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Advanced Decision-Making with Interval-Valued Probabilistic Linguistic Fuzzy Sets

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Abstract: Decision-making under uncertainty is a fundamental challenge in various fields, requiring methods that effectively capture probabilistic uncertainty and decision-maker hesitancy. This study introduces Interval-Valued Probabilistic Uncertain Linguistic *q*-Rung Orthopair Fuzzy Sets (IVPULq-ROFSs), a novel framework that extends existing fuzzy set models by incorporating interval-valued probabilities and linguistic hesitancy. Based on this model, we develop two advanced Multi-Attribute Group Decision-Making (MAGDM) methods: (1) an aggregation-based approach, and (2) a TODIM-based decision method. To validate the effectiveness of the proposed methods, we apply them to autonomous vehicle sensor configuration selection, demonstrating enhanced decision robustness and reliability. The results indicate that IVPULq-ROFSs outperform conventional fuzzy models in handling complex and uncertain decision scenarios. Future research will explore algorithmic optimization, real-time applications, and integration with deep learning models for enhanced decision intelligence.

Keywords: autonomous vehicle sensor configurations; hesitancy degrees; interval-valued probabilistic uncertain linguistic *q*-rung orthopair fuzzy sets; multi-attribute group decision-making

1 INTRODUCTION

Decision-making is an integral part of everyday life, essential for individuals, groups, and organizations across various contexts. Whether it involves resource allocation in industry, supplier selection in commerce [1, 2], or disease diagnosis in healthcare [3, 4], decision-making fundamentally entails evaluating, ranking, and ultimately choosing the optimal solution from a set of alternatives using specific methodologies or techniques. Depending on the approach used, decision-making can be categorized into Bayesian decision theory [5], Markov decision process [6], multi-objective decision-making [7], and multi-criteria decision-making [8]. These methods have been widely applied in real-world scenarios, significantly enhancing decision-making efficiency and intelligence across different sectors.

For example, Zou [9] established a hybrid decision-making model integrating soft probability and Bayesian decision theory to improve decision support in medical diagnosis when exact probabilities are unavailable. Zhu et al. [10] introduced a mean-field Markov decision process model to optimize decision-making in ride-sourcing markets by using spatial-temporal subsidies to address supply-demand imbalance, allowing the platform to balance revenue and service rate while enabling drivers to make income-maximizing decisions. Tansar et al. [11] developed a multi-objective decision-making framework to optimize the implementation of green-grey infrastructures for urban flood damage reduction and urban drainage system resilience. Lo et al. [12] proposed a two-stage multi-criteria decision-making approach to simultaneously address sustainable supplier selection and transportation planning optimization in multi-level supply chain networks.

In recent years, multi-attribute group decision making (MAGDM) has gained increasing attention from researchers due to its ability to comprehensively assess alternatives across multiple dimensions [13, 14]. This approach offers broader applicability and aligns more closely with real-world decision-making scenarios. Typically, MAGDM involves a group of decision makers (DMs) evaluating a set of alternatives by analysing them from various attribute perspectives and expressing their judgments in a structured manner. These individual assessments are then aggregated using suitable methods to derive an overall evaluation. Due to its advantages, MAGDM has been widely applied in diverse fields, including risk assessment [15, 16], charging station site selection [17, 18], and so on.

However, despite the progress made in MAGDM, existing fuzzy set theories still face limitations in handling certain complex decision-making environments. To address this gap, this paper proposes a novel fuzzy set theory, interval-valued probabilistic uncertain linguistic *q*-rung orthopair fuzzy sets (IVPULq-ROFSs), that enhances the expression of expert decision opinions in MAGDM problems. The primary contributions of this paper are as follows: (1) This paper proposed a new tool for depict decision makers' evaluation information, called interval-valued probabilistic uncertain linguistic *q*-rung orthopair fuzzy sets (IVPULq-ROFSs). IVPULq-ROFSs leverage the advantages of existing fuzzy sets and demonstrate significant superiority in expressing expert decision opinions within the context of complex MAGDM problems. (2) This paper proposes the concept of IVPULq-ROFSs and proposed two novel MAGDM methods. (3) Finally, the two methods are used to solve a realistic MAGDM problem so that the validity of two methos is proved.

The rest of this paper is organized as follows. Section 2 recalls the related works of the MAGDM under uncertain conditions. Section 3 proposes the definition of IVPULq-ROFSs and discusses their properties. Section 4 proposes two novel MAGDM methods based on IVPULq-ROFSs. Section 5 uses the two methods to solve a real decision-making problem. Conclusions and future research can be found in Section 6.

2 RELATED WORKS

2.1 Decision-Making under Complex and Uncertain Conditions

Numerous methods are proposed to improve decision-making under complex and uncertain conditions. Researchers have developed various MAGDM models by integrating fuzzy set theories, aggregation operators, and consensus measures to enhance decision-making performance. Arya & Kumar [19] constructed a MAGDM model by integrating q-rung orthopair fuzzy entropy and divergence measures with a TODIM-VIKOR approach to address supplier selection problems in supply chain management, ensuring reliable and effective decision-making under uncertainty. Li & Qiao [20] established a MAGDM model to address decision-making under incomplete information, with an application to healthcare waste treatment technology selection. Sun & Ma [21] introduce a multi-attribute group decision-making (MAGDM) model for linguistic preference relations by defining similarity-based consensus measures and an acceptance consensus threshold, enabling the evaluation of individual and collective preference consistency to improve group decision-making outcomes.

2.2 MAGDM Methods under Fuzzy Decision-Making Circumstances

Recently, some new MAGDM methods under fuzzy decision-making circumstances have been proposed. Farid et al. [22] introduced the concept of q-rung orthopair fuzzy rough sets, investigated their aggregation operators, based on which a novel MAGDM method has been developed. The performance of the new MAGDM method is illustrated by solving an urban climate change policies evaluation problem. He et al. [23] studied the MAGDM method based on hesitant triangular fuzzy weighted average operator and applied it in selecting a proper green retailer. Under linear diophantine fuzzy sets, Asif et al. [24] developed a series of aggregation operators based on Aczel-Alsina. Seikh & Chatterjee [25] investigated MAGDM problems under level-based interval-valued spherical fuzzy sets, introduced a combined decision-making method, i.e., MEREC-VIKOR. The combined method was successfully applied in evaluation of sustainable strategies for electric vehicle adoption. Wang [26] introduced a Pythagorean cubic fuzzy sets-based MAGDM method, and it is used for solving a supplier chain management problem. Javed et al. [27] introduced a series of T-spherical fuzzy Heronian mean operators based on Dombi t-norm and t-conorm, and used them in solving a post-flood road rehabilitation problem. The above mentioned publications have revealed the good performance of fuzzy sets and their extensions in representing decision makers' evaluation values in complicated decision-making environment. Meanwhile, some other fuzzy sets have been widely applied in AMGDM problems, such as circular intuitionistic fuzzy sets [28], complex q-rung orthopair fuzzy sets [29], linguistic Z-numbers [30], etc. All these fuzzy sets theories have good performance in representing decision makers' evaluation information in MAGDM. However, there are some circumstances that these theories cannot handle.

The above studies highlight the effectiveness of fuzzy sets and their extensions in representing decision-makers' evaluations in complex decision-making environments. Despite these advancements, existing fuzzy set theories still encounter challenges in adequately capturing expert evaluation information in highly complex MAGDM scenarios. To bridge this gap, this paper introduces IVPULq-ROFSs, a new fuzzy set approach that offers a more precise representation of decision-makers' opinions. By leveraging the strengths of existing fuzzy set methods while addressing their limitations, this new approach aims to enhance decision-making accuracy and robustness in MAGDM applications.

3 INTERVAL-VALUED PROBABILISTIC UNCERTAIN LINGUISTIC *Q*-RUNG ORTHOPAIR FUZZY SETS

Let X be a fixed set and $\hat{S} = \{s_\delta | \delta = 1, 2, ..., t\}$ be a continuous linguistic term set (LTS) with odd cardinality. Then the interval-valued probabilistic uncertain linguistic q-rung orthopair fuzzy set (IVPULq-ROFS)

$$\hat{D} = \left\{x, \hat{h}_{\hat{D}}(x) \middle| \hat{p}_{\hat{D}}(x), \hat{g}_{\hat{D}}(x) \middle| \hat{v}_{\hat{D}}(x) \middle| x \in X\right\} \tag{1}$$

where $\hat{h}_{\hat{D}}(x)$ and $\hat{g}_{\hat{D}}(x)$ represents the two uncertain linguistic variables on $\hat{S}$, denoting the member degree (MD) and non-member degree (NMD) of the element $x \in X$. In addition, $\left[\xi^L, \xi^U\right] \in \hat{h}_{\hat{D}}(x)$, $\left[\zeta^L, \zeta^U\right] \in \hat{g}_{\hat{D}}(x)$, $\left(\xi^U\right)^+ = \cup_{\left[\xi^L, \xi^U\right] \in \hat{h}_{\hat{D}}(x)} \max\left(\xi^U\right)$ and $\left(\zeta^U\right)^+ = \cup_{\left[\zeta^L, \zeta^U\right] \in \hat{g}_{\hat{D}}(x)} \max\left(\zeta^U\right)$. Moreover, $\hat{p}_{\hat{D}}(x) = \left[\hat{p}^L_{\hat{D}}(x), \hat{p}^U_{\hat{D}}(x)\right]$, $\hat{v}_{\hat{D}}(x) = \left[\hat{v}^L_{\hat{D}}(x), \hat{v}^U_{\hat{D}}(x)\right]$ are two collection interval values, denoting the interval-valued probabilistic information of $\hat{h}_{\hat{D}}(x)$ and $\hat{g}_{\hat{D}}(x)$, satisfying that $\hat{p}^L_{\hat{D}}(x) = \inf\left(\hat{p}_{\hat{D}}(x)\right)$, $\hat{p}^U_{\hat{D}}(x) = \sup\left(\hat{p}_{\hat{D}}(x)\right)$, $\hat{v}^L_{\hat{D}}(x) = \inf\left(\hat{v}_{\hat{D}}(x)\right)$ and $\hat{v}^U_{\hat{D}}(x) = \sup\left(\hat{v}_{\hat{D}}(x)\right)$. Furthermore, the IVPULq-ROFS $\hat{D}$ should satisfy that

$$\begin{aligned} & 0 \le \xi^L, \xi^U, \zeta^L, \zeta^U \le t, \left(\left(\xi^U\right)^+\right)^q + \left(\left(\zeta^U\right)^+\right)^q \le t^q \\ & \hat{p}_{\hat{D}}(x), \hat{v}_{\hat{D}}(x) \subseteq [0, 1] \\ & \sum_{\left[\hat{p}^L_{\hat{D}}(x), \hat{p}^U_{\hat{D}}(x)\right] \in \hat{p}_{\hat{D}}(x)} \hat{p}^U_{\hat{D}}(x) = 1, \\ & \sum_{\left[\hat{v}^L_{\hat{D}}(x), \hat{v}^U_{\hat{D}}(x)\right] \in v_{\hat{D}}(x)} \hat{v}^U_{\hat{D}}(x) = 1 \end{aligned} \tag{2}$$

For convenience, the order pair $\hat{d} = \left(\hat{h}_{\hat{D}}(x) \middle| \hat{p}_{\hat{D}}(x), \hat{g}_{\hat{D}}(x) \middle| \hat{v}_{\hat{D}}(x)\right)$ is called the interval-valued probabilistic uncertain linguistic q-rung orthopair fuzzy element (IVPULq-ROFE), which can be denoted by $\hat{d} = \left(\hat{h} \middle| \hat{p}_h, \hat{g} \middle| \hat{v}_g\right)$. Then, the $\hat{h} | \hat{p}_h, \hat{g} | \hat{v}_g$ represent the interval-valued probabilistic uncertain linguistic q-rung orthopair fuzzy membership elements (IVPULq-ROFMEs) and interval-valued probabilistic uncertain linguistic q-rung orthopair fuzzy non-membership elements (IVPULq-ROFNMEs). Finally, the basic operations of IVPULq-ROFEs are depicted as follows.

Let $\hat{d}=\left(\hat{h}\left|\hat{p}_h, \hat{g}\right|\hat{v}_g\right)$, $\hat{d}_1=\left(\hat{h}_1\left|\hat{p}_{h_1}, \hat{g}_1\right|\hat{v}_{g_1}\right)$ and $\hat{d}_2=\left(\hat{h}_2\left|\hat{p}_{h_2}, \hat{g}_2\right|\hat{v}_{g_2}\right)$ be any three IVPULq-ROFEs with a continuous linguistic term set $\hat{S}=\left\{s_\delta \left| \delta=1, 2, ..., t\right.\right\}$, and κ be a positive real number, then we have

(1)

$$\hat{d}_1 \oplus \hat{d}_2=\cup_{\left[\xi_1^L, \xi_1^U\right]\in\hat{h}_1, \left[\xi_2^L, \xi_2^U\right]\in\hat{h}_2, \left[\zeta_1^L, \zeta_1^U\right]\in\hat{g}_1, \left[\zeta_2^L, \zeta_2^U\right]\in\hat{g}_2}$$

$$\left\{\left\{\begin{matrix}\left[s_{\left(\left(\xi_1^L\right)^q+\left(\xi_2^L\right)^q-\frac{\left(\xi_1^L\xi_2^L\right)^q}{t^q}\right)^{\frac{1}{q}}}, s_{\left(\left(\xi_1^U\right)^q+\left(\xi_2^U\right)^q-\frac{\left(\xi_1^U\xi_2^U\right)^q}{t^q}\right)^{\frac{1}{q}}}\right] \\ \left\|\left[\hat{p}^L_{\left[\xi_1^L, \xi_1^U\right]}\hat{p}^L_{\left[\xi_2^L, \xi_2^U\right]}, \hat{p}^U_{\left[\xi_1^L, \xi_1^U\right]}\hat{p}^U_{\left[\xi_2^L, \xi_2^U\right]}\right]\right.\end{matrix}\right\}\right\},$$

$$\left\{\begin{matrix}\left[s_{\left(\frac{\zeta_1^L\zeta_2^L}{t}\right)}, s_{\left(\frac{\zeta_1^U\zeta_2^U}{t}\right)}\right] \\ \left\|\left[\hat{v}^L_{\left[\zeta_1^L, \zeta_1^U\right]}\hat{v}^L_{\left[\zeta_2^L, \zeta_2^U\right]}, \hat{v}^U_{\left[\zeta_1^L, \zeta_1^U\right]}\hat{v}^U_{\left[\zeta_2^L, \zeta_2^U\right]}\right]\right.\end{matrix}\right\}\Bigg\}$$

(2)

$$\hat{d}_1 \otimes \hat{d}_2=\cup_{\left[\xi_1^L, \xi_1^U\right]\in\hat{h}_1, \left[\xi_2^L, \xi_2^U\right]\in\hat{h}_2, \left[\zeta_1^L, \zeta_1^U\right]\in\hat{g}_1, \left[\zeta_2^L, \zeta_2^U\right]\in\hat{g}_2}$$

$$\left\{\left\{\begin{matrix}\left[s_{\left(\frac{\xi_1^L\xi_2^L}{t}\right)}, s_{\left(\frac{\xi_1^U\xi_2^U}{t}\right)}\right] \\ \left\|\left[\hat{p}^L_{\left[\xi_1^L, \xi_1^U\right]}p^L_{\left[\xi_2^L, \xi_2^U\right]}, \hat{p}^U_{\left[\xi_1^L, \xi_1^U\right]}\hat{p}^U_{\left[\xi_2^L, \xi_2^U\right]}\right]\right.\end{matrix}\right\}\right\},$$

$$\left\{\begin{matrix}\left[s_{\left(\left(\zeta_1^L\right)^q+\left(\zeta_2^L\right)^q-\frac{\left(\zeta_1^L\zeta_2^L\right)^q}{t^q}\right)^{\frac{1}{q}}}, s_{\left(\left(\zeta_1^U\right)^q+\left(\zeta_2^U\right)^q-\frac{\left(\zeta_1^U\zeta_2^U\right)^q}{t^q}\right)^{\frac{1}{q}}}\right] \\ \left\|\left[\hat{v}^L_{\left[\zeta_1^L, \zeta_1^U\right]}\hat{v}^L_{\left[\zeta_2^L, \zeta_2^U\right]}, \hat{v}^U_{\left[\zeta_1^L, \zeta_1^U\right]}\hat{v}^U_{\left[\zeta_2^L, \zeta_2^U\right]}\right]\right.\end{matrix}\right\}\Bigg\}$$

(3) $\kappa\hat{d}=\cup_{\left[\xi^L, \xi^U\right]\in\hat{h}, \left[\zeta^L, \zeta^U\right]\in\hat{g}}$

$$\left\{\left\{\begin{matrix}\left[s_{t\left(1-\left(1-\frac{\left(\xi^L\right)^q}{t^q}\right)^{\kappa}\right)^{\frac{1}{q}}}, s_{t\left(1-\left(1-\frac{\left(\xi^U\right)^q}{t^q}\right)^{\kappa}\right)^{\frac{1}{q}}}\right] \\ \left\|\left[\hat{p}^L_{\left[\xi^L, \xi^U\right]}, \hat{p}^U_{\left[\xi^L, \xi^U\right]}\right]\right.\end{matrix}\right\}\right\},$$

$$\left\{\left[s_{t\left(\frac{\zeta^L}{t}\right)^{\kappa}}, s_{t\left(\frac{\zeta^U}{t}\right)^{\kappa}}\right]\right\}\left\|\left[\hat{v}^L_{\left[\zeta^L, \zeta^U\right]}, \hat{v}^U_{\left[\zeta^L, \zeta^U\right]}\right]\right\}\Bigg\}$$

(4) $\hat{d}^{\kappa}=\cup_{\left[\xi^L, \xi^U\right]\in\hat{h}, \left[\zeta^L, \zeta^U\right]\in\hat{g}}$

$$\left\{\left\{\left[s_{t\left(\frac{\xi^L}{t}\right)^{\kappa}}, s_{t\left(\frac{\xi^U}{t}\right)^{\kappa}}\right]\right\}\left\|\left[\hat{v}^L_{\left[\xi^L, \xi^U\right]}, \hat{v}^U_{\left[\xi^L, \xi^U\right]}\right]\right\},$$

$$\left\{\begin{matrix}\left[s_{t\left(1-\left(1-\frac{\left(\zeta^L\right)^q}{t^q}\right)^{\kappa}\right)^{\frac{1}{q}}}, s_{t\left(1-\left(1-\frac{\left(\zeta^U\right)^q}{t^q}\right)^{\kappa}\right)^{\frac{1}{q}}}\right] \\ \left\|\left[\hat{p}^L_{\left[\zeta^L, \zeta^U\right]}, \hat{p}^U_{\left[\zeta^L, \zeta^U\right]}\right]\right.\end{matrix}\right\}\Bigg\}$$

Example 1. A hospital is considering a telemedicine platform to enhance remote patient care. The decision-making process is influenced by multiple criteria, such as user-friendliness (C_1), data security (C_2), and cost-effectiveness (C_3). Assume that there an expert is invited to assess the continuous linguistic term set S = { s_0 = "Extremely poor", s_1 = "Very poor", s_2 = "Poor", s_3 = "Fair", s_4 = "Good", s_5 = "Very good", s_6 = "Extremely good"}. Then, his/her opinion of the platform A_1 under C_1 is $\hat{d}_1=\left\{\left\{\left[s_5, s_6\right]\left|\left[0.8,1\right]\right.\right\}, \left\{\left\{\left[s_2, s_3\right]\left|\left[0.9, 1\right]\right.\right\}\right\}\right\}$; opinion of the platform A_1 under C_2 is

$$\hat{d}_2 = \left\{ \begin{array}{l} \left\{ \left(\left[s_4, s_5 \right] \middle| \left[0.3, 0.6 \right] \right), \left(\left[s_5, s_6 \right] \middle| \left[0.6, 0.8 \right] \right) \right\}, \\ \left\{ \left[s_3, s_4 \right] \middle| \left[0.7, 1 \right] \right\} \end{array} \right\};$$

opinion of the platform A_1 under C_3 is

$$\hat{d}_3 = \left\{ \begin{array}{l} \left\{ \left[s_2, s_3 \right] \middle| \left[0.6, 1 \right] \right\}, \\ \left\{ \left(\left[s_3, s_4 \right] \middle| \left[0.6, 0.7 \right] \right), \left(\left[s_4, s_5 \right] \middle| \left[0.7, 0.8 \right] \right) \right\} \end{array} \right\}.$$

In addition, the weight of criteria is $\varsigma = \left(0.3, 0.3, 0.4 \right)^T$.

Then, we can get the following results based on the above aggregation rules.

$$(1) \quad \hat{d}_1 \oplus \hat{d}_2 = \left\{ \begin{array}{c} \left\{ \begin{array}{l} \left(\left[s_{5.2463}, s_6 \right] \middle| \left[0.24, 0.6 \right] \right), \\ \left(\left[s_{5.5497}, s_6 \right] \middle| \left[0.48, 0.8 \right] \right) \end{array} \right\}, \\ \left\{ \left[s_1, s_2 \right] \middle| \left[0.63, 1 \right] \right\} \end{array} \right\}$$

$$(2) \quad \hat{d}_1 \otimes \hat{d}_2 = \left\{ \begin{array}{c} \left\{ \begin{array}{l} \left(\left[s_{3.3333}, s_5 \right] \middle| \left[0.24, 0.6 \right] \right), \\ \left(\left[s_{4.1667}, s_6 \right] \middle| \left[0.48, 0.8 \right] \right) \end{array} \right\}, \\ \left\{ \left[s_{3.1302}, s_{4.2328} \right] \middle| \left[0.63, 1 \right] \right\} \end{array} \right\}$$

$$(3) \quad 0.4\hat{d}_3 = \left\{ \begin{array}{c} \left\{ \left[s_{1.5920}, s_{2.3973} \right] \middle| \left[0.6, 1 \right] \right\}, \\ \left\{ \begin{array}{l} \left(\left[s_{4.5471}, s_{5.1017} \right] \middle| \left[0.6, 0.7 \right] \right), \\ \left(\left[s_{5.1017}, s_{5.5780} \right] \middle| \left[0.7, 0.8 \right] \right) \end{array} \right\} \end{array} \right\}$$

$$(4) \quad \left(\hat{d}_3 \right)^{0.4} = \left\{ \begin{array}{c} \left\{ \left[s_{3.8664}, s_{4.5471} \right] \middle| \left[0.6, 1 \right] \right\}, \\ \left\{ \begin{array}{l} \left(\left[s_{2.3973}, s_{3.2327} \right] \middle| \left[0.6, 0.7 \right] \right), \\ \left(\left[s_{3.2327}, s_{4.1618} \right] \middle| \left[0.7, 0.8 \right] \right) \end{array} \right\} \end{array} \right\}$$

Then, a method of ranking any two IVPULq-ROFEs is developed based on the concept of score function and accuracy function of IVPULq-ROFE. Let $d = \left(\hat{h} \middle| \hat{p}_h, \hat{g} \middle| \hat{v}_g \right)$ be an IVPULq-ROFE with a continuous linguistic term set $\hat{S} = \left\{ s_\delta \middle| \delta = 1, 2, ..., t \right\}$, then the score function of $\hat{d}$ is

$$S\left(\hat{d} \right) = s_{\left(\frac{\left(2t^q + \sum_{l=1}^{\#\hat{h}} \left(\left(\xi_l^L \right)^q \hat{p}^L_{\left[\xi^L, \xi^U \right]} + \left(\xi_l^U \right)^q \hat{p}^U_{\left[\xi^L, \xi^U \right]} \right) \right)}{4} - \frac{\sum_{y=1}^{\#\hat{g}} \left(\left(\varsigma_y^L \right)^q \hat{v}^L_{\left[\varsigma^L, \varsigma^U \right]} + \left(\varsigma_y^U \right)^q \hat{v}^U_{\left[\varsigma^L, \varsigma^U \right]} \right)}{4} \right)^{\frac{1}{q}}} \tag{3}$$

where $\#\hat{h}$ and $\#\hat{g}$ denoted the number of elements in $\hat{h}$ and $\hat{g}$. In addition, the accuracy function is

$$H\left(\hat{d} \right) = s_{\left(\frac{\sum_{l=1}^{\#\hat{h}} \left(\left(\xi_l^L \right)^q \hat{p}^L_{\left[\xi^L, \xi^U \right]} + \left(\xi_l^U \right)^q \hat{p}^U_{\left[\xi^L, \xi^U \right]} \right)}{4} + \frac{\sum_{y=1}^{\#\hat{g}} \left(\left(\varsigma_y^L \right)^q \hat{v}^L_{\left[\varsigma^L, \varsigma^U \right]} + \left(\varsigma_y^U \right)^q \hat{v}^U_{\left[\varsigma^L, \varsigma^U \right]} \right)}{4} \right)^{\frac{1}{q}}} \tag{4}$$

where $\#\hat{h}$ and $\#\hat{g}$ also denoted the number of elements in $\hat{h}$ and $\hat{g}$.

Based on the above definition of score function and accuracy function, the comparing method between any two IVPULq-ROFEs is shown as follows. Let $\hat{d}_1 = \left(\hat{h}_1 \middle| \hat{p}_{h_1}, \hat{g}_1 \middle| \hat{v}_{g1} \right)$ and $\hat{d}_2 = \left(\hat{h}_2 \middle| \hat{p}_{h_2}, \hat{g}_2 \middle| \hat{v}_{g2} \right)$ be any two IVPULq-ROFEs with a continuous linguistic term set $\hat{S} = \left\{ s_\delta \middle| \delta = 1, 2, ..., t \right\}$, then

(1) If $S\left(\hat{d}_1 \right) > S\left(\hat{d}_2 \right)$, then $\hat{d}_1 > \hat{d}_2$;

(2) If $S\left(\hat{d}_1 \right) = S\left(\hat{d}_2 \right)$, then

If $H\left(\hat{d}_1 \right) > H\left(\hat{d}_2 \right)$, then $\hat{d}_1 > \hat{d}_2$;

If $H\left(\hat{d}_1 \right) = H\left(\hat{d}_2 \right)$, then $\hat{d}_1 = \hat{d}_2$.

To aggregate the IVPULq-ROFEs information, the aggregation operators of IVPULq-ROFEs are proposed. Let us assume there is a set $\hat{d}_i = \left(\hat{h}_i \middle| \hat{p}_{h_i}, \hat{g}_i \middle| \hat{v}_{gi} \right) (i = 1, 2, ..., n)$ with a continuous linguistic term set $\hat{S} = \left\{ s_\delta \middle| \delta = 1, 2, ..., t \right\}$. The weight of each $\hat{d}_i$ is denoted by $\varsigma = \left(\varsigma_1, \varsigma_2, ..., \varsigma_n \right)$ and satisfies $\varsigma_i \in \left[0, 1 \right] \sum_{i=1}^n \varsigma_i = 1$. Therefore, the interval-valued probabilistic uncertain linguistic *q*-rung orthopair fuzzy weighted average (IVPULq-ROFWA) operator is defined as

$$\text{IVPULq-ROFWA}\left(\hat{d}_1, \hat{d}_2, ..., \hat{d}_n \right) = \oplus_{i=1}^n \varsigma_i \hat{d}_i \tag{5}$$

when $\varsigma = \left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n} \right)^T$, then the IVPULq-ROFWA transfers to the interval-valued probabilistic uncertain linguistic *q*-rung orthopair fuzzy average (IVPULq-ROFA) operator

$$\text{IVPULq-ROFA}\left(\hat{d}_1, \hat{d}_2, ..., \hat{d}_n \right) = \frac{1}{n} \oplus_{i=1}^n d_i \tag{6}$$

Assume that $\hat{d}_i = \left(\hat{h}_i \middle| \hat{p}_{h_i}, \hat{g}_i \middle| \hat{v}_{g_i}\right)$ be a set of IVPULq-ROFSs, then the aggregation values of IVPULq-ROFWA and IVPULq-ROFA are also an IVPULq-ROFE. Based on the basic operations of IVPULq-ROFEs, the aggregation results of $\hat{d}_i = \left(\hat{h}_i \middle| \hat{p}_{h_i}, \hat{g}_i \middle| \hat{v}_{g_i}\right)$ can be obtained, and

$$\text{IVPULq-ROFWA}\left(\hat{d}_1, \hat{d}_2, ..., \hat{d}_n\right) = \oplus_{i=1}^{n} \varsigma_i \hat{d}_i = \cup_{\left[\xi_i^L, \xi_i^U\right]\in\hat{h}_i, \left[\zeta_i^L, \zeta_i^U\right]\in\hat{g}_i}$$

$$\left\{\left\{\left[s_{t\left(1-\prod_{i=1}^{n}\left(1-\frac{\left(\xi_i^L\right)^q}{t^q}\right)^{\varsigma_i}\right)^{\frac{1}{q}}}, s_{t\left(1-\prod_{i=1}^{n}\left(1-\frac{\left(\xi_i^U\right)^q}{t^q}\right)^{\varsigma_i}\right)^{\frac{1}{q}}} \right] \middle| \left[\prod_{i=1}^{n}\hat{p}^L_{\left[\xi_i^L, \xi_i^U\right]}, \prod_{i=1}^{n}\hat{p}^U_{\left[\xi_i^L, \xi_i^U\right]}\right]\right\},\right.$$

$$\left.\left\{\left[s_{t\prod_{i=1}^{n}\left(\frac{\zeta_i^L}{t}\right)^{\varsigma_i}}, s_{t\prod_{i=1}^{n}\left(\frac{\zeta_i^L}{t}\right)^{\varsigma_i}} \right] \middle| \left[\prod_{i=1}^{n}\hat{v}^L_{\left[\zeta_i^L, \zeta_i^U\right]}, \prod_{i=1}^{n}\hat{v}^U_{\left[\zeta_i^L, \zeta_i^U\right]}\right]\right\}\right\}$$

when $\varsigma = \left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right)^T$, then

$$\text{IVPULq-ROFA}\left(\hat{d}_1, \hat{d}_2, ..., \hat{d}_n\right) = \frac{1}{n}\oplus_{i=1}^{n} \hat{d}_i = \cup_{\left[\xi_i^L, \xi_i^U\right]\in\hat{h}_i, \left[\zeta_i^L, \zeta_i^U\right]\in\hat{g}_i}$$

$$\left\{\left\{\left[s_{t\left(1-\prod_{i=1}^{n}\left(1-\frac{\left(\xi_i^L\right)^q}{t^q}\right)^{\frac{1}{n}}\right)^{\frac{1}{q}}}, s_{t\left(1-\prod_{i=1}^{n}\left(1-\frac{\left(\xi_i^U\right)^q}{t^q}\right)^{\frac{1}{n}}\right)^{\frac{1}{q}}} \right] \middle| \left[\prod_{i=1}^{n}\hat{p}^L_{\left[\xi_i^L, \xi_i^U\right]}, \prod_{i=1}^{n}\hat{p}^U_{\left[\xi_i^L, \xi_i^U\right]}\right]\right\},\right.$$

$$\left.\left\{\left[s_{t\prod_{i=1}^{n}\left(\frac{\zeta_i^L}{t}\right)^{\frac{1}{n}}}, s_{t\prod_{i=1}^{n}\left(\frac{\zeta_i^U}{t}\right)^{\frac{1}{n}}} \right] \middle| \left[\prod_{i=1}^{n}\hat{v}^L_{\left[\zeta_i^L, \zeta_i^U\right]}, \prod_{i=1}^{n}\hat{v}^U_{\left[\zeta_i^L, \zeta_i^U\right]}\right]\right\}\right\}$$

Furthermore, the interval-valued probabilistic uncertain linguistic q-rung orthopair fuzzy weighted geometric average (IVPULq-ROFWGA) operator can be obtained

$$\text{IVPULq-ROFWGA}\left(\hat{d}_1, \hat{d}_2, ..., \hat{d}_n\right) = \otimes_{i=1}^{n} \hat{d}_i^{\varsigma_i} \tag{7}$$

when $\varsigma = \left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right)^T$, then the IVPULq-ROFWGA transfers to the interval-valued probabilistic uncertain linguistic q-rung orthopair fuzzy geometric average (IVPULq-ROFGA) operator

$$\text{IVPULq-ROFGA}\left(\hat{d}_1, \hat{d}_2, ..., \hat{d}_n\right) = \otimes_{i=1}^{n} \hat{d}_i^{\frac{1}{n}} \tag{8}$$

Then, the results are

$$\text{IVPULq-ROFWGA}\left(\hat{d}_1, \hat{d}_2, ..., \hat{d}_n\right) = \otimes_{i=1}^{n} \hat{d}_i^{\varsigma_i} = \cup_{\left[\xi_i^L, \xi_i^U\right]\in\hat{h}_i, \left[\zeta_i^L, \zeta_i^U\right]\in\hat{g}_i}$$

$$\left\{\left\{\left[s_{t\prod_{i=1}^{n}\left(\frac{\xi_i^L}{t}\right)^{\varsigma_i}}, s_{t\prod_{i=1}^{n}\left(\frac{\xi_i^U}{t}\right)^{\varsigma_i}} \right] \middle| \left[\prod_{i=1}^{n}\hat{p}^L_{\left[\xi_i^L, \xi_i^U\right]}, \prod_{i=1}^{n}\hat{p}^U_{\left[\xi_i^L, \xi_i^U\right]}\right]\right\},\right.$$

$$\left.\left\{\left[s_{t\left(1-\prod_{i=1}^{n}\left(1-\frac{\left(\zeta_i^L\right)^q}{t^q}\right)^{\varsigma_i}\right)^{\frac{1}{q}}}, s_{t\left(1-\prod_{i=1}^{n}\left(1-\frac{\left(\zeta_i^U\right)^q}{t^q}\right)^{\varsigma_i}\right)^{\frac{1}{q}}} \right] \middle| \left[\prod_{i=1}^{n}\hat{v}^L_{\left[\zeta_i^L, \zeta_i^U\right]}, \prod_{i=1}^{n}\hat{v}^U_{\left[\zeta_i^L, \zeta_i^U\right]}\right]\right\}\right\} \tag{9}$$

where $\varsigma = \left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right)^T$, then

$$\text{IVPULq-ROFGA}\left(\hat{d}_1, \hat{d}_2, ..., \hat{d}_n\right) =$$

$$\otimes_{i=1}^{n} \hat{d}_i^{\frac{1}{n}} = \cup_{\left[\xi_i^L, \xi_i^U\right]\in \hat{h}_i, \left[\zeta_i^L, \zeta_i^U\right]\in \hat{g}_i}$$

$$\left\{\left(\left[s_{t\prod_{i=1}^{n}\left(\frac{\xi_i^L}{t}\right)^{\frac{1}{n}}}, s_{t\prod_{i=1}^{n}\left(\frac{\xi_i^U}{t}\right)^{\frac{1}{n}}} \right] \left| \left[\prod_{i=1}^{n}\hat{p}^L_{\left[\xi_i^L, \xi_i^U\right]}, \prod_{i=1}^{n}\hat{p}^U_{\left[\xi_i^L, \xi_i^U\right]}\right]\right.\right)\right\},$$

$$\left\{\left[s_{t\left(1-\prod_{i=1}^{n}\left(1-\frac{\left(\zeta_i^L\right)^q}{t^q}\right)^{\frac{1}{n}}\right)^{\frac{1}{q}}}, s_{t\left(1-\prod_{i=1}^{n}\left(1-\frac{\left(\zeta_i^U\right)^q}{t^q}\right)^{\frac{1}{n}}\right)^{\frac{1}{q}}} \right] \left[\prod_{i=1}^{n}\hat{v}^L_{\left[\zeta_i^L, \zeta_i^U\right]}, \prod_{i=1}^{n}\hat{v}^U_{\left[\zeta_i^L, \zeta_i^U\right]}\right]\right\}$$

$$\left\{\left[s_{t\left(1-\prod_{i=1}^{n}\left(1-\frac{\left(\zeta_i^L\right)^q}{t^q}\right)^{\frac{1}{n}}\right)^{\frac{1}{q}}}, s_{t\left(1-\prod_{i=1}^{n}\left(1-\frac{\left(\zeta_i^U\right)^q}{t^q}\right)^{\frac{1}{n}}\right)^{\frac{1}{q}}} \right] \left[\prod_{i=1}^{n}\hat{v}^L_{\left[\zeta_i^L, \zeta_i^U\right]}, \prod_{i=1}^{n}\hat{v}^U_{\left[\zeta_i^L, \zeta_i^U\right]}\right]\right\} \quad (10)$$

4 NOVEL MAGDM METHODS BASED ON IVPULq-ROFSs

In this section, we attempt to develop novel MAGDM methods based on the proposed IVPULq-ROFSs theory. We firstly introduce the framework of a MAGDM method under interval-valued probabilistic uncertain linguistic q-rung orthopair fuzzy decision environment. Let us consider a typical MAGDM problem under IVPULq-ROFSs. We assume that there are m alternatives, i.e., $\{A_1, A_2, …, A_m\}$ and several decision makers are invited to express their evaluation values under attributes which can be denoted as $\{G_1, G_2, …, G_n\}$. Weight vector of attributes is $w = (w_1, w_2, …, w_n)^T$, such that $\sum_{j=1}^{n} w_j = 1$. Let $\tilde{S} = \{s_\alpha | 0 \le \alpha \le l\}$ be a pre-defined continuous linguistic term set. For attribute $G_j(j = 1, 2, …, n)$ of alternative $A_i(i = 1, 2, …, n)$, decision makers use IVPULq-ROFE $\tilde{d}_{ij} = \left(\tilde{h}_{ij} \left| p_{\tilde{h}_{ij}}, \tilde{g}_{ij} \right| \tilde{t}_{\tilde{g}_{ij}}\right)$ to denote their evaluation values. In the following, we present two novel approaches to solve this problem. We first propose an aggregation operator based IVPULq-ROFWA operator and IVPULq-ROFWGA operator.

4.1 The First Approach: Approach A

In this section, we propose an aggregation operator based MAGDM method.

Step 1. Standardize the original decision matric. In real MAGDM problems, there are usually two types of attributes. Therefore, the original decision matrix should be standardized according to

$$\tilde{d}_{ij} = \begin{cases} \left(\tilde{h}_{ij} \left| p_{\tilde{h}_{ij}}, \tilde{g}_{ij} \right| \tilde{t}_{\tilde{g}_{ij}}\right) \\ \left(\tilde{g}_{ij} \left| \tilde{t}_{\tilde{g}_{ij}}, \tilde{h}_{ij} \right| p_{\tilde{h}_{ij}}\right) \end{cases} \quad (11)$$

Step 2. For alternative $A_i(i = 1, 2, …, m)$, use the IVPULPFWA operator

$$\tilde{d}_i = \text{IVPULPFWA}\left(\tilde{d}_{i1}, \tilde{d}_{i2}, ..., \tilde{d}_{\text{in}}\right) \quad (12)$$

or the IVPULPFWG operator

$$\tilde{d}_i = \text{IVPULPFWG}\left(\tilde{d}_{i1}, \tilde{d}_{i2}, ..., \tilde{d}_{\text{in}}\right) \quad (13)$$

to calculate the comprehensive evaluation value of each alternative.

Step 3. Compute the score values of alternatives.

Step 4. Rank alternatives according to their score values.

4.2 The Second Approach: Approach B

In this subsection, we propose a TODIM based MAGDM method.

Step 1. Standardize the original decision matric, which is the same as the step 1 in Approach A.

Step 2. Compute the relative weight of attribute G_j to the reference attribute G_r according to the following formula

$$w_{jr} = w_j / w_r \quad (14)$$

where w_j denotes the weight of G_j and $w_r = \max\left\{w_j \mid j = 1, 2, ..., n\right\}$.

Step 3. Compute the dominance of each alternative A_i over each alternative A_k according to

$$\vartheta\left(A_i, A_k\right) = \sum_{j=1}^{n}\phi_j\left(A_i, A_k\right), i, s = 1, 2, ..., m \tag{15}$$

wherein

$$\phi_j\left(A_i, A_s\right) = \begin{cases} \sqrt{w_{jk} / \sum_{j=1}^{n} w_{jk}}\, d\left(\tilde{d}_{ij}, \tilde{d}_{kj}\right) & \text{if } \tilde{d}_{ij} > \tilde{d}_{kj} \\ 0 & \text{if } \tilde{d}_{ij} = \tilde{d}_{kj} \\ -\frac{1}{\theta}\sqrt{\left(\sum_{j=1}^{n} w_{jk}\right) / w_{jk}}\, d\left(\tilde{d}_{ij}, \tilde{d}_{kj}\right) & \text{if } \tilde{d}_{ij} < \tilde{d}_{kj} \end{cases} \tag{16}$$

wherein $d\left(\tilde{d}_{ij}, \tilde{d}_{kj}\right)$ denotes the distance between $\tilde{d}_{ij}$ and $\tilde{d}_{kj}$, $\phi_j\left(A_i, A_k\right)$ denotes the contributions of the attribute G_j to the function $\vartheta\left(A_i, A_k\right)$, and the parameter θ represents the attenuation factor of the losses.

Step 4. Compute the overall prospect value of alternative A_i based on the following formula

$$\phi\left(A_i\right) = \frac{\sum_{k=1}^{m}\vartheta\left(A_i, A_k\right) - \min_i\left\{\sum_{k=1}^{m}\vartheta\left(A_i, A_k\right)\right\}}{\max_i\left\{\sum_{k=1}^{m}\vartheta\left(A_i, A_k\right)\right\} - \min_i\left\{\sum_{k=1}^{m}\vartheta\left(A_i, A_k\right)\right\}} \tag{17}$$

Step 5. Rank alternatives according to their overall prospect values.

5 APPLICATIONS OF THE NEW MAGDM METHODS

Example 2: With the rapid advancement of artificial intelligence, sensor technology, and high-performance computing, autonomous vehicles have emerged as a transformative innovation in modern transportation. As global interest in self-driving technology grows, automakers, technology firms, and governments are actively investing in research and development to enhance safety, efficiency, and reliability. However, achieving full autonomy requires robust and intelligent sensor configurations capable of operating seamlessly across diverse and often unpredictable environments. A leading autonomous vehicle technology company is currently in the process of selecting the optimal sensor configuration for its next-generation self-driving system. Following extensive technical feasibility studies and preliminary field testing across six distinct climate zones and twelve urban environments, the company has shortlisted six sensor arrays $A_i(i = 1, 2, 3, 4, 5, 6)$ for final evaluation. To ensure a comprehensive and objective assessment, a series of DMs, including robotics engineers, AI specialists, automotive safety consultants, and systems integration engineers, has been assembled to evaluate the six configurations based on four critical attributes: Detection accuracy in adverse weather conditions (G_1); Power efficiency and energy consumption (G_2); Redundancy capabilities and fault tolerance (G_3); Cost-to-performance ratio (G_4). The relative importance of these attributes is represented by a weight vector $\varsigma = (0.1, 0.4, 0.2, 0.3)^T$. Given the complexity of real-world conditions and the inherent uncertainty in performance predictions, the DMs utilize a continuous linguistic term set $S = \{ s_0$ = "Extremely poor", s_1 = "Very poor", s_2 = "Poor", s_3 = "Slightly poor", s_4 = "Fair", s_5 = "Slightly good", s_6 = "Good", s_7 = "Very good", s_8 = "Extremely good"$\}$ to express their evaluations. To further accommodate variability in expert assessments and account for uncertainty, these evaluations are represented using IVPULq-ROFEs. The comprehensive decision matrix, incorporating the assessments of all DMs, is detailed in Tab. 1.

Table 1 The original decision matrix

	A_1
G_1	$\left\{\left\{[s_2, s_3] \mid [0.6, 1]\right\}, \left\{([s_4, s_5] \mid [0.2, 0.3]), ([s_5, s_7] \mid [0.6, 0.7])\right\}\right\}$
G_2	$\left\{\left\{([s_3, s_4] \mid [0.6, 0.8]), ([s_3, s_6] \mid [0.1, 0.2])\right\}, \left\{([s_5, s_6] \mid [0.8, 1])\right\}\right\}$
G_3	$\left\{\left\{([s_3, s_4] \mid [0.3, 1])\right\}, \left\{([s_5, s_6] \mid [0.6, 1])\right\}\right\}$
G_4	$\left\{\left\{([s_1, s_2] \mid [0.2, 0.4]), ([s_2, s_3] \mid [0.2, 0.3]), ([s_3, s_4] \mid [0.1, 0.3])\right\}, \left\{([s_3, s_7] \mid [0.8, 1])\right\}\right\}$
	A_2
G_1	$\left\{\left\{([s_4, s_5] \mid [0.5, 1])\right\}, \left\{([s_2, s_4] \mid [0.1, 0.3]), ([s_5, s_6] \mid [0.5, 0.7])\right\}\right\}$
G_2	$\left\{\left\{([s_1, s_3] \mid [0.7, 1])\right\}, \left\{([s_2, s_6] \mid [0.1, 0.2]), ([s_4, s_5] \mid [0.7, 0.8])\right\}\right\}$.
G_3	$\left\{\left\{([s_2, s_3] \mid [0.4, 0.5]), ([s_4, s_5] \mid [0.3, 0.5])\right\}, \left\{([s_2, s_3] \mid [0.6, 1])\right\}\right\}$
G_4	$\left\{\left\{([s_4, s_6] \mid [0.6, 1])\right\}, \left\{([s_2, s_5] \mid [0.8, 1])\right\}\right\}$
	A_3
G_1	$\left\{\left\{([s_3, s_5] \mid [0.7, 1])\right\}, \left\{([s_3, s_6] \mid [0.9, 1])\right\}\right\}$

Table 1 The original decision matrix - continuation

G_2	$\{\{([s_1, s_3]\|[0.9, 1])\}, \{([s_3, s_4]\|[0.6, 0.7]), ([s_5, s_6]\|[0.2, 0.3])\}\}$
G_3	$\{\{([s_5, s_6]\|[0.1, 1])\}, \{([s_3, s_4]\|[0.9, 1])\}\}$
G_4	$\{\{([s_2, s_4]\|[0.5, 1])\}, \{([s_4, s_6]\|[0.1, 0.4]), ([s_3, s_7]\|[0.4, 0.6])\}\}$
	A_4
G_1	$\{\{([s_1, s_4]\|[0.7, 0.8]), ([s_5, s_6]\|[0.1, 0.2])\}, \{([s_2, s_4]\|[0.8, 1])\}\}$
G_2	$\{\{([s_3, s_4]\|[0.7, 1])\}, \{([s_5, s_6]\|[0.4, 1])\}\}$
G_3	$\{\{([s_2, s_5]\|[0.5, 1])\}, \{([s_5, s_6]\|[0.4, 1])\}\}$
G_4	$\{\{([s_3, s_4]\|[0.7, 1])\}, \{([s_1, s_4]\|[0.1, 0.2]), ([s_5, s_7]\|[0.7, 0.8])\}\}$

5.1 The Decision-Making Results by Approach A

Step 1. It is obvious that all attributes are benefit type and the decision matrix does not need to be standardized.

Step 2. For each alternative, we use the IVPULq-ROFWA operator to compute the overall preference value of each alternative and the results are shown as follows (without loss of generality, we assume q = 1)

$\tilde{d}_1$ = {{[$s_{2.3590}$, $s_{3.3212}$]|[0.0216, 0.3200], [$s_{2.5602}$, $s_{3.5523}$]|[0.0216, 0.2400], [$s_{2.8771}$, $s_{3.8742}$]|[0.0108, 0.2400], [$s_{2.3590}$, $s_{4.2081}$]|[0.0036, 0.0800], [$s_{2.5602}$, $s_{4.3699}$]|[0.0036, 0.0600], [$s_{2.8771}$, $s_{0.6028}$]|[0.0018, 0.0600]}, {[$s_{4.0438}$, $s_{6.3326}$]|[0.0768, 0.3000], [$s_{4.1814}$, $s_{6.4807}$]|[0.2304, 0.7000]}};

$\tilde{d}_2$ = {{[$s_{3.1106}$, $s_{4.7705}$]|[0.0840, 0.5000], [$s_{3.5529}$, $s_{5.1141}$]|[0.0630, 0.5000]}, {[$s_{2.0000}$, $s_{4.4542}$]|[0.0048, 0.0600], [$s_{2.3784}$, $s_{4.2557}$]|[0.0336, 0.2400], [$s_{2.2947}$, $s_{4.7335}$]|[0.0240, 0.1400], [$s_{2.7288}$, $s_{4.5226}$]|[0.1680, 0.5600]}};

$\tilde{d}_3$ = {{[$s_{3.1893}$, $s_{4.6829}$]|[0.0315, 1.0000]}, {[$s_{3.3178}$, $s_{4.8990}$]|[0.0486, 0.2800], [$s_{3.0000}$, $s_{5.1706}$]|[0.1944, 0.4200], [$s_{3.7697}$, $s_{5.4216}$]|[0.0162, 0.1200], [$s_{3.4087}$, $s_{5.7222}$]|[0.0648, 0.1800]}};

$\tilde{d}_4$ = {{[$s_{2.5756}$, $s_{4.2928}$]|[0.1715, 0.8000], [$s_{3.2581}$, $s_{4.6870}$]|[0.0245, 0.2000]}, {[$s_{2.4811}$, $s_{4.8990}$]|[0.0128, 0.2000], [$s_{4.3579}$, $s_{5.9589}$]|[0.0896, 0.8000]}};

Step 3. Compute the score values of alternatives and we can obtain the following results

$$S(\tilde{d}_1) = s_{4.8910}, S(\tilde{d}_2) = s_{5.7522}, S(\tilde{d}_3) = s_{5.4684},$$
$$S(\tilde{d}_4) = s_{5.3279}$$

Step 4. Rank alternatives according to their score values and we can obtain $A_2 \succ A_3 \succ A_4 \succ A_1$. Hence A_2 is the best alternative.

In step2, if the IVPULq-ROFWGA operator is used to aggregate the attribute values of alternatives, then we can obtain the following results

$\tilde{d}_1$ = {{[$s_{1.9218}$, $s_{3.0058}$]|[0.0216, 0.3200], [$s_{2.4495}$, $s_{3.4641}$]|[0.0216, 0.2400], [$s_{2.8230}$, $s_{3.8311}$]|[0.0108, 0.2400], [$s_{1.9218}$, $s_{3.3265}$]|[0.0036, 0.0800], [$s_{2.4495}$, $s_{3.8337}$]|[0.0036, 0.0600], [$s_{2.8230}$, $s_{4.2398}$]|[0.0018, 0.0600]}, {[$s_{4.3153}$, $s_{6.4416}$]|[0.0768, 0.3000], [$s_{4.4737}$, $s_{6.5959}$]|[0.2304, 0.7000]}};

$\tilde{d}_2$ = {{[$s_{2.3784}$, $s_{4.1282}$]|[0.0840, 0.5000], [$s_{2.8284}$, $s_{4.6905}$]|[0.0630, 0.5000]}, {[$s_{2.0000}$, $s_{4.8622}$]|[0.0048, 0.0600], [$s_{2.6936}$, $s_{4.4895}$]|[0.0336, 0.2400], [$s_{2.7847}$, $s_{5.1745}$]|[0.0240, 0.1400], [$s_{3.2873}$, $s_{4.8539}$]|[0.1680, 0.5600]}};

$\tilde{d}_3$ = {{[$s_{2.2474}$, $s_{4.2598}$]|[0.0315, 1.0000]}, {[$s_{3.3999}$, $s_{5.2287}$]|[0.0486, 0.2800], [$s_{3.0000}$, $s_{5.8765}$]|[0.1944, 0.4200], [$s_{3.9847}$, $s_{5.6534}$]|[0.0162, 0.1200], [$s_{3.6782}$, $s_{6.1845}$]|[0.0648, 0.1800]}};

$\tilde{d}_4$ = {{[$s_{2.2990}$, $s_{4.2295}$]|[0.1715, 0.8000], [$s_{2.9267}$, $s_{4.4947}$]|[0.0245, 0.2000]}, {[$s_{3.8462}$, $s_{5.2287}$]|[0.0128, 0.2000], [$s_{4.7325}$, $s_{6.2928}$]|[0.0896, 0.8000]}};

The score values of the four alternatives are

$$S(\tilde{d}_1) = s_{4.7838}, S(\tilde{d}_2) = s_{5.5461}, S(\tilde{d}_3) = s_{5.2436},$$
$$S(\tilde{d}_4) = s_{5.1991}$$

Therefore, the ranking order of the four candidates is $A_2 \succ A_3 \succ A_4 \succ A_1$ and A_2 is the best alternative.

5.2 The Decision-Making Results by Approach B

Step 1. It is obvious that all attributes are benefit type and the decision matrix does not need to be standardized.

Step 2. Compute the relative weight of attribute c_j to the reference attribute c_r according to Eq. (14)

$$w_{jr} = (0.4286, 0.7143, 0.7143, 1)^T$$

Step 3. Compute the dominance of each alternative A_i over each alternative A_k according to Eq. (15) (Without loss of generality, we assume $\theta = 1$, $q = 3$)

$$\vartheta(A_i, A_k) = \begin{bmatrix} 0 & -1.0119 & -0.9289 & -0.8756 \\ 0.2436 & 0 & -0.2217 & -0.0386 \\ 0.1959 & -0.4196 & 0 & -0.1236 \\ 0.2116 & -0.8652 & -0.4914 & 0 \end{bmatrix},$$
$$i, k = 1, 2, 3, 4$$

Step 4. Compute the overall prospect value of alternative A_i according to Eq. (17)

$$\phi(A_1) = -1.7834, \phi(A_2) = 0.4405, \phi(A_3) = 0.1175,$$
$$\phi(A_4) = -0.2599$$

Step 5. Rank alternatives according to their prospect values and we can obtain $A_2 \succ A_3 \succ A_4 \succ A_1$, which indicates that A_2 is the best alternative.

5.3 Sensitivity Analysis

(1) The analysis of parameter q in the Approach A

From the definition of IVPULq-ROFEs and the decision-making process of approach A, it is evident that the parameter q plays a crucial role in influencing the decision-making outcomes. To further investigate its impact, we analyse the results obtained using the IVPULq-ROFWA and IVPULq-ROFWGA operators under different values of q. The findings are illustrated in Fig. 1 and Fig. 2. As shown in Fig. 1, when $q = 1$, the ranking order of alternatives is $A_2 \succ A_4 \succ A_3 \succ A_1$; when $q > 1$, the ranking remains consistently $A_2 \succ A_3 \succ A_4 \succ A_1$. Similarly, Fig. 2 demonstrates that regardless of the value of parameter q, the ranking order remains unchanged as $A_2 \succ A_3 \succ A_4 \succ A_1$. Notably, in all cases, the optimal alternative is consistently A_2, highlighting the strong stability and robustness of the proposed approach A when applied with the IVPULq-ROFWA and IVPULq-ROFWGA operators. This stability suggests that decision-makers can adjust the parameter q within a reasonable range based on the specific requirements of real-world scenarios, ensuring adaptability while maintaining reliable decision outcomes.

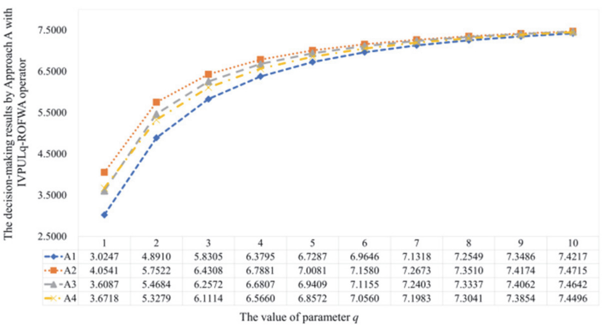

Figure 1 The decision-making results by Approach A

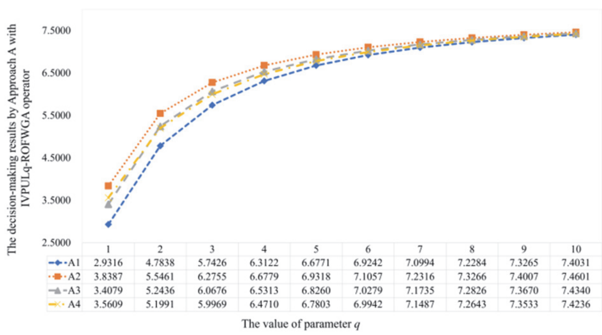

Figure 2 The decision-making results by Approach A

(2) The analysis of parameter θ in the Approach B

From the decision-making process of Approach B, we observe that the parameter θ plays a crucial role in influencing the final decision outcomes. To further examine its impact, we conduct an in-depth analysis, with the results presented in Fig. 3. Our findings reveal that as θ increases, the decision outcomes for each alternative also show an upward trend, indicating that this parameter significantly affects the final rankings. Specifically, when $\theta > 0.4$, the ranking of alternatives remains stable as $A_2 \succ A_3 \succ A_4 \succ A_1$. However, when $\theta = 0.4$, alternatives A_2 and A_3 become equally ranked, leading to the order $A_2 = A_3 \succ A_4 \succ A_1$; when $\theta = 0.3$, the ranking shifts to $A_3 \succ A_2 \succ A_4 \succ A_1$; while $\theta = 0.1$ or 0.2, the order changes further to $A_3 \succ A_4 \succ A_2 \succ A_1$.

These results highlight the flexibility of the proposed decision-making method under Approach B, as the parameter θ allows for adaptability in ranking outcomes based on varying levels of influence. To ensure a stable and rational decision-making process, it is advisable to set θ within an appropriate range. If maintaining a consistent ranking order is a priority, selecting $\theta > 0.4$ is recommended, as this range provides stability in alternative rankings. However, if sensitivity analysis is required to explore different ranking patterns, lower values of $\theta = [0.2, 0.3]$ can be considered to allow for greater differentiation in decision outcomes. The choice of θ should ultimately align with the decision-making organizers' preference for stability versus adaptability in the ranking process.

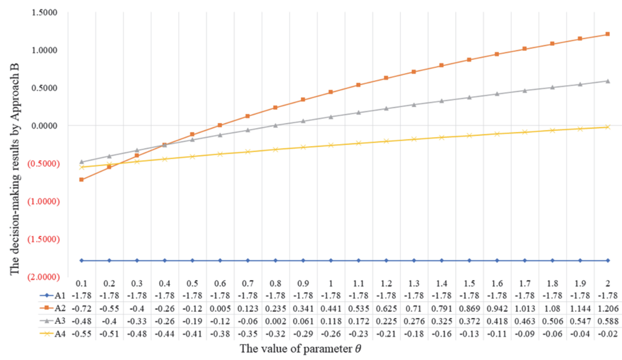

Figure 3 The decision-making results by Approach B

5.4 Comparative Analysis

To better demonstrate the advantages and superiorities of our proposed methods, we compare the advantages and disadvantages of the proposed method in this paper with the existing fuzzy decision-making models, as shown in Tab. 2.

Table 2 The comparative analysis of the existing models

Model	Uncertainty Representation	Flexibility in Aggregation	Computational Complexity	Handling of Hesitancy
q-ROFSs [22]	Strong representation of uncertainty	Moderate, requires predefined parameters	Moderate	Strong
HTFSs [23]	Captures hesitation in triangular form	Moderate, relies on weighting functions	High	Strong
IVPULq-ROFSs (Proposed Model)	Enhanced uncertainty representation with IV-PUL q-rung extension	High, supports advanced aggregation operator	Moderate, optimized computational efficiency	Strong, effectively models hesitant evaluations

Based on the comparative analysis in Tab. 2, we find that the proposed IVPULq-ROFSs model offers several key advantages over existing fuzzy decision-making approaches. First, it provides enhanced uncertainty representation by integrating interval-valued and possibility mechanisms with q-rung orthopair fuzzy sets, allowing for more accurate modeling of vague and imprecise opinions. Second, it ensures greater flexibility in aggregation operations, enabling more comprehensive information fusion in multi-attribute group decision-making problems. Additionally, IVPULq-ROFSs balances computational efficiency and complexity, making it well-suited for large-scale decision-making scenarios without excessive computational overhead. Moreover, the model effectively captures hesitancy and uncertainty, which is essential in real-world decision-making environments where experts may express incomplete or conflicting evaluations.

6 CONCLUSIONS

This study introduces IVPULq-ROFSs, a novel fuzzy set model designed to handle uncertainty and hesitancy in multi-attribute decision-making problems. We develop two decision-making approaches, an aggregation-based method and a TODIM-based approach, and validate them through an autonomous vehicle sensor configuration selection problem. The key findings demonstrate that the proposed IVPULq-ROFSs framework enhances decision robustness by outperforming conventional fuzzy models, offering a more refined representation of probabilistic hesitancy in decision-making. Additionally, the newly developed aggregation operators effectively combine linguistic probabilistic information, improving decision accuracy. The model's versatility makes it applicable to various domains, including supply chain optimization, risk assessment, medical decision-making, and intelligent transportation systems. From a managerial perspective, the IVPULq-ROFSs framework enhances decision-making by improving risk assessment and reducing uncertainty in complex environments. Its ability to aggregate hesitant and probabilistic information enables more transparent and reliable decisions, which is crucial for strategic planning. Additionally, its flexibility allows decision-makers to adapt to changing conditions, ensuring more robust and informed choices.

While IVPULq-ROFSs provide a powerful decision-making tool, future research should focus on optimizing computational efficiency for real-time applications, expanding its use in multi-criteria decision-making across finance, healthcare, and smart cities, and exploring AI-driven extensions for automated decision support systems. In finance, IVPULq-ROFSs can be applied to risk assessment and portfolio optimization under uncertain conditions, addressing conflicting opinions and market unpredictability by incorporating probabilistic uncertainty and linguistic hesitancy. In healthcare, the model offers a robust framework for treatment selection by integrating expert opinions from doctors, specialists, and patients. It effectively captures linguistic hesitancy, facilitating decisions that consider varying levels of trust, knowledge, and uncertainties in treatment outcomes. For urban planning, IVPULq-ROFSs can aid in selecting sustainable policies for smart cities, where environmental, economic, and social uncertainties intersect. This capability is crucial for decision-makers dealing with complex, conflicting criteria and diverse stakeholder interests. Moreover, AI-driven extensions of IVPULq-ROFSs could enable the development of automated decision support systems that autonomously process inputs, learn from historical data, and adapt to changing conditions, ideal for applications in smart cities, robotics, and autonomous vehicles. By integrating probabilistic uncertainty with fuzzy logic, this research advances multi-attribute group decision-making methodologies, offering a new direction for uncertainty-aware decision science.

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