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# A REALIZED COVARIANCE APPROACH IN REEXAMINING CRYPTO AND FX CURRENCIES AS SAFE HAVENS AND HEDGES

# PRISTUP REALIZIRANE KOVARIJANCIJE U ISPITIVANJU KRIPTO I FX VALUTA KAO SIGURNIH UTOČIŠTA I ZAŠTITA

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**ABSTRACT:** This research paper deals with the field of financial econometrics. Two financial market phenomena are investigated: covolatility and safe haven assets using time series analysis and high-frequency data on financial assets. The main objective is re-examining the safe haven and hedge properties of two currency classes, distinguished by the regulation and centralization level. While the role of a long-term hedge and safe haven during crisis periods, such as COVID, has been explored in the literature for various assets, this paper fills a significant gap by utilizing intraday transaction data and addresses numerous issues related to integrated covariance estimation. By sampling the data so frequently we get more complete information on price movement and trading activity. The two-phase analysis led to the achievement of the goal - the answer to the main hypothesis: The robust two times scaled estimator is superior to other considered covolatility estimators. This hypothesis leads to the identification of a safe haven currency in comparison with the general market movements. We determine the best covolatility estimator and identify Swiss Franc as the best performing safe haven currency compared to general market movements and confirm that Bitcoin has the characteristics of safe haven currency.

**KEYWORDS:** safe haven, FX, cryptocurrency, high-frequency observations, realized covariance

**JEL CLASSIFICATION:** C58, G15, P45

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**SAŽETAK:** Ovaj istraživački rad bavi se područjem financijske ekonometrije. Istražuju se dva fenomena financijskog tržišta: kovolatilnost i sigurna utočišta (engl. *safe haven*) koristeći analizu vremenskih serija i visokofrekventne podatke o financijskim instrumentima. Glavni je cilj ponovno ispitati svojstva sigurnog utočišta i zaštite (engl. *hedge*) za dvije klase valuta, razlikovane prema razini regulacije i centralizacije. Iako je u literaturi već istražena uloga dugoročne zaštite i sigurnog utočišta tijekom kriznih razdoblja, poput pandemije bolesti COVID-19, za različite klase imovine, ovaj rad popunjava važnu prazninu korištenjem intradnevnih transakcijskih podataka i rješava niz izazova povezanih s procjenom integrirane kovarijance. Čestim uzorkovanjem podataka dobiva se potpunija informacija o kretanju cijena i tržišnoj aktivnosti. Dvofazna analiza dovela je do ostvarenja cilja – odgovora na glavnu hipotezu: Robusni procjenitelj s dvjema skalama superioran je u odnosu na ostale razmatrane procjenitelje kovolatilnosti. Ova hipoteza omogućuje identifikaciju valute sigurnog utočišta u usporedbi s općim tržišnim kretanjima. Određujemo najbolji procjenitelj kovolatilnosti i utvrđujemo da je švicarski franak najučinkovitija valuta sigurnog utočišta u odnosu na opća tržišna kretanja te potvrđujemo da bitcoin posjeduje karakteristike valute sigurnog utočišta.

**KLJUČNE RIJEČI:** sigurno utočište, devize, kriptovaluta, opažanja visoke frekvencije, realizirana kovarijanca

**JEL KLASIFIKACIJA:** C58, G15, P45

## 1. INTRODUCTION

During the last few decades we have been witnesses to a series of rolling crises starting from dot-com bubble, 2008 global financial crisis, COVID crisis, energy crisis, European sovereign debt crisis, Russo-Ukrainian War and Chinese property sector crisis. Historically, currencies such as the Swiss Franc (CHF) and the US Dollar (USD) have been viewed as primary safe havens due to their perceived stability, strong institutional backing, and liquidity. These currencies serve as a refuge for investors looking to hedge against market downturns, providing a measure of protection against volatility in riskier asset classes.

According to Gorton (2017) one of the main characteristics of safe haven assets is "information insensitivity". "Information insensitivity" in relation to safe assets means that they are relatively immune information asymmetries and privileged information so that traders and investors can trade in them without special concern for adverse selection issues. It aligns with Caballero and Simsek (2013) perspective that "simple" assets hold special intrinsic value during economic turmoil, where an asset's safety is contingent on others perceiving it as secure. Safe assets play a pivotal role in the economy, driven by nonpecuniary returns, often termed "convenience yield", in the form of liquidity and safety. Such assets offer a convenience yield and they include but are not limited to global leading currencies, U.S. T-bills and asset backed securities.

The main approach to safe assets, emphasizing the convenience yield aspect, primarily focuses on US issued debt. The simple reason for this is their vast liquidity and safety which are of vital importance for the investors whose equilibrium price is driven and determined by changes in their supply. The interest in and the need for secure assets has increased since the global financial crisis of 2008. In the realm of such assets, the "equilibrium" safe real interest rate experiences a decline, falling below zero and the actual rate due to excess demand, especially when nominal rates reach the zero lower bound, restricting central banks from reducing real rates even further. In this scenario, the appreciation of the currency associated with the safe asset serves as an adjustment mechanism, commonly referred to as the "paradox of the reserve currency" (Habib & Stracca, 2012). A phenomenon called "a safety trap" equilibrium situation arises when prices and exchange rates fail to clear the market of safe assets by reducing demand. It is crucial to effectively manage risk in order to optimize portfolio performance during market distress. We apply dynamic hedging framework to test how well the foreign exchange asset deemed to be safe haven affects risk-return portfolio profile.

Notably, safe haven currencies typically belong to major currency pairs with higher liquidity compared to more regional or local currency pairs. In times of heightened risk aversion among market participants market liquidity usually decreases. In such circumstances investors may find themselves compelled to speculate in liquid currencies, further elevating the status of the most liquid currencies and this by definition pushes them to status of safe assets. In situations involving relative safety, low risk and independent policies, the key determinants for a currency to be considered a safe haven include sound fundamentals of a country, such as a robust economic policy and strong trade positions. In theory, currencies from countries with these favourable attributes could be viewed as safe currencies. Alternatively, the status of safe-haven currencies is traditionally associated with countries boasting a large current account surplus, low sovereign risks, and/or a significant share in global trade.

On the other hand, the currencies which are tied to high-interest rates which is usually indicative of high inflation or high inflation expectation, are not likely to be considered safe-haven currencies. In periods of elevated risk appetite (risk on), investors typically leverage low-interest rate currencies to acquire high-interest rate currencies. Conversely, during risk-averse periods (risk off), they divest from the high-interest rate currency, often a riskier one, and reacquire the low-interest rate safe-haven currency. The commonality among the US Dollar, Euro, Swiss Franc and Japanese Yen lies in their low-interest rates, which is one of the key features of safe-haven currencies. Another determining factor is classifying currencies as safe havens is the current account balance of the issuing country. Countries with substantial deficits are more prone to depreciation compared to those with a surplus, indicating a higher level of exports relative to imports. However, in the globalized landscape of the 21st century external vulnerability related to net foreign position can sometimes be outweighed by the size of a country's capital market.

During risk off periods investors tend to shift their funds towards safe haven currencies. In such times markets are perceived as excessively volatile, causing even relatively secure assets to be viewed as precarious. Consequently, the liquidity rises for safe haven currencies, i.e. the US Dollar, Swiss Franc and Japanese Yen (standing out as the most prominent ones). Increased volatility in U.S. capital market can dictate the future monetary policies of major central banks, as the capital shifts from one asset to another. It is important to monitor the behavior of safe haven currencies and their characteristics in order to better understand the triggers of future policies. Safe-haven currencies, such as the US Dollar, Swiss Franc, and Japanese Yen, have historically been perceived as reliable stores of value during times of crisis. However, with the emergence of cryptocurrencies like Bitcoin, the landscape of safe-haven assets has evolved, introducing new contenders. Can we determine the potential safe haven currency that is a better safe haven asset over the observation period based on the defined benchmark Robust two times scaled estimator of covolatility?

Besides the U.S. Dollar, Swiss Franc and Japanese Yen which are well established in the literature as safe haven currencies i.e. a reliable stores of value during uncertain economic times cryptocurrencies also have the potential to act as safe haven asset due to their decentralized nature and lack of regulation. They have been discussed as a potential safe haven asset, especially in a recent years when their popularity spiked. Cryptocurrencies are not tied to any particular nation or organization, which makes them less vulnerable to government action or geopolitical threats. However, they are still a new and untested asset class, and their volatility could make them a risky investment. The suitability of cryptocurrencies for this purpose is still debatable, but investors need to properly weigh the risks and potential rewards before making an investment. Both cryptocurrencies and FX are influenced by macroeconomic factors like inflation, interest rates, and geopolitical events. Analysing these markets together provides a comprehensive view of global capital flows. This research examines which potential safe haven currency is a better safe haven asset over the observation period based on the defined benchmark Robust two times scaled estimator of covolatility. The methodology for determining the target relies on covolatility. This research utilizes realized covariance estimators. The integrated covariance (true but unknown covariance of the two populations) is estimated using high frequency data. It is understood that this is the covariance of returns between two types of financial assets, identified by the concept of covolatility. There are several approaches that do not use high-frequency data, such as historical covolatility (estimating the covariance by one number based on the selected observation period), heteroscedastic covolatility (estimating the time-varying covariance using multivariate GARCH models), etc. However, in this research, estimators of realized covariance are used for the same reason. In addition to microstructural noise and price jumps, an additional problem to consider in choosing the most appropriate estimator of realized covariance is the non-synchronization of prices observed at unequal intervals, which is a major challenge. In the current literature, there is no strict consensus on the superiority of the realized covariance estimator, so the results of this research paper will make an important contribution because our results will suggest which estimators are appropriate for each analyzed market and how to solve the problem of microstructural noise, price jumps and non-synchronization, which are closely related to the selection of the optimal sampling frequencies on both the slow and fast time scale. There is no consensus on which asset is better safe haven, especially with respect to cryptocurrencies such as Bitcoin, as stated in Wen et al. (2022). We will provide a part of the answer in this research paper. One of the main objectives of this study is to determine the best covariance estimator for both synchronized and unsynchronized high frequency data. According to the results of the simulation study, the best covariance estimator is used for real high frequency data set. The specific objectives of this study are:

a) To examine the covariance estimators ($rBPCov_t^{\Delta}$, $rCov_t^{\Delta}$, $rHYCov_t^{\Delta,\theta}$, $rThresholdCov_t^{k,h}$, $rTSCov_t^{\Delta,k}$ and $rRTSCov_t^{\Delta,k,\theta}$) for both synchronized and unsynchronized high-frequency data.

b) To compare the covariance estimators' performances basing on measures of fit i.e. Relative bias and Root mean squared error (RMSE) for both simulations of synchronized and unsynchronized high-frequency data.

c) To examine the impact of jumps on the model on all of the six covariance estimators for both synchronized and unsynchronized high-frequency data.

d) Using the obtained results on covariance estimators on real world high-frequency data we explore which type of an asset is a better safe haven. Negative correlation with general market indicates safe haven characteristics as in Kaczmarek et al. (2022).

The rest of the paper is structured as follows. The next section, (2), provides the theoretical background on the covariance estimators and discusses the reviewed literature on covariance estimators as well as recent literature on safe haven assets. Section (3) discusses the methodology of the simulation design, and analysis of the measure accuracy of each estimator. Section (4) discusses the results on the realized covariance estimator based on simulated high-frequency data. Section (5) gives practical application of realized covariance estimator benchmark on real world high-frequency data. The last section concludes.

## 2. RELATED LITERATURE OVERVIEW

In this paper, we investigate which currencies are better safe havens using the methodology of best-realized covariance estimator by comparing three FX currencies and Bitcoin cryptocurrency. As the cryptocurrency market is ever-evolving, including Bitcoin as a cryptocurrency representative in our analysis was crucial. Ozer-Imer and Ozkan (2014) studied the behavior of major reserve currencies such as the U.S. Dollar, the Euro, the Japanese Yen, and the British Pound, as well as other currencies like the Swiss Franc and the Australian Dollar during times of distress such as the global financial crisis of 2008. Performing statistical analysis on a comprehensive dataset to assess the safe haven properties of different currencies, they concluded that there is an inverse relationship between volatility and the duration of the crisis. Cho and Han (2021) investigate whether some currencies maintain their value during market stresses. Their research showed that currencies from countries with better macroeconomic conditions have better characteristics of safe haven currencies, especially the Swiss Franc and the Japanese Yen. Kim and Lee (2023) focused on trying to find currencies that exhibit safe haven characteristics during times of financial turmoil. They concluded that safe haven currencies, the U.S. Dollar, the Japanese Yen, the Swiss Franc, and the British Pound, were protected from the shocks during market distress. This is the reason we included these currencies into our analysis. Gozbasi et al. (2021) examines whether Bitcoin and cryptocurrencies preserve value during market stresses. They also contribute to the understanding of whether cryptocurrencies can be safe haven currencies. Karim et al. (2022) highlight that cryptocurrencies are very important for investors because they help to diversify risk in portfolios containing standard currencies in times of market uncertainty. Defined by their ability to retain or appreciate in value amid crises, these currencies exhibit low or negative correlations with equities and other risk assets, providing stability during downturns Tronzano (2023), Feder-Sempach et al. (2024). Political stability, low inflation and generally good macroeconomic conditions underpin the safe haven status of currencies Feder-Sempach et al. (2024), Feng et al. (2024). Research has shown that including safe haven currencies in portfolios reduces Value-at-Risk (VaR) by 10–30% during crises Tronzano (2023), Feng et al. (2024). On the other hand, hedging outcomes depend on the geographic area. Safe haven characteristics are dependent on time and type of crisis. As an example, the JPY proved to be a good hedge in the Global Financial Crisis but not so effective in the Eurozone Debt Crisis Tronzano (2023), Feder-Sempach et al. (2024). Emerging markets exhibit different dynamics as oil exporting countries benefit more from crude oil as a hedge, while developed countries depend on traditional currencies Feng et al. (2024), Liu et al. (2020). In one of the studies, although it is said to be a safe haven digital currency, Bitcoin is only a good short-term hedge for the USD according to, Conlon et al. (2024). A similar pattern has been shown for the Swiss Franc, where they are good hedges in portfolios but only in the short term, while in long crises they show unstable ratio hedges, Feng et al. (2024). Even though recent literature has concluded that specifically, Bitcoin and the Swiss Franc have characteristics of a safe haven compared to traditional assets such as stocks and bonds, there is no resolution on which asset is a "more" safe haven. This is what motivated us to investigate which asset is a better safe haven. This research can be valuable for investors, policymakers as well as portfolio managers. Policymakers and portfolio managers must prioritize dynamic, data-driven strategies to harness their full potential. Boudt and Zhang (2015) propose a two-time scale approach where the short-term volatility captures the impact of jumps, while the long-term volatility captures the underlying continuous price dynamics. Using the Robust two-times scaled estimator, they are estimating the covariance matrix of asset returns that is robust to the presence of jumps. Barndorff-Nielsen and Shephard (2004) introduced the concept of realized covariation as an alternative measure of covariance for financial assets. The authors propose a method for estimating the realized covariation, the sum of the squared high-frequency returns for two assets. They used realized covariance between the returns of two assets in order to calculate covolatility. Their paper also discussed applications in the finance industry, such as the benefits of using realized covariation in volatility forecasting and portfolio optimization. Palandri (2006) focused on addressing the challenges of estimating realized covariance when working with high-frequency financial data that is asynchronous and contaminated by market microstructure noise. They propose a consistent covariance estimator that builds on the covariance estimators of Hayashi and Yoshida (2005) and Corsi et al. (2010). The pseudo-aggregation is introduced to deal with asynchronous data. Hayashi and Yoshida (2005) introduced a method to align the non-synchronous observations to a standard time grid. This alignment allows for a consistent covariance matrix estimation by ensuring that the observations are effectively synchronized. For these reasons we use the robust covariance estimation to examine the safe haven characteristics of analysed currencies.

## 3. METHODOLOGY

The initial goal of this research is to define and discover the best covariance estimator for synchronous and asynchronous high-frequency data. In some real-world scenarios, data or events may not occur synchronously due to various reasons such as delays or processing time variations. Asynchronous simulation allows us to model and analyze such situations more accurately. Our research provides a detailed analysis and structurally two-part process of determining the best covariance estimator. Firstly, we employ a simulation design to provide a comprehensive assessment of the accuracy of each estimator. Secondly, we use the conclusions of the simulation results to analyze high-frequency real-world data (see Appendix for descriptive statistics of observed real-world data). The simulation part allows us to have controlled conditions where we can manipulate variables to better assess the effectiveness of each estimator. Also, it enables us to establish benchmarks for comparison with real-world data.

We start this study with the analysis of the role of the Swiss Franc (CHF) as a dynamic hedge for a diversified global investment portfolio. The portfolio consists of U.S. equities (S&P 500), European equities (DAX), emerging markets (EM), Bitcoin (BTC), and gold. We used historical daily price data from January 1, 2020, to the present and calculated daily logarithmic returns for each asset. The portfolio was constructed using fixed weights: 25% S&P 500, 25% DAX, 20% EM, 20% Bitcoin, and 10% gold. To model a dynamic hedge, we used a rolling 60-day window to estimate hedge ratio between the portfolio returns and CHF returns. The hedge ratio was computed as the negative covariance between the portfolio and CHF returns divided by the variance of CHF returns over the same window. This dynamic hedge ratio was updated daily and applied to the portfolio to create a hedged return series. The 60-day rolling window is taken because the balanced period is long enough to generate a stable covariance while remaining sensitive to short-term market changes. As seen in Figure 1 the hedged and unhedged portfolios were plotted together for comparison purposes. The results show that the hedged portfolio achieved higher cumulative returns and lower volatility compared to the unhedged portfolio. This shows that the application of dynamic hedging improves portfolio conditions in the observed period.

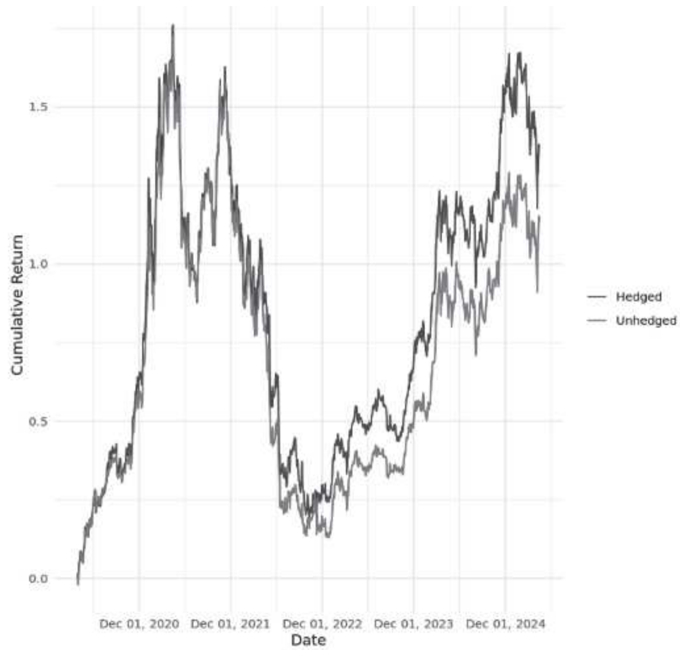


**Figure 1.** Comparison of Cumulative Returns between Hedged and Unhedged Portfolios

To ensure the accuracy of our simulations, we follow a methodology similar to the one proposed by Boudt and Zhang (2015) and Barndorff-Nielsen et al. (2011). We simulate for days with a 1-second increment between finite data samples. Figure 2 shows a 10-day subsample of the simulated data, visually representing how the data is structured and how each estimator is evaluated. The simulation design enables us to compare the performance of each estimator in different scenarios and to identify any potential biases or limitations in their accuracy. This approach ensures that the results of our research are robust and reliable.

$$\begin{aligned} d\tilde{X}_{1t} &= \gamma_{x_1}\sigma_t^{x_1}dB_t^{x_1} + \sqrt{1-\gamma_{x_1}^2}\sigma_t^{x_1}dW_t + dZ_t^{x_1}, \\ d\tilde{X}_{2t} &= \gamma_{x_2}\sigma_t^{x_2}dB_t^{x_2} + \sqrt{1-\gamma_{x_2}^2}\sigma_t^{x_2}dW_t + dZ_t^{x_2}, \\ \sigma_t^{x_1} &= e^{(\beta_0+\beta_1 v_t^{x_1})}, \qquad dv_t^{x_1} = \alpha v_t^{x_1}dt + dB_t^{x_1}, \\ \sigma_t^{x_2} &= e^{(\beta_0+\beta_1 v_t^{x_2})}, \qquad dv_t^{x_2} = \alpha v_t^{x_2}dt + dB_t^{x_2} \end{aligned} \quad (1)$$

In this study, we are dealing with a system of stochastic differential equations. The equations are represented by $\sigma_t^{x_1} = e^{(\beta_0+\beta_1 v_t^{x_1})}$ and $dv_t^{x_1} = \alpha v_t^{x_1}dt + dB_t^{x_1}$ for $X_1$ as well as $\sigma_t^{x_2} = e^{(\beta_0+\beta_1 v_t^{x_2})}$ and $dv_t^{x_2} = \alpha v_t^{x_2}dt + dB_t^{x_2}$ for $X_2$. We use Boudt and Zhang (2015) assumptions that include $B_{X_1} \perp B_{X_2}, B_{X_1} \perp W$, and $B_{X_2} \perp W$. They assumed that the factors of asset groups $X_{it}, i = 1,2$ are independent of each other and each is also independent of market wide shock *W*. Moreover, the parameters $(\beta_0, \beta_1, \alpha, \gamma_{x_1}, \gamma_{x_2})$ are set to $(\frac{5}{16}, \frac{1}{8}, \frac{1}{40}, 0.3, 0.3)$. The initial value of $v_t^{x_1}$ and $v_t^{x_2}$ for each day is drawn from je normal distribution $N(0,\frac{-1}{2\alpha})$. The spot correlation between the continuous part of the log-price changes is $\rho = 0.91$. To simulate the independent noise, we assume that $\varepsilon_t^{X_i} \sim N\left(0, \sigma_t^{x_i}\right), i =$ . The system of differential equations can be simulated using the Euler scheme with an increment of 1 second per tick, as described in Boudt and Zhang (2015). It is worth noting that we aim to simulate the behavior for

$$X_{it} = \tilde{X}_{it} + \varepsilon_t^{X_i}, i = 1,2.$$

In order to find a solution for the system of differential equations presented in equation (1) we rely on the variables $X_{1t}$ and $X_{2t}$. We followed the methodology from Boudt and Zhang (2015). One method for obtaining a solution through the application of Ito's lemma. Ito's lemma is applied to transform the original variables so that the joint system of stochastic differential equations can be rewritten in terms of functions whose drift and diffusion components become tractable. This allows the system to be integrated explicitly, yielding closed-form solutions. This technique allows us to rewrite the original equations in a way that is more amenable to finding a solution. By utilizing this approach, we can arrive at solution that is both accurate and reliable.

$$\tilde{X}_{it} = \tilde{X}_0 + \gamma_{x_i}\sigma_t^{x_i}B_t^{x_i} + \sqrt{1-\gamma_{x_i}^2}\sigma_t^{x_i}W_t + Z_t^{x_i}, i = 1,2 \qquad (2)$$

with $B_t^{x_i} \sim N(0,t), W_t \sim N(0,t)$ and $Z_t^{x_i} \sim \text{Poisson}(t)\,, Z_0^{x_i} = 0, i = 1,2.$

In order to replicate the values of $X_{it}, i = 1,2,$ we employ several stochastic processes. Specifically, we utilize $B_t^{x_i}, i = 1,2,$ which conforms to a normal distribution characterized by mean of zero and a standard deviation equal to the square root of *t* Additionally, we incorporate $W_t$ which is also a normal distribution with a mean of zero and a standard deviation of *t* Finally, we consider $Z_0^{x_i}, i = 1,2$ which follows a Poisson distribution with parameter value of *t* . It is important to note that $Z_0^{x_i}, i = 1,2$ is set to zero as an initial condition. By leveraging these stochastic processes, we are able to effectively simulate $X_{it}, i = 1,2$ with high degree of accuracy.

$$X_{it} = \underbrace{\tilde{X}_0 + \gamma_{x_i}\sigma_t^{x_i}B_t^{x_i} + \sqrt{1-\gamma_{x_i}^2}\sigma_t^{x_i}W_t + Z_t^{x_i}}_{\tilde{X}_{it}} + \varepsilon_t^{X_i}, i = 1,2 \qquad (3)$$

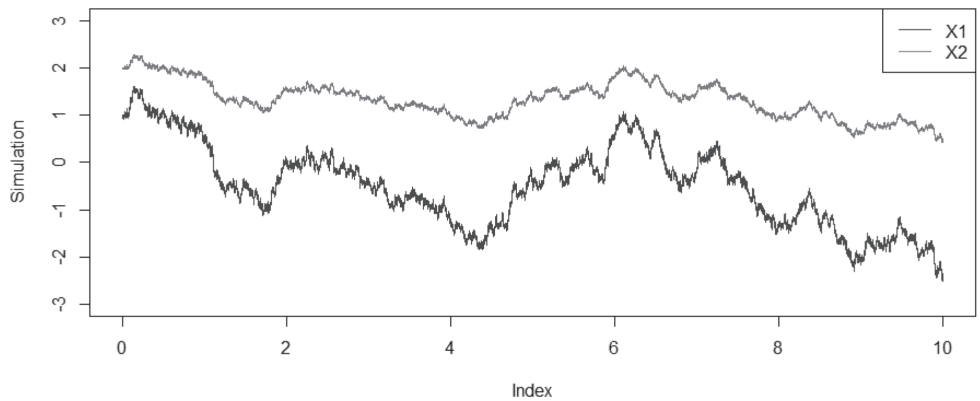


**Figure 2.** Simulation of high-frequency data, 10-day subsample

In the simulation study, the estimators from Čuljak et al. (2022) realized covariance version, as seen in (4), are compared to determine if the proposed has the best accuracy among alternative competitors of realized covariance. Realized covariance estimators are analyzed in this research as they can handle non-synchronous trading by considering all available intra-day observations for each asset, even if they do not correspond to the same time points. Also, they allow for incorporating more granular information, i.e., high-frequency data.

$$rCov_t^{\Delta} = \sum_{i=1}^{n_t} r_{t_ih}^2, h = 1,2$$

$$rBPCov_t^{\Delta} = \frac{\pi}{2}\sum_{i=2}^{n_t} |r_{t_{i-1}h}||r_{t_ih}|, h = 1,2$$

$$rTSCov_t^{\Delta,k} = \frac{1}{1-\frac{n_t-k+1}{n_tk}}\left(\frac{1}{k}\sum_{j=1}^{k}\sum_{i=1}^{n_t} r_{t_{ij}h}^2 - \frac{n_t-k+1}{n_tk}\sum_{i=1}^{n_t} r_{t_ih}^2\right), h = 1,2$$

$$rRTSCov_t^{\Delta,k,\theta} = \frac{c_\theta}{1-\frac{n_t-k+1}{n_tk}}\left(\frac{1}{k}\sum_{j=1}^{k}\sum_{i=1}^{n_t} r_{t_{ij}h}^2 I_i(\theta) - \frac{n_t-k+1}{n_tk}\sum_{i=1}^{n_t} r_{t_ih}^2 I_i(\theta)\right), h = 1,2 \qquad (4)$$

$$rHYCov_t^{\Delta,\theta} = \sum_{i=1}^{n_t} r_{t_ih}^2 I_i(\theta), h = 1,2$$

$$rThresholdCov_t^{k,h} = \sum_{i=1}^{M} r_{(k)t_i} I(TR_M) r_{(\mathrm{h})t_i} I(TR_M)$$

Six realized covariance estimators are examined in this research paper in order to determine which can be used as a benchmark. They are presented in (4). Threshold covariance $(rThresholdCov_t^{k,h})$ estimator incorporates univariate jump detection rules to mitigate the impact of jumps on the covariance estimate. By utilizing these rules, it aims to reduce the influence of jumps on the estimated covariance matrix. This approach allows the $rThresholdCov^{k,h}{}_t$ estimator to remain feasible even in high-dimensional settings.

However, it is important to note that the estimator may be less robust to capturing small co-jumps, which are simultaneous jumps or changes in multiple variables. The threshold value $TR_M$ is set to $9\beta^{-1}$times the daily realized bipower variation of an asset $k$, as suggested in Jacod and Todorov (2009), where $M$ is the number of intraday returns. Realized covariance estimator $(rCov_t^\Delta)$calculates realized covariance between two assets by multiplying the realized variances of the two assets with their corresponding cross-product of returns. The estimator is robust to the microstructural noise depending on the sampling frequency on a slow and fast time scale. It is important to choose an appropriate time window or sampling frequency, as different time scales may provide different levels of information and sensitivity to noise. Price jumps contaminate the estimations with $rCov_t^\Delta$. Therefore modifications of the $rCov_t^\Delta$ have been introduced, like Realized bipower covariance estimator $rBPCov_t^\Delta$. The advantage of $rBPCov_t^\Delta$ is that it provides a consistent estimate of the true covariance, even in the presence of price jumps. It is robust to price jumps but is still affected by microstructural noise and non-linear dependencies between asset returns. $rBPCov_t^\Delta$ takes into account both the sign and magnitude of the price changes as it calculates the realized bipower covariance estimate by summing the absolute product of consecutive returns for each asset $h$, multiplied by a constant term $(\pi/2)$. In order to avoid lack of data but still remain unbiased and asymptotically consistent estimator the Two times scaled covariance estimator was introduced ($rTSCov_t^{\Delta,k}$). As seen in the equation (4) it consists of two parts. First, it represents the average of the squared values of $r_{tijh}$over $k$ iterations and $n_t$ observations at time *t*. Second part represents a scaled version of the squared values of $r_{tijh}$ over $n_t$ observations at time *t*. This term is used to adjust for bias in the estimator. Therefore it is robust to microstructural noise and by including the scaling factors in the estimator, the bias can be reduced. Due to not being jump robust the modification of $rTSCov_t^{\Delta,k}$ was introduced. The Robust two times scaled covariance estimator ($rRTSCov_t^{\Delta,k,\theta}$) is a robust version of Two times scaled covariance estimator. It is microstructural noise and jump robust. First term of the equation, as seen in (4) represents the sum of squared returns of the $h$-th asset, weighted by the indicator function $I_i(\theta)$, across the $k$ historical observations. Second term represents the sum of squared returns of the *h*-th asset, weighted by the indicator function $I_i(\theta)$, across all $n_t$ observations, where $\frac{n_t-k+1}{n_t k}$ factor accounts for the bias correction. Last realized covariance estimator for comparison is Hayashi-Yoshida covariance estimator ($rHYCov_t^{\Delta,\theta}$ introduced by Hayashi and Yoshida (2005). As presented in (4), the $rHYCov_t^{\Delta,\theta}$ uses an indicator function $I_i(\theta)$ in order to truncate the effects of price jumps. Due to usage of real world 1 minute high-frequency data in second phase of this research, for the simulation study, we set fast time scale at 1 and slow time scale at 20 for the $rTSCov_t^{\Delta,k}$ and $rRTSCov_t^{\Delta,k,\theta}$ realized covariance estimators. As we simulated 1 second data the fast time scale is set as 1 and slow time scale was defined by the similar design in Čuljak et al. (2022) where optimal sampling frequency on a slow time scale was evaluated and recommended.

## 4. RESULTS

In the first part the realized covariance estimators are analyzed based on simulated high-frequency data. The frequency of the simulated data is 1 second for $S = 100$ days. The observed scenarios are presented in the Table 1 and Table 2, respectively. First the synchronized high-frequency data is used to analyze the accuracy of the estimators. Synchronized means that the data was used immediately after the simulation without tempering with the frequency between two time series $X_1$ and $X_2$ using the default frequency of 1 second between the observations. The relative bias as $\text{Relative bias} = \frac{1}{S}\sum_{i=1}^{S} \frac{\widehat{\mathrm{IC}}_i - \mathrm{IC}_i}{\mathrm{IC}_i}$ and Root mean squared error as $\mathrm{RMSE} = 100\sqrt{\frac{1}{S}\sum_{i=1}^{S} \left(\widehat{\mathrm{IC}}_i - \mathrm{IC}_i\right)^2}$ are calculated for each estimator in order to measure accuracy as used in Boudt and Zhang (2015) and Čuljak et al. (2022). As presented in the Table 1 the lowest relative bias had the Robust two times scaled estimator ($rRTSCov_t^{\Delta,k,\theta}$) in case when there were no jumps present in the simulated data. In the presence of jumps the values of relative bias increase showing that the accuracy of estimators decreases. The increase of bias due to jumps is apparent from the results in Table 1. It is apparent that all of the estimators are underestimating the true value of integrated covariance $\mathrm{IC}_i$. When there are no jumps present in the simulated high-frequency data the results show that the bias is less present with estimators $rBPCov_t^{\Delta}$, $rCov_t^{\Delta}$, $rHYCov_t^{\Delta,\theta}$, and $rThresholdCov_t^{k,h}$. The Robust two times scaled estimator ($rRTSCov_t^{\Delta,k,\theta}$) which is robust to price jumps has shown not much of an increase in bias regardless of the intensity of the jumps in the simulated data.

Results in Table 2 indicate that the Relative bias is impacted by jumps, as Relative bias estimates increase with an increase or inclusion of jumps for both simulations of synchronized and unsynchronized high-frequency data. This result is the same with other measures of fit like Root mean squared error (RMSE) though the associated estimated for RMSE for unsynchronized high-frequency data with or without jumps do not differ very much. However, this is not a surprise since in general the Relative bias and RMSE as statistical properties and measures of fit are different quantitatively. The asynchronicity in the high-frequency data we consider is motivated by the work of Hansen and Lunde (2005). Hansen and Lunde (2005) considered two independent Poisson process sampling schemes to generate the times of the actual observations, the same simulation design we followed. There is no evidence that including jumps improves the covariance estimators in terms of lowering RMSE, as seen in both synchronized and unsynchronized high-frequency data, the covariance estimators' RMSE increased as jumps are included on the model. We would opt for other measures of fit like Multiple R-squared and Adjusted R-squared that may give more statistical evidence as we include jumps in the model. Statistically the closer the Multiple R-squared and Adjusted R-squared of the model to 1, the better the model.

We analyzed the six covariance estimators that is $rBPCov_t^{\Delta}$, $rCov_t^{\Delta}$, $rHYCov_t^{\Delta,\theta}$,

$rThresholdCov_t^{k,h}$, $rTSCov_t^{\Delta,k}$, and $rRTSCov_t^{\Delta,k,\theta}$ for both synchronized and unsynchronized high-frequency data. Our simulation results of unsynchronized high-frequency data show that $rRTSCov_t^{\Delta,k,\theta}$ is the best covariance estimator since it has the smallest Relative bias for all cases (with or without jumps).

While for simulation results of synchronized high-frequency data, $rRTSCov_t^{\Delta,k,\theta}$ $rRTSCov_t^{\Delta,k,\theta}$ has the smallest Relative bias compared to other covariance estimators with or without jumps. This result is supportive to our hypothesis since the associated RMSE of $rRTSCov_t^{\Delta,k,\theta}$ is smaller compared to the RMSE of other covariance estimators as jumps are included in the model. These results show that $rRTSCov_t^{\Delta,k,\theta}$ is valid to be defined as a benchmark and superior to other covariance estimators.

**Table 1.** Simulation results of synchronized high-frequency data

| | **Relative bias** | **RMSE** |
|---|---|---|
| **No jumps** | | |
| $rBPCov_t^{\Delta}$ | -0.5462356 | 2.961614 |
| $rCov_t^{\Delta}$ | -0.5462497 | 2.961672 |
| $rHYCov_t^{\Delta,\theta}$ | -0.5462244 | 2.961672 |
| $rThresholdCov_t^{k,h}$ | -0.5462497 | 2.961672 |
| $rTSCov_t^{\Delta,k}$ | -0.1913763 | 2.162879 |
| $rRTSCov_t^{\Delta,k,\theta}$ | -0.1816072 | 2.154633 |
| Small jumps | | |
| $rBPCov_t^{\Delta}$ | -0.6795248 | 3.552866 |
| $rCov_t^{\Delta}$ | -0.6796252 | 3.552007 |
| $rHYCov_t^{\Delta,\theta}$ | -0.6795684 | 3.552058 |
| $rThresholdCov_t^{k,h}$ | -0.6796252 | 3.552007 |
| $rTSCov_t^{\Delta,k}$ | -0.1934863 | 3.228447 |
| $rRTSCov_t^{\Delta,k,\theta}$ | -0.1837072 | 3.224062 |
| Large jumps | | |
| $rBPCov_t^{\Delta}$ | -0.7455796 | 4.099145 |
| $rCov_t^{\Delta}$ | -0.7454552 | 4.099058 |
| $rHYCov_t^{\Delta,\theta}$ | -0.7453924 | 4.098981 |
| $rThresholdCov_t^{k,h}$ | -0.7454552 | 4.099058 |
| $rTSCov_t^{\Delta,k}$ | -0.1952669 | 3.329447 |
| $rRTSCov_t^{\Delta,k,\theta}$ | -0.1888425 | 3.325062 |

**Table 2.** Simulation results of unsynchronized high-frequency data

| | **Relative bias** | **RMSE** |
|---|---|---|
| **No jumps** | | |
| $rBPCov_t^{\Delta}$ | -1.05273 | 3.321425 |
| $rCov_t^{\Delta}$ | -1.052616 | 3.321758 |
| $rHYCov_t^{\Delta,\theta}$ | -1.05262 | 3.321736 |
| $rThresholdCov_t^{k,h}$ | -1.052616 | 3.321758 |
| $rTSCov_t^{\Delta,k}$ | -0.2568758 | 2.163979 |
| $rRTSCov_t^{\Delta,k,\theta}$ | -0.2479691 | 2.157633 |
| Small jumps | | |
| $rBPCov_t^{\Delta}$ | 1.246548 | 4.112031 |
| $rCov_t^{\Delta}$ | 1.246545 | 4.111846 |
| $rHYCov_t^{\Delta,\theta}$ | 1.247246 | 4.111849 |
| $rThresholdCov_t^{k,h}$ | 1.246545 | 4.111846 |
| $rTSCov_t^{\Delta,k}$ | 1.258619 | 2.254377 |
| $rRTSCov_t^{\Delta,k,\theta}$ | 1.295133 | 2.247213 |
| Large jumps | | |
| $rBPCov_t^{\Delta}$ | 1.598908 | 4.626791 |
| $rCov_t^{\Delta}$ | 1.599028 | 4.626593 |
| $rHYCov_t^{\Delta,\theta}$ | 1.599747 | 4.626648 |
| $rThresholdCov_t^{k,h}$ | 1.599028 | 4.626593 |
| $rTSCov_t^{\Delta,k}$ | 1.368619 | 3.427033 |
| $rRTSCov_t^{\Delta,k,\theta}$ | 1.296134 | 3.422213 |

## 5. APPLICATION

Analysing the volatility of cryptocurrencies and FX helps traders and analysts design strategies for managing risk across asset classes with a holistic view. Both cryptocurrencies and FX markets are globally accessible, with participants ranging from retail traders to institutional investors, creating constant price discovery. As a results these markets are highly sensitive to macro events and news cycles. By analysing shared sources of volatility, traders and analysts can build strategies that exploit the overlaps between cryptocurrencies and FX markets. For instance, they might use volatility indices, correlation analysis, or portfolio optimization to balance exposure to both markets. We examine which potential safe haven currency is a better safe haven asset than the observation period based on the defined benchmark Robust two times scaled estimator of covolatility and utilizing realized, covariance estimators. We estimated the proportion of negative correlation using Robust two times scaled estimator, which would be an indicator of which asset is “more” safe among the observed “safe haven” currencies. A negative correlation indicates safe haven characteristics as in Kaczmarek et al.

(2022). The Robust two times scaled estimator correlation between the asset and the general market is shown in Figures 3, 4, 5 and 6 for Bitcoin, Japanese Yen, U.S. Dollar and Swiss Franc, respectively. As presented in the Table 3 the results have shown that Bitcoin had 49.35% estimated negative correlation with the general market in observed period and Japanese Yen had 29.79%. The U.S. Dollar had 46.06%, and Swiss Franc had the highest estimated proportion of 50.46%. This estimated negative correlation clearly indicates that Bitcoin and Swiss Franc had the highest reverse movement from the general market in the observed period from June 2013 until May 2022. These results are a novelty in recent literature because of the use of benchmark realized covariance estimator – Robust two times scaled.

**Table 3.** Robust two times scaled estimator negative correlation with the general market

| | BTC | JPY | USD | CHF |
|---|---|---|---|---|
| Negative correlation | 49.35% | 29.79% | 46.06% | 50.46% |

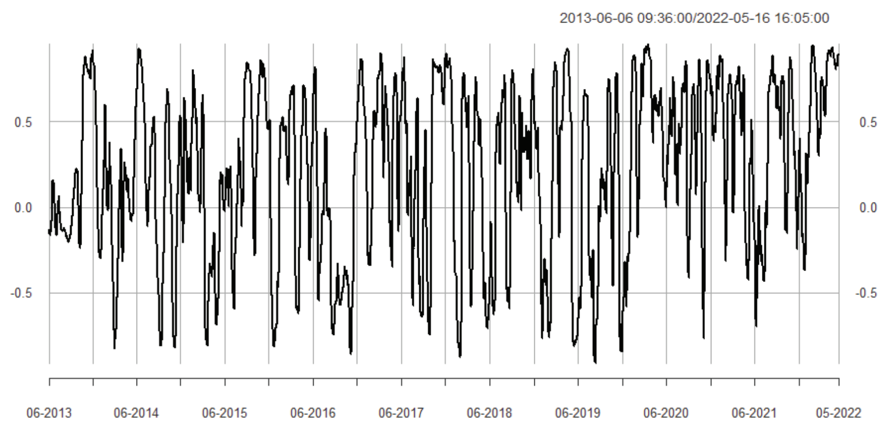


**Figure 3.** Robust two times scaled estimator correlation between Bitcoin and S&P500

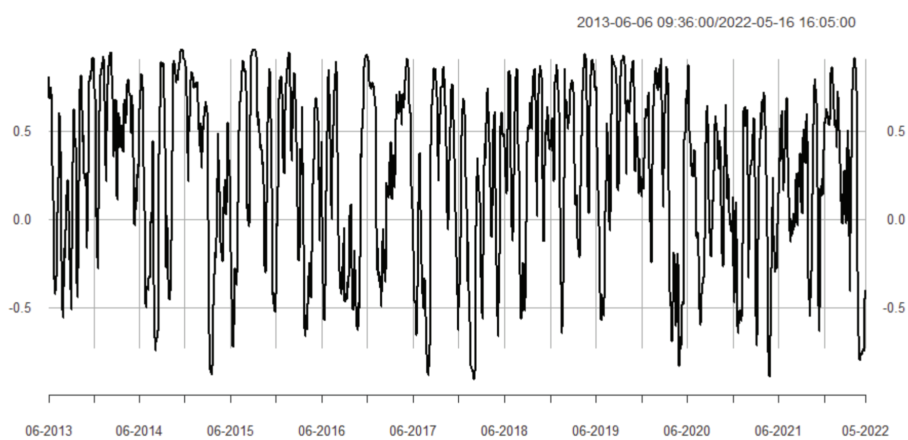


**Figure 4.** Robust two times scaled estimator correlation between Japanese Yen and S&P500

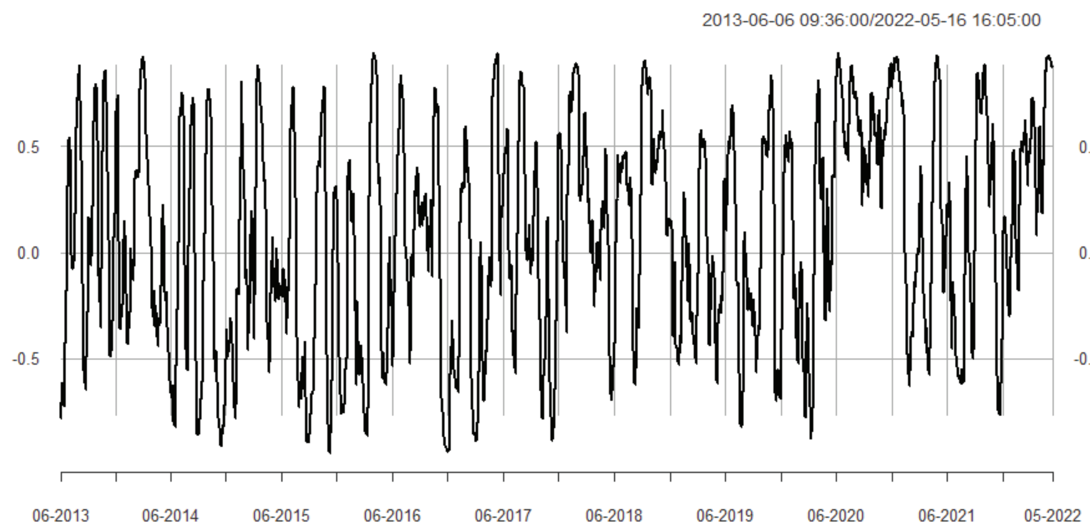


**Figure 5.** Robust two times scaled estimator correlation between U.S Dollar and S&P500

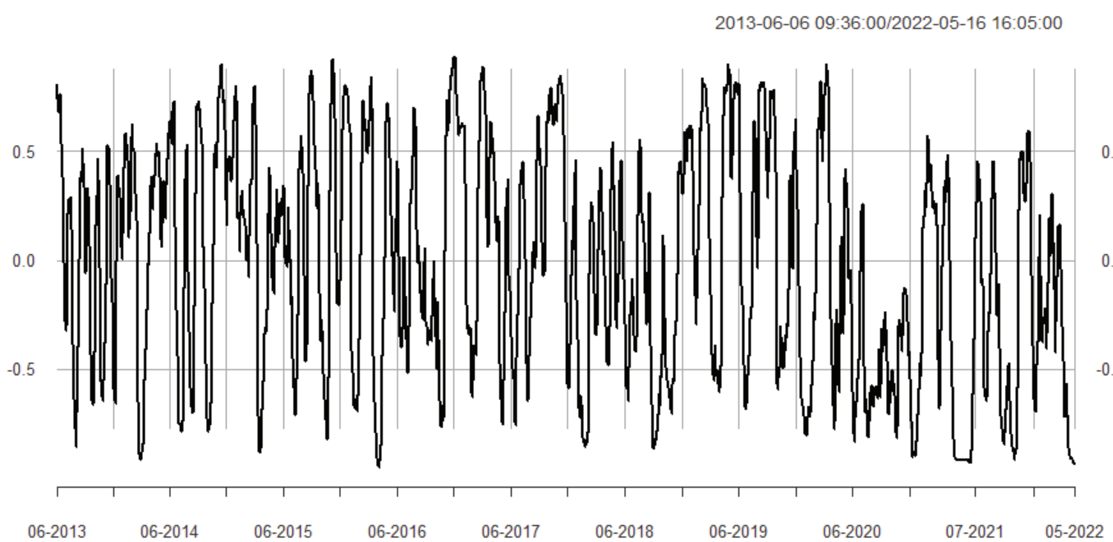


**Figure 6.** Robust two times scaled estimator correlation between Swiss Franc and S&P500

In order to determine the safe haven characteristics from the estimated negative correlation for each observed currency, we investigate further with a Chi-square test and one-tailed Z-test.

We continued with the Chi-square test in order to compare the difference in population proportions between four groups, i.e., proportions of estimated negative correlation of four different currencies with the general market (S&P500 index). We test the null hypothesis: "There is no significant difference between the observed proportions." Whereas the alternative hypothesis is: "There is at least one proportion different from others." Our test resulted in a p-value of 0.5998; therefore, we cannot reject the hypothesis on a 5% significance level. In other words, there is no statistically significant difference among the proportions we were comparing. There is no statistical evidence to conclude that the proportions differ from each other. This shows us that all of the examined currencies, including Bitcoin, have characteristics of safe haven currency, as stated in the recent literature.

In order to elaborate more, we examined further and conducted a one-tailed Z-test. We were specifically interested in determining if the estimated proportion of negative correlation for each currency is significantly greater than the null hypothesis value. The null hypothesis value is the estimated proportion of negative correlation using Robust two-times scaled estimator. We performed the one-tailed Z-test for each observed currency at a 5% significance level. The null hypothesis states that the expected proportion of negative correlation is exactly the estimated proportion of negative correlation using Robust two-times scaled estimator. The alternative view concludes that the proportion of negative correlation is significantly higher than the estimated one. The resulting p-values are presented in the Table 4.

The p-values are 0.4951 for Bitcoin, 0.9812 for the Japanese Yen, 0.951 for the U.S. Dollar, and 0.00021 for the Swiss Franc. Hence, we reject the null hypothesis in the case of the Swiss Franc, where we can conclude that the proportion is significantly higher than the specified tested value.

**Table 4.** Results of one-tailed Z-test of examining significance of estimated proportion of negative correlation

| | BTC | JPY | USD | CHF |
|---|---|---|---|---|
| p-value | 0.4951 | 0.9812 | 0.951 | 0.00021*** |

***p < 0.001, **p < 0.05, *p < 0.1

Therefore, the Swiss Franc exhibits the best safe-haven characteristics among competing alternative currencies. We have shown that cryptocurrencies, particularly Bitcoin, exhibit the characteristics of a safe haven currency compared to established safe haven currencies; it can be concluded that Bitcoin and the Swiss Franc outperform the Japanese Yen and the U.S. Dollar in terms of safe haven characteristics, as they have a higher estimated negative correlation to the general market. The results of the hedging strategy demonstrate that incorporating a dynamic hedge using a safe haven asset, such as the Swiss Franc (CHF), significantly reduces portfolio volatility while enhancing risk-adjusted returns. From an economic perspective, this suggests that hedging can be an effective tool for mitigating systemic risk, particularly in times of market stress or economic downturns. By stabilizing returns and reducing the likelihood of significant losses, hedging strategies provide investors with a more resilient portfolio, thereby offering a safeguard against unforeseen economic shocks and improving overall financial security.

## 6. CONCLUSIONS

Since the start of the 21st century, it has become harder to create the best investment portfolios and protect against market risks. We have seen a series of crises, starting with the dot-com bubble, then the 2008 financial crisis, followed by the COVID crisis, the energy crisis, and the European debt crisis. Right now, we are facing the Russo-Ukrainian War and issues in China's property market. The ongoing and serious issues are mainly caused by better links between different markets and much stronger relationships between various assets across regions or worldwide. These features show that the capital market is becoming more global and connected, which increases the risk of a serious financial crisis happening worldwide. The connections between countries make it harder to spread out investments, which reduces the chance to use hedging strategies. The impact of global financial contagion affects regular changes in investment portfolios and ways to reduce risks. It also challenges the ideas behind modern portfolio theory and the idea of spreading investments across different countries to lower risk. The unstable situation and constant worry about financial problems spreading are big reasons why more people are looking to invest in safe assets. Safe haven assets are interesting to study because they offer protection during tough financial times.

In our research, we focus on a special class of safe-haven assets, safe-haven currencies, including the most famous digital asset/currency, Bitcoin. In particular, we examine which potential safe haven currency is the best safe haven asset over the observation period based on the defined benchmark Robust two times scaled estimator of covolatility. The robust two-times-scaled covariance estimator proposed by Zhang (2011) has been repeatedly evaluated very positively in the recent literature, but no comprehensive comparison has been made between the realized covariance estimators to determine which one has the best accuracy. We used the high-frequency simulation study and analysed the reduced relative bias and root mean squared error, which showed the superiority of Zhang's Robust two times scaled covariance estimator. As a second contribution, we used the Robust two times scaled covariance estimator for a comparative analysis to determine which currency has the best safe haven characteristics over a long period of time from June 2013 to May 2022, which includes periods of severe market distress.

Our results show that cryptocurrencies, particularly Bitcoin, exhibit the characteristics of a safe-haven currency compared to established safe-haven currencies. Our analysis shows that Bitcoin and the Swiss Franc outperform the Japanese Yen and the U.S. Dollar regarding safe-haven characteristics, as they have a higher estimated negative correlation to the general market. The best-performing safe-haven currency is the Swiss Franc, using the estimated correlation with the benchmark Robust two-times scaled estimator. The findings from our study provide valuable guidance for portfolio managers, policymakers, and retail investors. The presented results can give a clear perspective on the macro implications of cryptocurrencies and whether the domestic institutions decide to regulate their status to guard sovereign currencies. The widespread adoption of cryptocurrencies could minimize the effectiveness of monetary policy in the long run but increase availability and usability for retail investors. Also, this research provides insight into investment opportunities for retail investors by introducing cryptocurrencies into their portfolios. Even though our findings contribute to the existing literature on the topic of safe haven assets, the main limit of this research is the use of only one cryptocurrency over a specified period as well as the omission from analysis of other assets that could be considered as "safe havens" such as gold and energy. Further research should investigate hedging portfolios with cryptocurrencies as opposed to just fiat currencies and include another type of non-traditional safe-have haven assets. The application of a dynamic hedging strategy utilizing the Swiss Franc (CHF) as a hedge asset has shown promising results in reducing portfolio volatility and enhancing risk-adjusted returns. This analysis underscores the value of CHF as a safe haven currency, particularly in periods of market uncertainty and economic turbulence. By integrating a hedge with a stable, negatively correlated asset, investors can mitigate risks associated with broader market fluctuations, ultimately contributing to more stable and resilient portfolio performance. The findings highlight the potential for currency-based hedging strategies to serve as an effective risk management tool in global investment portfolios.

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## APPENDIX

**Table 1.** Descriptive statistics of real world high-frequency dataset

| | Min | 1st Qu | Median | Mean | 3rd Qu | Max |
|---|---|---|---|---|---|---|
| BTC | 1.06 | 653.93 | 6488.12 | 12348.58 | 11407.83 | 68997.76 |
| JPY | 75.6 | 92.13 | 106 | 102.22 | 111.32 | 131.33 |
| USD | 1.034 | 1.126 | 1.203 | 1.229 | 1.328 | 1.514 |
| CHF | 0.71 | 0.9224 | 0.9635 | 0.9648 | 0.9938 | 1.1958 |
| S&P500 | 666.8 | 1359.7 | 2065.9 | 2221.4 | 2802.7 | 4817.7 |