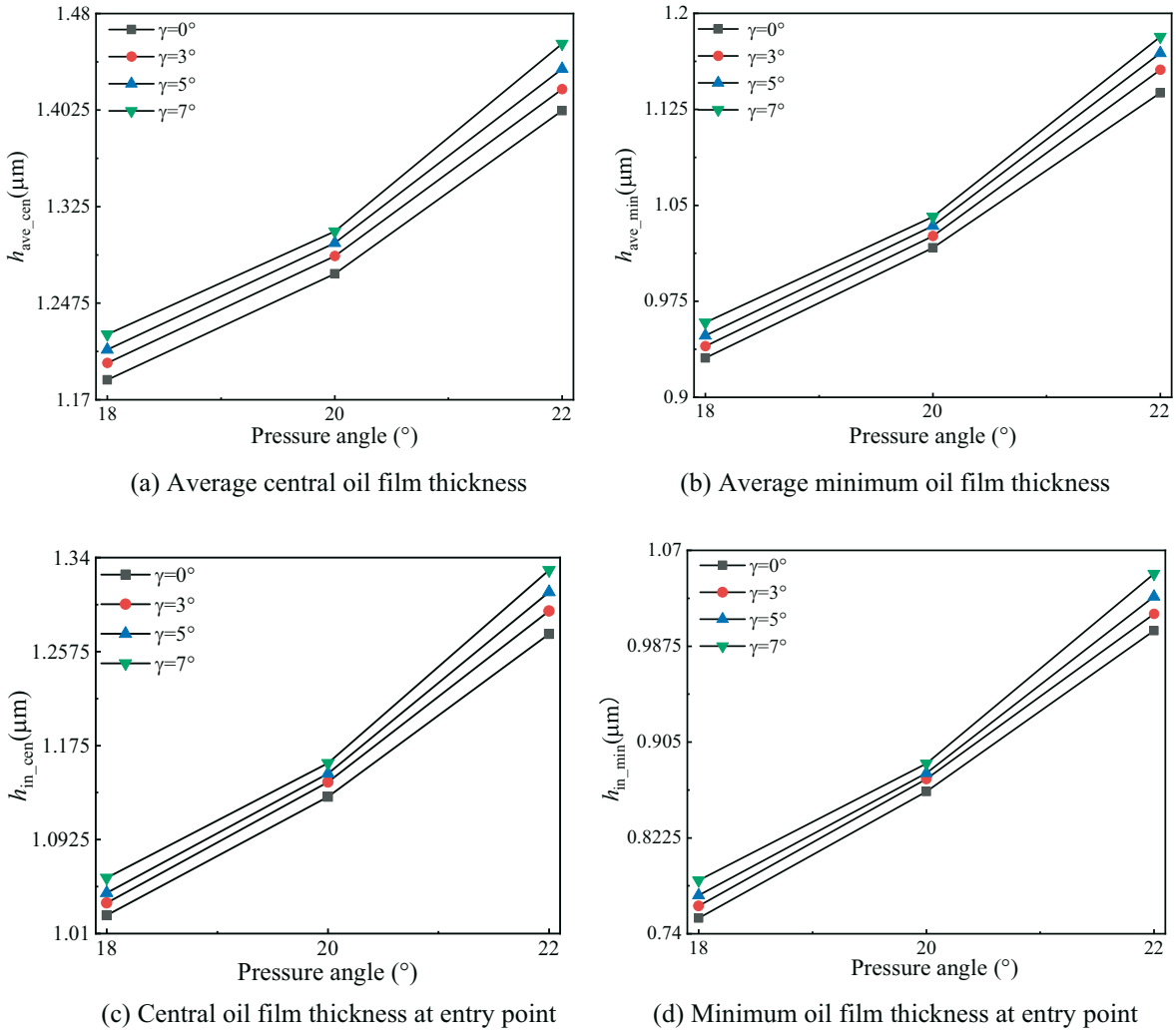


Fig. 10 $\alpha = 22^{\circ}$. Changes of central oil film thickness and minimum oil film thickness under the action of the cutter head tilt angle

From Figures 8 to 10, it can be observed that as the pressure angle increases from 18° to 20° and 22°, both the minimum oil film thickness and the central oil film thickness of the EHL during the meshing cycle of the VH-CATT cylindrical gear pair increase with the cutterhead inclination angle. On average, the central oil film thickness and minimum oil film thickness increase by approximately 0.95% and 0.55%, respectively, over the meshing process. The increase in central oil film thickness due to the cutterhead inclination angle is particularly notable. Specifically, the average central oil film thickness increases by 0.70%, 0.82%, and 1.34% as the pressure angle rises, while the oil film thickness at the meshing point centre increases by 0.51%, 0.65%, and 1.31%.

This trend highlights the significant impact of the pressure angle on the lubrication performance of the gear pair. A larger pressure angle improves load distribution and contact geometry, leading to a thicker and more stable oil film. The combined effect of the increased pressure angle and cutterhead inclination optimises hydrodynamic lubrication, reducing friction and wear while enhancing the gear pair's operational efficiency and durability.

Additionally, the influence of the pressure angle of the VH-CATT cylindrical gear on the oil film thickness of EHL is further investigated by taking into account the cutterhead inclination angles as outlined in Table 1 ($\Delta\gamma$). The average oil film thickness on the tooth surface and the oil film thickness of the meshing point under various pressure angle parameters during the meshing process of the VH-CATT cylindrical gear pair are depicted in Fig. 11.



(a) Average central oil film thickness

(b) Average minimum oil film thickness

(c) Central oil film thickness at entry point

(d) Minimum oil film thickness at entry point

Fig. 11 The variation of average oil film thickness and the oil film thickness at the entry point under the action of the pressure angle

As shown in Figures 11(a) and 11(b), the EHL oil film thickness during the meshing process of the VH-CATT cylindrical gear pair increases with the modulus under the same cutterhead inclination angle. The average increases in the minimum oil film thickness and central oil film thickness are 33.99% and 25.05%, respectively. Figures 8(a), 9(a), and 10(a) show that the central oil film thickness increases with the pressure angle, while Figures 8(b), 9(b), and 10(b) reveal a similar trend for the minimum oil film thickness. When the cutterhead inclination angles are $\gamma = 0^{\circ}$, $\gamma = 3^{\circ}$, $\gamma = 5^{\circ}$, $\gamma = 7^{\circ}$, the average increases in the minimum oil film thickness are 22.24%, 22.99%, 23.27%, and 23.30%, respectively, while the central oil film thickness increases by 24.06%, 24.71%, 25.24%, and 25.49%.

Additionally, when the pressure angle increases from 20°to 22°, the growth in the minimum oil film thickness is more significant than when it increases from 18°to 20°. Under the same cutterhead inclination angles, the average increases in the minimum oil film thickness are 13.00%, 13.24%, 13.53%, and 13.57%, respectively.

In summary, the minimum and central oil film thicknesses of EHL increase with both the modulus and pressure angle. These findings indicate that increasing the pressure angle promotes the formation of a more effective lubrication film at the gear meshing points, enhancing the overall lubrication performance.

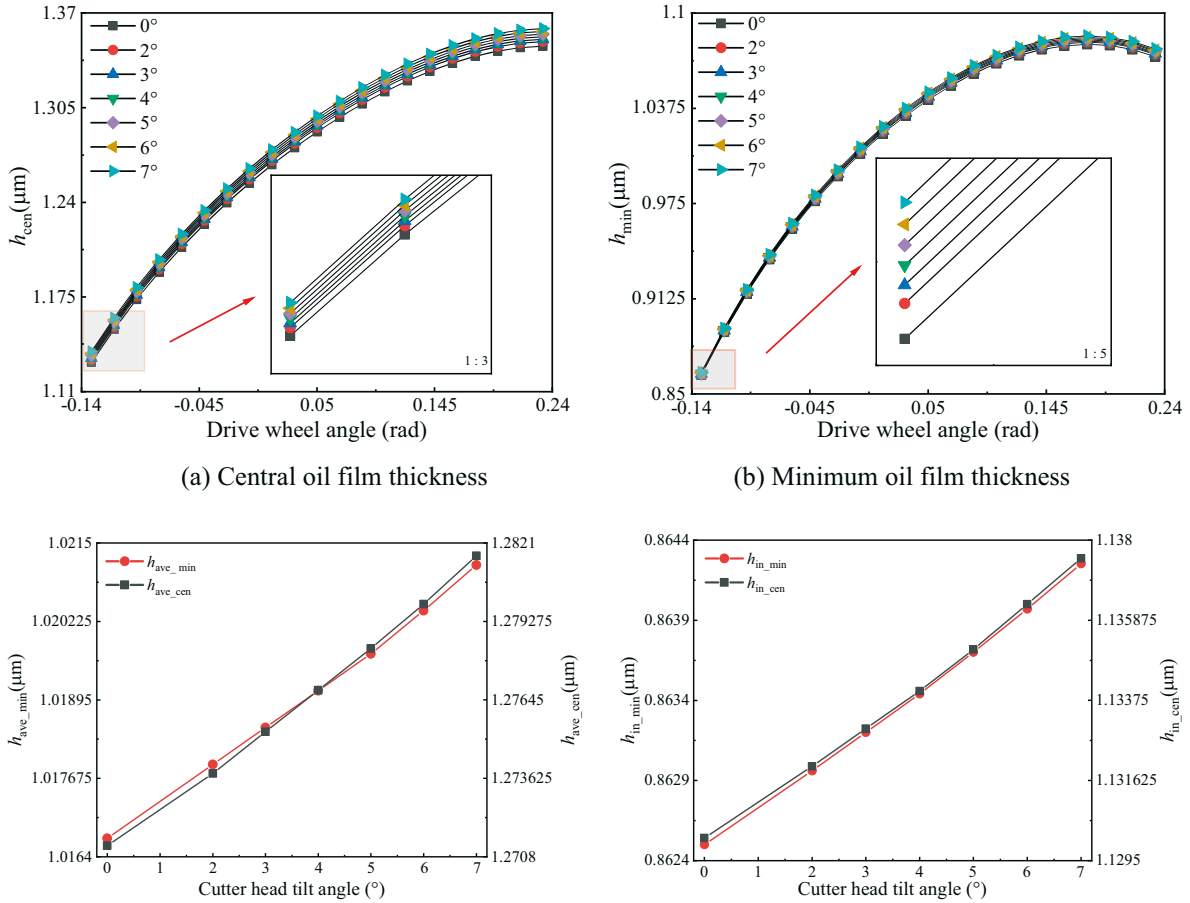
As the pressure angle increases, both the central oil film thickness and the minimum oil film thickness of EHL increase. This is primarily due to the pressure angle's influence on the

contact geometry of the gear tooth surface. A larger pressure angle promotes the formation of an elliptical contact area, resulting in a sharper contact line and a more concentrated contact zone. This localised concentration of contact stress, within a certain range, enhances the conditions for oil film formation.

Furthermore, the increased pressure angle causes the oil film to be distributed across more contact areas on the tooth surface, contributing to a thicker and more uniform oil film. The pressure angle also directly affects key parameters such as entrainment velocity and the comprehensive curvature radius. Specifically, a higher pressure angle increases the entrainment velocity while reducing the comprehensive curvature radius. These changes improve the hydrodynamic effects near the meshing point, further facilitating the development of a stable and robust oil film between the gear tooth surfaces. This enhanced lubrication reduces friction, wear, and the risk of contact fatigue, thereby improving gear performance and durability.

5.3 The influence of tooth line radius parameters on the minimum oil film thickness and the central oil film thickness

According to Table 1, if $\alpha = 20^{\circ}$ and $m = 8$ mm, the remaining parameters are unchanged. Firstly, the variation law of $h_{\min}$ and h_{cen} of the tooth surface under the inclination angle of the cutter head during the meshing process of the VH-CATT cylindrical gear pair is obtained as shown in Figs. 12–14.



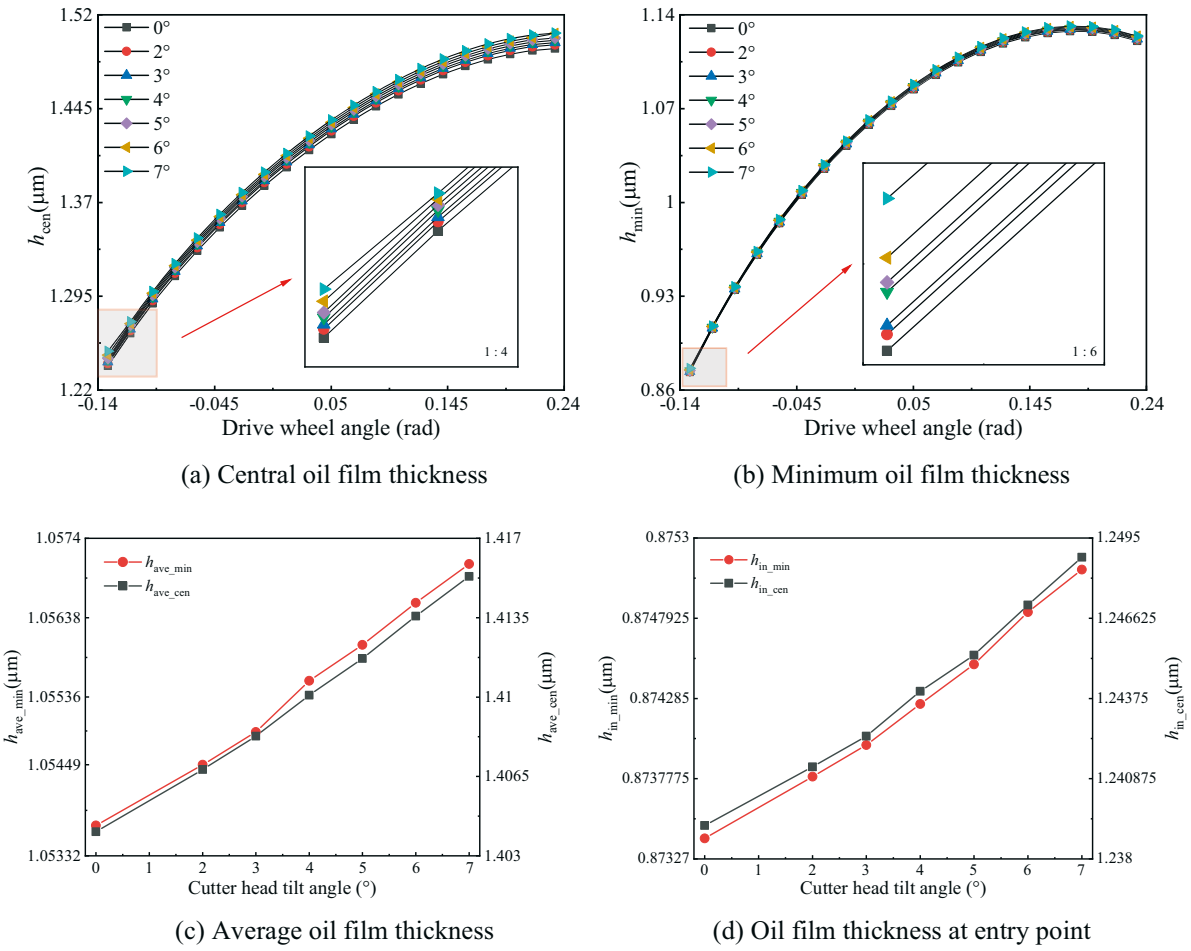
(a) Central oil film thickness

(b) Minimum oil film thickness

(c) Average oil film thickness

(d) Oil film thickness at entry point

Fig. 12 $R_T = 200$ mm. Changes of central oil film thickness and minimum oil film thickness under the action of the cutter head tilt angle



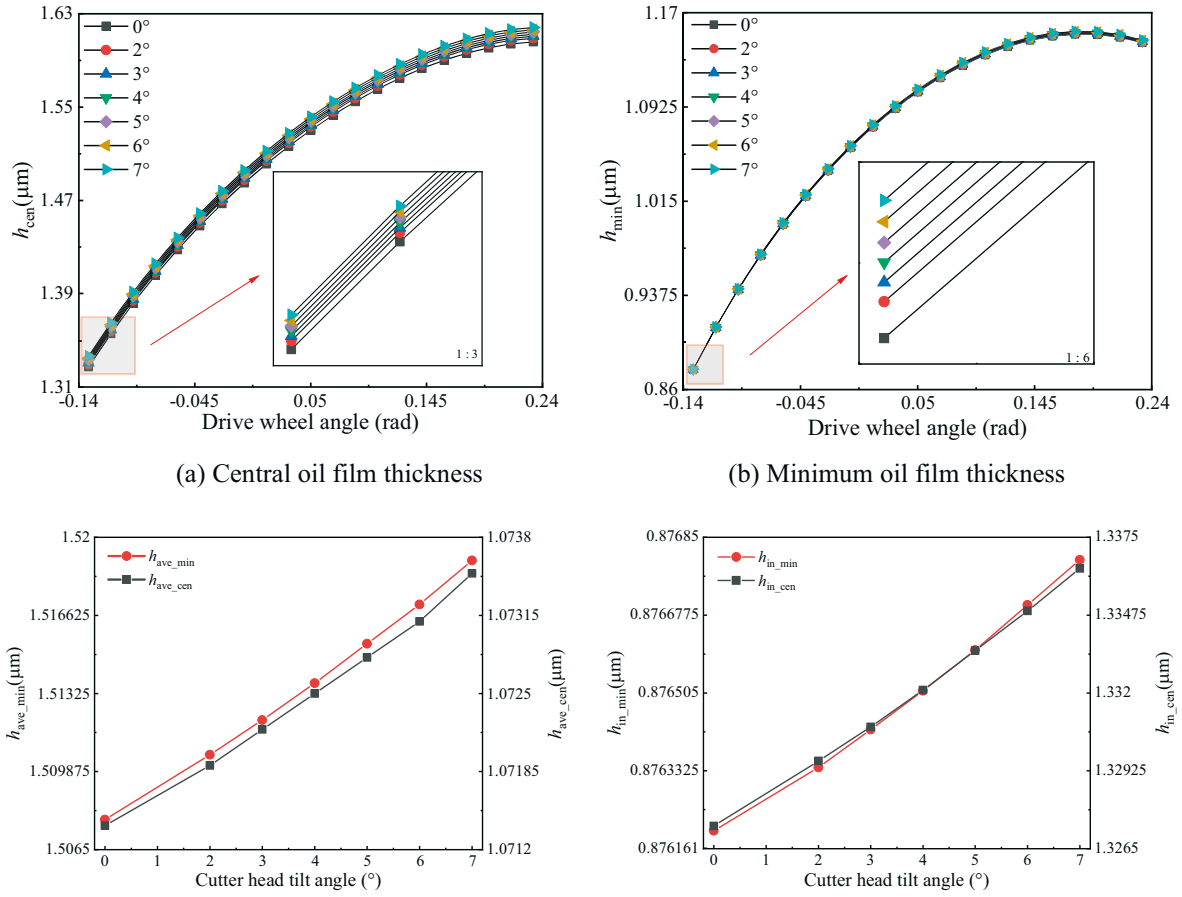
(a) Central oil film thickness

(b) Minimum oil film thickness

(c) Average oil film thickness

(d) Oil film thickness at entry point

Fig. 13 R_T = 300 mm. Changes of central oil film thickness and minimum oil film thickness under the action of cutter head tilt angle



(a) Central oil film thickness

(b) Minimum oil film thickness

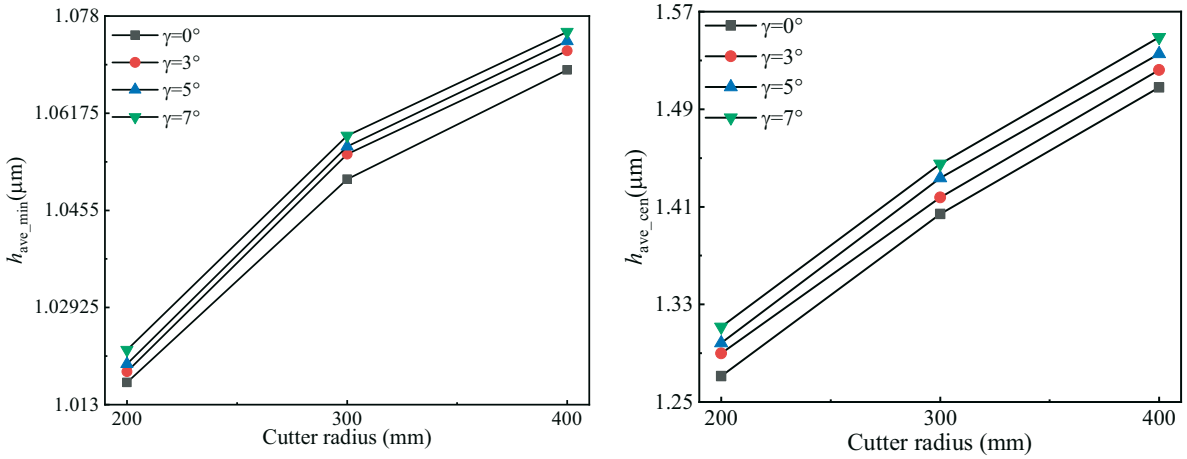
(c) Average oil film thickness

(d) Oil film thickness at entry point

Fig. 14 R_T = 400 mm. Changes of central oil film thickness and minimum oil film thickness under the action of the cutter head tilt angle

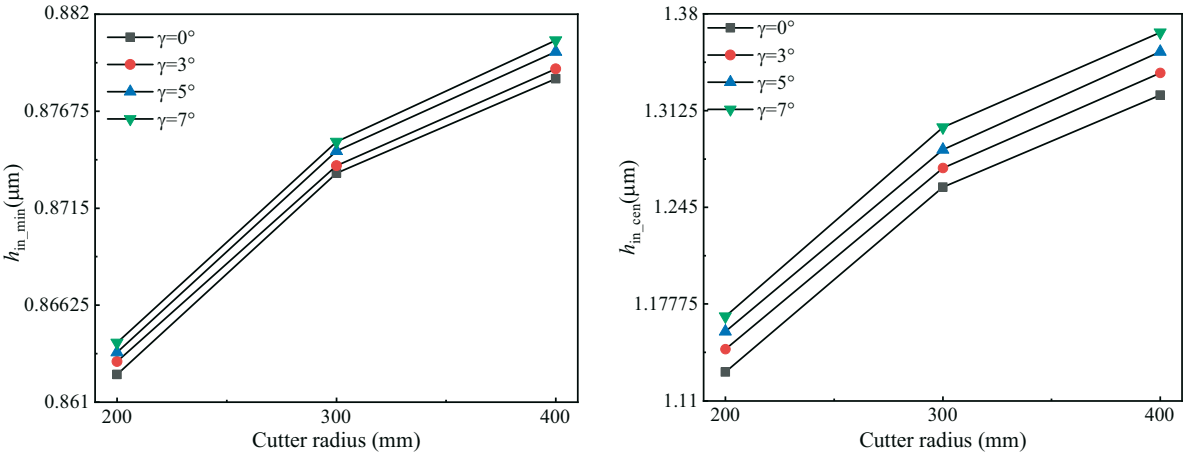
From Figs. 12 to 14, as the radius of the tooth line varies at 200 mm, 300 mm, and 400 mm, the EHL h_{min} and h_{cen} of the VH-CATT cylindrical gear pair increase with the inclination angle of the cutter head during a meshing cycle. The average increase in the meshing process is 1.20% and 0.54%, respectively. The largest increase in the central oil film thickness occurs under the effect of the tool tilt angle. Specifically, h_{ave_cen} increases by 0.82%, 3.70%, and 1.60%, respectively, while the increase of oil film thickness in the centre of the meshing point is 2.14%, 5.22%, and 4.77%, respectively.

Furthermore, the study also investigates the influence of the tooth line radius of the VH-CATT cylindrical gear on the EHL oil film thickness by taking into account the cutter head inclination angle, as shown in Table 1 ($\Delta\gamma$). Fig. 15 depicts the variations in h_{ave} on the tooth surface and at the meshing point under the tooth line radius parameter during the meshing process of the VH-CATT cylindrical gear pair.



(a) Average central oil film thickness

(b) Average minimum oil film thickness



(c) Central oil film thickness at entry point

(d) Minimum oil film thickness at entry point

Fig. 15 The variation of the average oil film thickness and the oil film thickness at the entry point under the action of the cutter head radius

Figure 15 shows that the EHL oil film thickness in the VH-CATT cylindrical gear pair increases as the tooth line radius grows, under the same cutter inclination angle. The average increases in minimum and central oil film thickness are 6.88% and 19.20%, respectively. Figures 12(a), 13(a), and 14(a) confirm that the central oil film thickness rises with the tooth line radius, while Figures 12(b), 13(b), and 14(b) show a similar trend for the minimum oil film thickness. For cutterhead inclination angles of $\gamma = 0^{\circ}$, $\gamma = 3^{\circ}$, $\gamma = 5^{\circ}$, $\gamma = 7^{\circ}$, average increases in the minimum oil film thickness are 5.14%, 5.27%, 5.30%, and 4.21%, while the central oil film thickness increases by 18.61%, 18.01%, 18.26%, and 19.10%.

When the tooth line radius increases from 200 mm to 300 mm, the central oil film thickness rises more significantly than from 300 mm to 400 mm. Under the same inclination angles, the average increases in central oil film thickness are 10.45%, 11.24%, 12.16%, and 13.42%, with corresponding increases at the meshing point of 9.65%, 12.85%, 13.23%, and 11.19%. These findings highlight the positive effect of tooth line radius on oil film formation and lubrication performance.

As the radius of the tooth line increases, both the average central oil film thickness and the average minimum oil film thickness of the EHL increase. This occurs because a larger tooth line radius alters the tooth surface curvature, reducing the primary curvature along the tooth line direction and increasing the overall radius of curvature. As a result, the tooth surface

becomes flatter, leading to an expanded contact area. The increased contact area ensures a more even distribution of force and load across the surface, enhancing the bearing capacity of the oil film. This improved load distribution not only reduces localised stress but also facilitates the formation of a thicker and more stable oil film. Therefore, increasing the radius of the tooth line effectively enhances lubrication performance, contributing to reduced wear and improved durability of the gear pair.

6. Conclusion

The findings of this study suggest that during the meshing process of VH-CATT cylindrical gear, the use of specific gear design parameters such as $m = 8$ mm, $\alpha = 22^\circ$, $R_T = 400$ mm, and $\gamma = 7^\circ$ can enhance the generation of oil film thickness at the meshing point, leading to optimal improvement outcomes. The key conclusions include:

(1) When the modulus remains constant, the EHL oil film thickness increases as the cutterhead inclination angle rises. For $m = 6$ mm, the cutter inclination angle has the greatest impact on both the average oil film thickness and the oil film thickness at the meshing point. Similarly, under the same inclination angles, the EHL oil film thickness significantly increases with a rise in the modulus. At $\gamma = 7^\circ$, the average oil film thickness and the oil film thickness at the contact point experience the most substantial increases due to the modulus.

(2) The gear's pressure angle positively influences the EHL oil film. With a constant pressure angle, the oil film thickness increases as the cutterhead tilt angle rises. At $\alpha = 22^\circ$, the cutter inclination angle leads to the most significant growth in the average and contact-point oil film thickness. Similarly, increasing the modulus under a fixed inclination angle substantially enhances the oil film. At $\gamma = 7^\circ$, the pressure angle drives the highest increase in both the average and contact-point oil film thickness.

(3) The tooth line radius plays a key role in enhancing the EHL oil film on the gear tooth surface. With a constant tooth line radius, the oil film thickness increases as the cutter tilt angle rises. At $R_T = 400$ mm, the cutter inclination angle drives the most significant growth in both the average and meshing-point oil film thickness. Similarly, for the same tilt angles, the oil film thickness increases with the modulus. At $\gamma = 7^\circ$, the tooth line radius has the greatest impact, resulting in the highest growth in the average and meshing-point oil film thickness.

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Nomenclature

Symbol	Description
γ	cutter inclination angle
K_i^a	principal curvature in the tooth line direction
K_i^b	principal curvature in the tooth profile direction
R_x	comprehensive curvature radius in the tooth profile direction
R_y	comprehensive curvature radius in the tooth line direction
v_p^T	velocity component of the driving wheel in the tangent plane
v_g^T	velocity component of the driven wheel in the tangent plane
v_r	relative sliding velocity in the tangent plane
u_e	entrainment velocity in the tangent plane
S	slip-roll ratio in the tangent plane
$h_{\min}$	the minimum oil film thickness of EHL
h_{cen}	the central oil film thickness of EHL
h_{ave}	average oil film thickness
$h_{\text{ave}-\min}$	average minimum oil film thickness
$h_{\text{ave}-\text{cen}}$	average central oil film thickness
h_{in}	oil film thickness at the meshing point
$h_{\text{in}-\min}$	minimum oil film thickness of the meshing point
$h_{\text{in}-cen}$	central oil film thickness of the meshing point

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