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FRACTIONAL PROGRAMMING IN OPTIMIZATION THE OPERATION EFFICIENCY OF SHIPOWNER

The paper aims at presenting the application of linear fractional programming in determining profitability as an index of operation efficiency. The mathematical model addressing the problem of cargo transport by a container ship on a selected route, where, certain number of containers of various masses and types are chosen out of the total number of containers available in the port of loading in order to achieve the maximum transport profitability, provided maximum payload and transport capacity of container ship, is formulated. The model was tested on real-life example of feeder ship "Lipa" owned by "Lošinjska plovidba", Rijeka, and operating on the Rijeka-Gioia Tauro route.

Key words: linear fractional programming, profitability, transport by container ship

1. INTRODUCTION

In economic literature and practice, the following criteria of efficiency are mostly applied [6, p. 328]: productivity, economic justifiability and profitability, expressing quantitative relations between effect of operating results and quantity of work process elements spent or invested.

Profitability is an index of efficiency showing the profit achieved per unit of invested capital, where the invested capital can also be time spent in achieving operating results. If operating revenues are greater than operating expenses, a company makes profit, while if operating revenues are smaller than operating expenses, a company suffers loss. In the first case, a company is said to be profit-making, while in the latter case it is an unprofitable one.

Since the above-mentioned indices are expressed as fractions, the maxim values of these indices can be determined by using linear fractional programming, provided that constraints are expressed in a linear form.

The objective of this paper is to show how to estimate the operation efficiency, taking into consideration maximum profitability as a criterion function which has to be optimized.

2. OPTIMIZATION OF THE SHIPOWNER OPERATION EFFICIENCY

Linear fractional programming is an area of optimization where a linear fractional objective function is an optimized subject to linear constraints. There are several methods for solving linear fractional programming problems (Dinkelbach method, Martos method, Charnes-Cooper method, Gilmore-Gomory method and others) [3, p. 31].

Martos method was chosen in this paper, representing a modification of the simplex method for linear programs, and using objective function gradient. In numerical examples, results of the structural variables are mainly decimal numbers. But, in practice, the nature of the problems which are to be optimized demands structural variables to be integer. In these cases, methods for integer fractional programming should be applied, such as branch-and-bound method, or the Gomory "cutting plane" method, adopted to the linear-fractional problem considered, see [1,5].

Problem description

The example for defining the optimal structure of container transport is illustrated through the operation of the container ship "Lipa", on the data basis provided by the owner "Lošinjska plovidba".

Namely, in early March 1999, the feeder service was established on the line (Rijeka-Ploče-Gioia Tauro-Malta), performed by the Croatian owner "Lošinjska plovidba", financially supported out of the Budget of the Republic of Croatia [4, p. 238]. The service has been maintained with one ship of approximately 200 TEU capacity (TEU is 20′ equivalent unit), scheduled once a week from the ports of Rijeka and Ploče to the largest Mediterranean ports of Malta and Gioia Tauro.

The feeder ship "Lipa" of 5,615 tons deadweight and the transport capacity of 250 TEU, carries four types of containers. The unit mass and revenue per one transported container are shown in Table 1.

Table 1. Unit mass and profit per one container

Type of container	Unit mass (in tons)	Profit per one container (in USD)
20′ (full)	25.0	150
40′ (full)	30.0	250
20′ (empty)	2.5	75
40′ (empty)	3.5	125

The maximum allowed mass of 20 feet full containers is 30 tons, while for the 40 feet full containers the maximum allowed mass is 35 tons. The average masses are considered in solving the problem, 25 and 30 tons for 20 and 40 feet containers, respectively.

The calculation considers one-way voyage on the direct Rijeka-Gioia Tauro line, without any other port of call. The voyage takes 42 hours, for which period respective ship operation costs amount to US$ 16,386. The waiting time costs in the ports of Rijeka and Gioia Tauro total to US$ 9,525, making the whole voyage cost of US$ 25,911.

The transshipment rate of one container crane is 35 TEU/h, running continuously, on the round-the-clock basis. Two container cranes are used parallelly. Therefore, the transshipment time per one container is 1/420 days.

The task is to determine the number of a particular container type to be transported that would yield maximum profitability per one voyage considering payload and transport capacity of a container ship.

Mathematical model formulation

As it can be seen from the problem described, this is the issue of the transport service production. Therefore, the profitability of the transport process will be defined as the ratio between profit yielded from the transport of various types of cargoes and funds invested for the realization of such a transport process.

If the following notation is introduced:

- x_j – quantity of j-th type of containers being loaded on board (TEU),
- Q_j – quantity of j-th type of containers available for loading on berth (TEU),
- f_j – transport price of j-th type of container (in currency units per TEU or per ton),
- g_j – mass of j-th type of container (tons),
- V – container ship deadweight (tons),
- W – ship transport capacity (TEU),

then the problem can be formulated as a linear programming model [5, p. 304-306]:

$$Max \sum_{j=1}^{n} f_j x_j \quad (1)$$

with constraints:

$$\begin{aligned} &\sum_{j=1}^{n} g_j x_j \le V \\ &\sum_{j=1}^{n} x_j \le W \\ &x_j \le Q_j \\ &x_j \ge 0, \quad j = 1,\dots,n. \end{aligned} \quad (2)$$

x_j - nonnegative integer values; $j=1,...,n$.

If $\sum_{j=1}^{n} Q_j \le W$ and $\sum_{j=1}^{n} g_j Q_j \le V$ the solution is trivial, i.e., all cargoes available on berth should be loaded on board.

However, the mentioned model (1)-(2) is not a real one because it does not take into consideration the time necessary to load cargo or the duration of a voyage between the loading port and the port of discharge. In order to formulate a model that would correspond to reality, additional elements are introduced:

- a_j – average number of TEUs of j-th type of container that can be loaded on board on the daily basis,
- C_1 – cost of a container ship stay in port (per day),
- C_2 – cost of a container ship during a voyage (per day),
- d – distance between ports (in nautical miles),
- s – speed of a container ship (in knots).

The total duration of the voyage and ship stay in a port during loading operations can be expressed as [5, p. 305]:

$$T=\frac{d}{s}+\sum_{j=1}^{n}\frac{x_j}{a_j}, \tag{3}$$

while the profit is

$$D=\sum_{j=1}^{n} f_j x_j - C_1\sum_{j=1}^{n}\frac{x_j}{a_j} - C_2\frac{d}{s}. \tag{4}$$

The operation efficiency cannot be defined by taking into consideration the total amount of profit, since yielding high profit over a (too) long period may not necessarily be acceptable. Therefore, it is necessary to calculate the profit per unit of time, i.e., in this case, *D/T*. Therefore, profitability of this line for the transport of various container types is expressed by the amount of profit per unit of time.

Considering (3) and (4), the mathematical model contains a fractional function thus becoming a linear fractional programming model:

$$Max\frac{\sum_{j=1}^{n} f_j x_j - C_1\sum_{j=1}^{n}\frac{x_j}{a_j} - C_2\frac{d}{s}}{\frac{d}{s}+\sum_{j=1}^{n}\frac{x_j}{a_j}} \tag{5}$$

with constraints:

$$\begin{aligned} &\sum_{j=1}^{n} g_j x_j \le V \\ &\sum_{j=1}^{n} x_j \le W \\ &x_j \le Q_j \\ &x_j \ge 0,\ j=1,\dots,n. \end{aligned} \tag{6}$$

x_j - nonnegative integer values; $j=1,\dots,n$.

The structure of a container ship transport is exclusively made of containers, and possibly of the transport of RO/RO cargo. Containers loaded on container ships are uniquely standardized (in terms of length, width, height), which enables setting conditions to be met for all cases when formulating a model.

To use such a mathematical model set for solving the problem of the transport of containers by sea, it is assumed that the maritime market offers enough containers of various types, masses and sizes, i.e., greater than the transport capacity of a ship. This assumption is by all means true for feeder ships, having a capacity of up to 1200 TEU, which results from their function of transporting containers from the so called hub terminals, where large container ships discharge more than 6000 TEU.

The mathematical model for the formulated problem is as follows:

$$R = \frac{150x_1 + 250x_2 + 75x_3 + 125x_4 - 25911}{1.74 + \frac{1}{420}x_1 + \frac{1}{420}x_2 + \frac{1}{420}x_3 + \frac{1}{420}x_4}$$

with constraints:

$$\begin{aligned} & x_1 + 2x_2 + x_3 + 2x_4 \le 250 \\ & 25x_1 + 30x_2 + 2.5x_3 + 3.5x_4 \le 5615 \qquad (7) \\ & x_1, x_2, x_3, x_4 \ge 0 \text{ and integer.} \end{aligned}$$

Notation x_j, $j=1,2,3,4$ refers to the container type as per Table 1, respectively. The numerator of function R represents the difference between the revenue of the transport of particular container type and total costs, including voyage costs and cost of the ship stay in a port. The denominator of function R represents the duration of a voyage and the transshipment time in a port for a particular container type, taking into consideration the transshipment capacity of a container crane, continuous operation and number of cranes simultaneously unloading a ship.

Solving technique

The problem was solved by using Martos method, i.e., modified simplex method. It is the criterion for selecting a new vector in the base that Martos developed and implemented in the simplex method algorithm, which represents his modification of the simplex method. For details on Martos method, please refer to Martić [3, p. 36, 37].

According to the simplex method, the starting point is one possible nondegenerated basic solution. The method then locates successively other basic feasible solutions having better values of the objective until optimal solution is obtained. The solution of the mathematical model (7) with the modified simplex method is shown in table 2.

In short, the solving technique is the following:

1) After the problem formulation, i.e. transforming a word problem into a mathematical model (6), it needs to be shifted into form where all constraints are modeled as equalities by introducing slack and surplus

variables where necessary. The introduction of slack and surplus variables alters neither the nature of the constraints nor the objective. Accordingly, such variables are incorporated into the objective function with zero coefficients. See details in [2,7].

2) The starting base consists of unit column vectors with the element one, and other elements zero (A_5, A_6). These vectors match to slack variables and surplus variables if there are any.
3) Elements in the row c_i are coefficients of structural variables (x_1, x_2, x_3 and x_4) from numerator of R (7), and elements in the row d_i are coefficients from denominator.
4) Row z_j^1 elements are obtained as the sum of the product of multiplication of coefficient c_i from the first vector in the base and the first element from vector of constraints and the product of multiplication of coefficient c_i from the second vector in the base and the second element from vector of constraints (i.e. $0\times250+0\times5615$).
5) Analogously, row z_j^2 elements are obtained, with the difference that instead of the coefficients c_i, coefficients d_i of the matching vectors in the base are used.
6) The elements of the row z_j^1-c_i result from the difference of the corresponding elements z_j^1 and c_i. By the same analogy, elements of the row z_j^2-d_i are calculated.
7) Row Δj elements are obtained as the cross-multiplication of the element in the column of the vector of constraints, row z_j^1-c_i, with the element in the column of the vector for which Δj is calculated, row z_j^2-d_i, i.e. [(-25 911× (-0.0024)]. After that, the element of the column A_0, row z_j^2-d_i, is multiplied with the element in the column of the vector for which Dj is calculated, and row z_j^1-c_i, i.e. [(1.74× (-150)]. Then, these two products of multiplication are subtracted . [(-25 911× (-0.0024)]- [(1.74× (-150)] = 323.2. It goes further by the same analogy.
8) The condition for the optimum solution is the following: $\Delta j \leq 0$, for each j. If this condition is not satisfied the base should be changed. The modification of the base and the calculation of the new basic solution elements is done in the same way as in the simplex method, see [7].

The modified simplex method asks for a larger number of operations than the ordinary simplex method, when applying the linear programming problem of the same size.

Table 2. Simplex tableau for the mathematical model (7)

c_i:			-25 911	150	250	75	125	0	0	R
	d_i:		1.74	0.0024	0.0024	0.0024	0.0024	0	0	
		Base	A_0	A_1	A_2	A_3	A_4	A_5	A_6	
0	0	A_5	250	1	2	1	2	1	0	125
0	0	A_6	5615	25	30	2.5	3.5	0	1	187
		z_j^1	0	0	0	0	0	0	0	
		$z_j^1-c_i$	-25911	-150	-250	-75	-125	0	0	
		z_j^2	0	0	0	0	0	0	0	
		$z_j^2-d_i$	1.74	-0.0024	-0.0024	-0.0024	-0.0024	0	0	
		Δ_j	$\frac{-25911}{1.74}$	323.2	497.2	192.7	279.7	0	0	
250	0.0024	A_2	125	1/2	1	1/2	1	1/2	0	250
0	0	A_6	1865	10	0	-12.5	-26.5	-15	1	186
		z_j^1	31 250	125	250	125	250	125	0	
		$z_j^1-c_i$	5 339	-25	0	50	125	125	0	
		z_j^2	0.3	0.0012	0.0024	0.0012	0.0024	0.0012	0	
		$z_j^2-d_i$	2.04	-0.0012	0	-0.0012	0	0.0012	0	
		Δ_j	$\frac{5339}{2.04}$	44.6	0	-108.4	-255	-248.6	0	
250	0.0024	A_2	31.75	0	1	1.125	2.325	1.25	-1/20	
150	0.0024	A_1	186.5	1	0	-1.25	-2.65	-1.5	1/10	
		z_j^1	35 912.5	150	250	93.75	183.75	87.5	2.5	
		$z_j^1-c_i$	10 001.5	0	0	18.75	58.75	87.5	2.5	
		z_j^2	0.5238	0.0024	0.0024	-0.0003	-0.0008	-0.0006	0.0001	
		$z_j^2-d_i$	46054	0	0	-0.0027	-0.0032	-0.0006	0.0001	
		Δj	$\frac{10001}{2.26}$	0	0	-69.4	-164.8	-203.75	-4.45	

Analysis of the optimal solution

According to the optimal program, the following results are obtained:

- $x_1 = 186.5$ containers of the first container type
- $x_2 = 31.75$ containers of the second container type
- $\Delta j =$ US$ 4,425 profit per one day.

As it can be seen, decimal results were obtained and, at the same time, structural variables represent the number of containers which has to be integer. In this case the branch-and-bound method is used for getting integer results, due to practical reasons. The branch-and-bound method is incorporated into the programme package WinGULF 3.1, see [1], for linear and linear-fractional problems.

With this programme, the result is as follows:

- x_1=186 containers
- x_2=32 containers
- R=US$ 4,413.66 - the optimal objective value.

Such a combination of cargo offers a full exploitation of the transport capacity and 5 tons of unused payload of container ship.

The maximum profitability of the Rijeka-Gioia Tauro line for the transport of the obtained combination of two types of full containers, expressed as profit per unit of time, i.e., profit per one day, amounts to US$ 4,413.66.

Since the function of a feeder ship is to deliver all containers from the home port to all other ports in the respective area, i.e., load all available cargoes regardless of profit yielded, the actual situation is generally worst than the obtained optimum programme. The optimum programme shows the potential maximum profitability, if the container types were selected arbitrarily. Therefore, the state subvention granted to such a service, which is the case in the Republic of Croatia, is quite justified.

The model presented in this paper could be the base to the shipowner "Lošinjska plovidba" at defining the amount of subvention, which is annually granted out of the Budget of the Republic of Croatia. Namely, the difference between the maximum profitability obtained in the model and the effectuated or expected amount of profitability of the container transport by ship is the so-called "lost profit", that should go to the shipowner.

Therefore, the obtained optimum solution represents the quantitative basis for the analysis of the results achieved by a container ship in the previous period and for the estimate of the expected future results.

3. CONCLUSION

In real life, it is often necessary to calculate the optimum productivity, economic justifiability and profitability as indices of the operation efficiency. Profitability is an especially important index of operation efficiency, representing the ratio between the profit yielded and the resources invested or consumed, i.e., time spent in the realization of the business result.

If these indices are chosen as a criteria, then the respective objective function in the mathematical model is the fractional function and the problem is solved by using the linear fractional programming.

In this paper the mathematical model for determining the optimum structure of a container ship, when the optimality criterion represents the maximum profitability, is presented. The model was tested on a real-life example of the feeder ship "Lipa" owned by "Lošinjska plovidba", Rijeka, and operating on the Rijeka-Gioia Tauro route.

The presented model can be used in operative planning, when making respective business decisions referring to the structure of cargo and by using other means of transport, aiming at obtaining a maximum profitability.

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Sažetak

RAZLOMLJENO PROGRAMIRANJE U OPTIMIZACIJI USPJEŠNOSTI POSLOVANJA BRODARA

U radu je prikazana primjena linearnog razlomljenog programiranja u određivanju rentabilnosti kao pokazatelja uspješnosti poslovanja. Postavljen je matematički model za problem prijevoza tereta kontejnerskim brodom kada se za odabranu morsku liniju od ukupnog broja kontejnera koji se nalaze u ukrcajnoj luci odabire određeni broj kontejnera raznih vrsta i masa s ciljem ostvarenja maksimalne rentabilnosti prijevoza i uz uvjet da se, što je više moguće, iskoristi nosivost i prijevozni kapacitet kontejnerskog broda. Model je testiran na primjeru prijevoza kontejnera feeder brodom "Lipa" brodara "Lošinjske plovidbe" iz Rijeke na liniji Rijeka-Gioia Tauro.

Ključne riječi: linearno razlomljeno programiranje, rentabilnost, prijevoz kontejnera brodom