THE SPECTRAL DISTRIBUTION METHOD IN BOSON SPACE

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1. Introduction

The spectral distribution method (SDM), introduced by Moszkowski[1] in problems of atomic spectroscopy, and developed by French and co-workers[2], allows one to obtain information on the energies and wave functions of systems requiring extremely large spectroscopic spaces.

The basic assumption in this method is, roughly speaking, that the level density tends to a Gaussian form, when the number of particles increases[3]; therefore, the low-order moments of the Hamiltonian operator carry the most important spectroscopic information. This assumption was checked and found satisfactory for fermion systems[4].

The aim of this paper is to discuss the validity of this hypothesis in the bosons space. It should be noted that, in this case, one may consider the number of bosons going to infinity, without increasing the number of single-particle levels (dense limit)[5].

2. Monopole Bosons

In the case of monopole bosons it is straightforward to show that the trace of a t-body operator 0(t), acting on a spectroscopic space of M bosons distributed over N single-particle levels, denoted by S(N,M), is given by[7,8]

$$\langle\langle 0(t)\rangle\rangle^{M}=\binom{M+N-1}{M-t}\sum_{\alpha_1<}\cdots\sum_{<\alpha_t}\sum_{(p)}p_1!\cdots p_t!\langle\alpha_1^{p_1}\cdots\alpha_t^{p_t}|0(t)|\alpha_1^{p_1}\cdots\alpha_t^{p_t}\rangle \qquad (1)$$

Expression (1) exhibits the characteristic features of a trace propagation, similar in some respects to the one derived for fermions[2]. The trace of 0(t) over S(N,t) propagates forward to S(N,M) by a binomial coefficient. However, as one cannot define a boson plenum state, the symmetry particle-hole does not hold and the "backward" propagation, typical for the fermion averages, does not occurs (see also Ref. [9]). As an example, we analyse the case of M noninteracting bosons. We calculate the first four cumulants of the level density, defined in the usual way as polynomials of the distribution moments $\mu_n(N,M)$. Though we restricted ourselves to the first four cumulants, it is generally believed that the knowledge of the reduced cumulants $\gamma_1=k_3/k_2^{3/2}$, $\gamma_2=k_4/k_2^2$ is sufficient for deciding whether the level density is approximately Gaussian or not. At a fixed number of levels and for a large number of particles the form parameters γ_1 and γ_2 vary slowly with M; this result had already been noted by the authors of Ref.[6] who made a numerical analysis of the non interacting boson spectra. If M goes to infinity γ_1 and γ_2 are not necessarily zero, their numerical values depending on the number of levels and on the form of the single-particle spectrum assumed. The level density may therefore show a noticeable departure from the Gaussian shape. For $M\to\infty$ one obtains for instance $\gamma_1=0$, $\gamma_2=-1.2$ if N=2.

3. Conclusions

Keeping in mind the results of section 2 we may conclude that, in the boson case, the presence of many single-particle levels seems to be essential in generating a normal level density, the number of particles playing a minor role.

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