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# The possibilities and effects of promoting logical-combinatorial thinking in initial mathematics teaching

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## Summary

*The purpose of the paper is to emphasize the importance of promoting and developing logical-combinatorial thinking in students at an early age, in initial mathematics teaching, and to point to certain specificities of this teaching model. With regard to that, the authors firstly provide the definition of the term logical-combinatorial thinking and stress the need for an early introduction of the elements of logic and combinatorics in the teaching process. Taking into consideration the specific nature of initial mathematics teaching and students' age, a special approach to the teaching content was designed, which involves a range of various combinatorial scenarios that can be used in the teaching process. Experimental research was carried out on a sample comprising 276 students in the fourth grade of primary school in the Republic of Serbia. The aim of the research was to examine if it is possible to develop logical-combinatorial thinking in students in mathematics teaching by selecting suitable content (tasks) which involve the application and solving of typical logical-combinatorial problems and contextual examples. The findings indicate that an appropriate selection of content can significantly influence the optimal development of logical-combinatorial thinking in students.*

***Keywords**: combinatorics; logic; mathematics; reasoning; task*

## Introduction

Efficient and socially acceptable basic mathematics education, which implies an increase in the overall educational potential of students, requires a large and elaborate structure of an educational process. Increased needs of modern individuals for understanding stochastic phenomena in the real world impose the need for development of the necessary functional knowledge and logical-combinatorial abilities as primary goals of mathematics education. Recognition of problems, an abundance of ideas, originality and variation of the basic knowledge during problem-solving process, as well as fluency and flexibility of mathematical thinking are all desirable characteristics of a mathematically well-educated individual. In that sense, combinatorial thinking, systematic exploration of possible solutions to problems, the ability to combine familiar concepts, opinions and rules to obtain new expressions are imperatives of the learning outcomes, even in initial mathematics teaching. Apart from that, understanding and determining statistical laws of probability in similar phenomena, frequent in everyday life, have been recognized as exceptionally significant content.

Starting from the fact that logical-combinatorial thinking and reasoning are at the core of mathematics instruction goals throughout the entire educational process, it can be assumed that there are pedagogical-psychological and didactic-methodological needs for and possibilities of promoting its development early, in the initial phases of the teaching process. Propaedeutic introduction of the content relating to the basic ideas and methods of combinatorics in primary mathematics instruction makes it possible to influence the development of students' mental, logical-combinatorial abilities and functional knowledge, which are necessary for solving various problems in everyday life. Considering the importance of promoting mathematical thinking in students at an early age, we focused this research on the theoretical and empirical consideration of the development of the elements of logical-combinatorial thinking in classroom teaching of mathematics. The paper examines whether mathematics teaching enriched by introducing the content which promotes the elements of logical-combinatorial thinking can be improved.

## Theoretical background of the paper

A special feature of mathematics instruction is a focused development of thinking skills in mastering a certain type of content. Great attention has been paid to students' acquisition of the necessary knowledge, as well as to their ability to detect, evaluate and apply the acquired knowledge logically.

Early introduction of the term and elements of logic and combinatorics in initial mathematics instruction is justified because logical-combinatorial thinking is an important component in the process of achieving mathematics learning outcomes. This means that we should not wait for the formal teaching of combinatorics as an isolated topic in secondary school; instead, we should offer students, as early as possible, tasks from interesting, familiar situations, in order to develop logical-combinatorial thinking ability (Zapata-Cardona, 2018).

By introducing the elements of combinatorics in initial teaching, the necessary conditions are ensured for creative observation, thinking, and behaviour, as well as for rational and causal explanation of stochastic phenomena, that is, for an early development of ideas about probability (English, 2005). Active student participation, risk-taking, making assumptions, and varying possible solutions are key features of instruction which includes the elements of combinatorics, and which contributes to the quality of the teaching process (Pinter Krekić, 2006). Such environment should be characterized by appropriate combinatorial problems which would promote students' thinking activity with the aim of promoting elastic thinking, sensitivity to problems, finding nonstandard answers, discovering new combinations and original ideas (Pinter, 1997) and finding logical explanations of these ideas (Syafitri et al., 2020).

Incikabi et al. (2013) define logical-combinatorial thinking as reasoning which involves consistent reasoning (according to Lazić et al., 2022), when the conditions set in the task require various options, permutations, combinations and relations which should be determined (Savenkov & Romanov, 2021). For Lockwood (2012), the basics of logical-combinatorial thinking entail conceptual logical analysis and students' thinking activity focused on finding the solutions to stimulating combinatorial problems, based on the procedures and activities of combinatorial enumeration and their understanding.

Various international programmes designed for measuring student achievement in mathematics within TIMSS (Trends in International Mathematics and Science Study) research and PISA (Programme for International Student Assessment) testify to the importance of this category of thinking, as well as to the need for promoting its development throughout the educational process. Important differences between Serbia and the countries which have scored better on these tests can be found precisely in the topics covered in mathematics instruction in the lower grades of primary school. These refer primarily to functional knowledge which includes quick thinking and reasoning skills (Milinković & Lazić, 2018).

Combinatorics, as a field of mathematics closely related to algebra, the theory of probability and geometry, has found wide application in various domains of life and work in the contemporary society. Recognizing and understanding the core of combinatorial problems has been considered extremely important for development of mathematical thinking and multiplicative reasoning, and has therefore been included in the core curricula in several European countries (Slovakia, Hungary, Germany, and Slovenia). The National Centre for Mathematics Education (NCTM, 2000) also emphasized the importance of early introduction of such content, recommending the introduction of discrete mathematics in the early mathematics learning and teaching curriculum.

On the other hand, an overview of the mathematics curriculum content in the Republic of Serbia reveals a slight representation of this kind of content, especially in the first cycle of primary education. The area of logic and combinatorics is considered too complex for classroom teaching, although there is a need and there are possibilities for their propaedeutic introduction, as they have a positive impact on the creation of mental images and mathematical concepts in children. Such content enables development of logical thinking and reasoning, as key abilities necessary in real life (Milinković & Lazić, 2018).

In the time of industrial revolution, a holistic development of each individual is imperative, as it promotes development of logical-combinatorial thinking, and application and interpretation of mathematics in various contexts. Since logical-combinatorial thinking and reasoning are important elements which can not be separated from learning mathematics (Hidayati et al., 2019), it is evident that it is the focus of mathematics instruction goals at all levels of education, and that it has been a subject of research in a broad scientific and education community.

Contemporary school should promote and develop logical-combinatorial thinking by providing students with numerous and diverse combinatorial situations in the teaching process (Melusova & Vidermanova, 2015). In order to develop logical-combinatorial thinking successfully, teachers need to integrate explicitly the abilities and skills of logical thinking into the entire knowledge acquisition process (Maričić et al., 2016). In that sense, it is necessary that initial mathematics instruction be associated with everyday situations in order to help students create a system of interrelated thinking operations and a whole that includes logic as a key skill of thinking activity. Progress in mathematical abilities development, and therefore in logical-combinatorial thinking, made through solving problem tasks has been detected in several similar studies (Anić & Pavlović Babić, 2015; Kadum, 2005; Mandak & Pavličić, 2016). They point out that problem solving as a process involves a range of methodical and logical operations which need to be performed to solve a problem task, and that it can not be examined isolated from thinking and learning. Such problem-based learning is characterized by an active mental process which promotes interpretation, information collection, and recognition of possible solutions to a problem. Problem-based learning improves logical thinking and mathematical communication skills (Anwarudin et al., 2020).

Care for early logical-combinatorial thinking development starts by promoting mathematics education through introduction of well-structured exercises and tasks which can be associated with other school subjects (Nunes & Csapó, 2011).

Some researchers interested in mathematics education, such as Graumann and Hackinga (according to Rezaie & Gooya, 2011), emphasize that learning about combinatorics concepts requires a special way of thinking. That is why combinatorics has a special role in mathematics teaching, as it is a significant factor for higher-order thinking skills development. These researchers claim that combinatorics is a special aspect of mathematical thinking and that it is an essential "tool" for solving combinatorial problems in geometry, which requires students to use their combinatorial skills, explore all possibilities, and find a systematic solution.

Research findings (Batanero et al., 1997) indicate that students' combinatorial ability is an important component of formal thinking. According to this study, even small children can use experimental or empirical probabilities. They conduct multiple experiments to examine how a certain phenomenon occurred, and by comparing them to the total number of experiments they obtain an experimental assessment of probability. It is claimed that primary school students should develop a thorough understanding of experiment limitation by comparing the experimental results with mathematically obtained probabilities, which often require combinatorial reasoning (Batanero et al., 1997).

Aini et al. (2020) emphasize that combinatorial reasoning is the basic competence each student should acquire in order to solve mathematical problems by using an appropriate strategy. During research, they broke down a complex task into several tasks based on the similar principle, and then conducted an interview to examine students' combinatorial thinking while solving the given tasks. They concluded that combinatorial thinking plays a key role in solving mathematical problems because it represents a cognitive system which includes a reasoning strategy. Such view implies that problem solving and logical-combinatorial thinking are two key aspects of mathematics learning goals (Aini et al., 2020).

The skills which students develop while applying combinatorial thinking in mathematics encompass assessment, generalization and systematic thinking (Maylisa et al., 2020). In a study conducted by Pinter-Krekić et al. (2015), strategies for solving combinatorial problems have been pointed out (permutations, combinations and variations), with special emphasis placed on the importance of methodical transformation of combinatorics content in mathematics teaching. The findings of research (Krpec, 2014) which was focused on combinatorial abilities of students in lower grades of primary school revealed that students face difficulties in solving combinatorial tasks. Oparnica et al. (2016) presented some combinatorial task models which can be easily integrated into the existing teaching content for lower grades of primary school, while Gordić et al. (2019) pointed to students' motivation for solving simple combinatorial tasks related to their timetable, grouping or the chosen subjects in initial mathematics teaching. Rakić et al. (2021) stated that some parts of the initial mathematics instruction teaching content enable teachers to promote logical-combinatorial thinking in students by applying various types of differentiated tasks. Such findings provide the necessary basis for further interpretation and examination of this current research problem.

## Methodology

The main aim of this research was to examine the possibility of developing elements of logical-combinatorial thinking in the fourth grade of primary school by designing and selecting suitable content. The research was based on the assumption that the elements of logical-combinatorial thinking can be developed by shaping the mathematical content in an appropriate manner and by engaging students in problem situations in which their logical-combinatorial thinking abilities become evident.

Research was conducted on a sample comprising students (N = 276) chosen from student population in the fourth grade of primary schools in the Republic of Serbia. The sample was randomly chosen – classes in schools were selected as well as all students comprising them. As the research was based on an experiment conducted in parallel groups, two equal groups of students were formed – the experimental group (E) and the control group (C). The experimental group was composed of six fourth-grade classes from two primary schools (N = 138), and the control group consisted of the same number of students (N = 138) from six fourth-grade classes from two primary schools (the city of Novi Sad).

To make the research groups equal, we used the following criteria: students' grade point average, their achievement in mathematics at the end of the third grade, and the results of the initial test (IT) in which they had to show logical thinking and combinatorics.

The experimental method with parallel group design was used in the research. The experimental programme was focused on the early promotion and improvement of students' logical-combinatorial thinking development, through the application of the specially designed approach to algebraic and geometric content. The programme was based on the creation, implementation and solving of various typical logical-combinatorial tasks given in mathematical context and real-world combinatorial problems. Both of these task types were equally represented during the research. The experimental programme included all curriculum topics for fourth-grade algebra and geometry curriculum. It was implemented during the second term throughout twenty-three Mathematics lessons, and was integrated into the regular curriculum students were following. According to the author's instructions, the programme was implemented by teachers who teach the fourth grade of primary school. The author was in constant contact with the teachers, followed the implementation process and provided support and assistance. During that period, the control group was following the regular curriculum. For the purpose of illustration, we will mention a few examples used in the experimental programme.

Example 1.

Example 1: *Write all three-digit numbers the sum of whose digits is 3. How many of them are there?*

Example 1 presented above belongs to the field of variations. While solving the problem, students primarily need to find all possible combinations of three numbers whose sum is 3, exhibiting fluent thinking. To do so, they make the following combinations: (3, 0, 0), (2, 1, 0) and (1, 1, 1). The following step is related to the field of permutations, where students write three-digit numbers for each combination of numbers: (3, 0, 0) – 300; (2, 1, 0) – 210, 201, 102, 120; (1, 1, 1) – 111. The example promotes logical-combinatorial thinking, an analysis of thinking operations, synthesis and comparison, creative observation and finding possible solutions.

Apart from such examples, given in a purely mathematical context, students were solving contextual tasks as well (Example 2).

Example 2: *There are 2 routes to get from Sombor to Novi Sad, and to get from Novi Sad to Niš there are 3 routes. In how many ways can we travel from Sombor to Niš via Novi Sad?*

In this example, students exhibit the skill of solving combinatorial problems, that is, they exhibit combinatorial thinking elements, their estimation ability, generalization, interpretation, and visualization of the given information (optical processing), variation and systematization of the possible solutions, logical reasoning, combinatorial reasoning, and the ability to draw conclusions.

We will also present an example from the field of geometry (Example 3).

Example 3. *Petra has placed in a stand the figures of various shapes: a square, a rectangle, a triangle and a circle. She has arranged a total of 6 figures next to each other. Each figure has exactly one shape as its base.*

Petra has revealed the following: All figures with the same number of sides are placed next to each other.

- Two figures have a square each as the base.
- The figures with more than 3 and less than 5 sides are placed between a square and a triangle.
- The figures with a circle as a base do not touch the squares.
- The figures in the middle (the 3$^{rd}$ and 4$^{th}$ in row) have a triangle and a rectangle as a base, but not in that order.

Based on the given information, write the order/arrangement of the figures according to the bases, as Petra has arranged them.

The solution: a square – a square – a rectangle – a triangle – a triangle – a circle.

The effects of the experimental programme were determined by employing examination technique. Two tests were performed: the initial test, before the experimental programme implementation, and the final test (FT), after the completed implementation of the experimental programme. For the purpose of this research, we designed knowledge tests containing 10 tasks each, in order to determine the difference in students' level of logical-combinatorial thinking while working on problem tasks: the initial test (IT) and the final test (FT). The final test (FT) included fourth-grade primary school algebra and geometry curriculum content. Apart from the typical combinatorial tasks (permutation, combination, and variation), the tests also include various real-world, context-based tasks which require logical-combinatorial thinking. These two groups of tasks were equal in number in both tests, containing 5 tasks each.

Firstly, preliminary testing was conducted on a sample comprising 45 students, after which corrections to the instrument were made and the final forms of the tests were created. During the testing stage, all students were put in almost the same testing conditions, while the solutions were checked in the same way, following the instructions based on the key in order to ensure the objectivity of the testing procedure. Logical and content test validation was performed, as well as validation of alignment of tests with the curriculum content. The instrument reliability was determined by calculating Cronbach's alpha coefficient (α = .88), which pointed to a high level of instrument reliability. Test discrimination was determined through item analysis. Coefficients of discriminative task value range from 0.16 to 0.25. The tasks with discriminative value lower than 0.20 were replaced by new tasks, after which discriminative value of each task was higher than 0.20.

The data obtained in the research were processed using the IBM Statistics SPSS23 software. One-way analysis of variance (ANOVA) was used to monitor the experimental programme effects in the experimental group, as well as for statistical control of the equality of research groups.

## Results

Descriptive statistics results (Table 1) indicate that the experimental and control groups are almost equal in the arithmetic means of the initial test results: the experimental group (M = 10.65, SD = 3.614), and the control group (M = 10.17, SD = 2.816). In the initial measurement of the logical-combinatorial thinking development stage, students in both groups scored only a half of the maximum number of points (20).

Table 1
*Descriptive statistics of the sample for initial and final test (IT and FT)*

| Test | Group | N | Mean | Std. Dev. | Std. Er. | 95% Confidence Interval for Mean | | Min | Max |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Lower Bound | Upper Bound | | |
| Initial test | Experimental group | 138 | 10.6486 | 3.613 | .307 | 10.0402 | 11.2569 | 1.00 | 19.00 |
| | Control group | 138 | 10.1667 | 2.815 | .239 | 9.6927 | 10.6406 | 4.00 | 18.00 |
| | Total | 276 | 10.4076 | 3.242 | .195 | 10.0234 | 10.7918 | 1.00 | 19.00 |
| Final Test | Experimental group | 138 | 15.7319 | 2.746 | .233 | 15.2696 | 16.1941 | 9.00 | 22.00 |
| | Control group | 138 | 10.0978 | 2.438 | .207 | 9.6874 | 10.5082 | 5.00 | 20.00 |
| | Total | 276 | 12.9149 | 3.831 | .230 | 12.4608 | 13.3689 | 5.00 | 22.00 |

After the initial testing, the experimental programme was introduced in the experimental group. The programme is based on solving combinatorial tasks, i.e., on promoting the logical-combinatorial thinking elements. On the other hand, the control group followed the set curriculum and the selected coursebook (the same as in the experimental group). The final test was administered after the experimental programme implementation had been completed. The comparison of the scores (Table 1) achieved by the experimental and the control group revealed that the experimental group scored a higher number of points in the final test after the implementation of the experimental programme, which is analysed through the presented arithmetic means – the experimental group (M = 15.73, SD = 2.746) and the control group (M = 10.09, SD = 2.438). We can also see that the experimental group's score was significantly higher in the final test than in the initial test, which can be ascribed to the effect of the experimental programme. The control group achieved almost the same score in the initial and in the final test.

The illustrated representations of the achievement distribution of the experimental and the control group in the plot diagram (Figure 1) show approximate equality. It can also be seen that the experimental group has a significantly larger range of the scores in comparison with the scores achieved by the control group, which are grouped to a larger extent. It can also be seen that the final test scores of both groups are symmetrically distributed, with approximately equal range. However, the experimental group's score was statistically significantly higher, both in comparison with the control group's score and the initial test score. At the same time, two students in the control group achieved a significantly better score in the final test, in comparison with other students in their group.

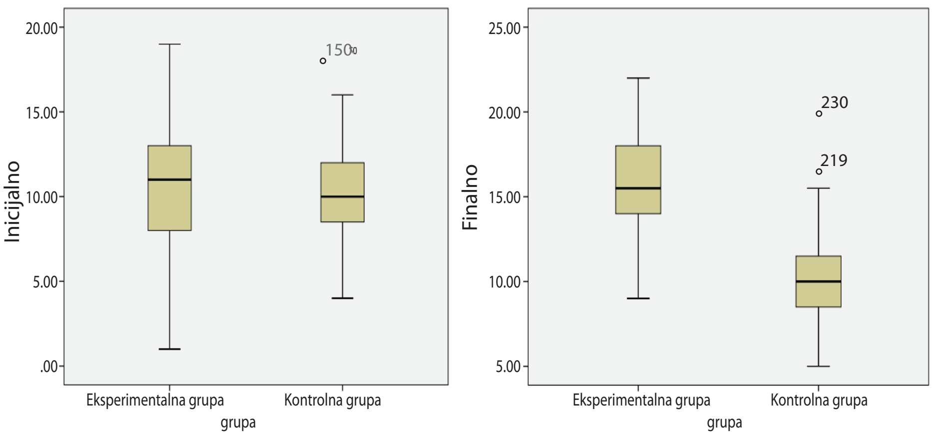


*Figure 1.* Graphical representation of the sample dispersion in the initial and the final test

Table 2
*Variance homogeneity test*

| | Levene's Statistics | df1 | df2 | Sig. |
|---|---|---|---|---|
| Initial testing | 8.586 | 1 | 274 | .004 |
| Final testing | 3.127 | 1 | 274 | .078 |

Levene's value of the initial testing (F(1.274) = 8.586; p = 0.004) indicates that the groups are not equal, while the in the final testing (F(1.274) = 3.127; p = 0.078), the assumption about variance homogeneity was not challenged. The groups were almost equal and the result obtained by variance analysis can be considered valid (Table 2).

The analysis of variance (ANOVA) statistical procedure was performed to examine if there are significant differences between the experimental and the control group in initial and final testing. The results obtained by ANOVA (Table 3) also reveal that both the experimental and the control group were equal in terms of the arithmetic means of the initial testing score, which is evident as there are no statistically significant differences between the experimental and the control group in initial testing. The value of the calculated variance in initial testing ($F(1.274) = 1.527$; $p = 0.218$) shows that there is no statistically significant difference between the groups. Taking into consideration the inequality of variance in initial testing, we performed Welch's t-test ($t(260) = 1.236$, $p = 0.218$), whose value shows that there is no statistically significant difference between the groups in initial testing.

Table 3

*Variance analysis of the initial and final test (ANOVA)*

| | Test | Sum of Squares | Df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| Initial testing | Between groups | 16.023 | 1 | 16.023 | 1.527 | .218 |
| | Within groups | 2875.371 | 274 | 10.494 | | |
| | Total | 2891.394 | 275 | | | |
| Final testing | Between groups | 2190.240 | 1 | 2190.240 | 324.830 | .000 |
| | Within groups | 1847.509 | 274 | 6.743 | | |
| | Total | 4037.749 | 275 | | | |

However, as was confirmed by ANOVA, the final test revealed that the experimental group had statistically significantly higher scores than the control group. The variance analysis result in final testing ($F(1.274) = 324.830$; $p = 0.000$) shows that there are statistically significant differences between the experimental group, which was involved in the experimental programme, and the control group, which followed the standard teaching methods. If we take into consideration the results obtained by descriptive statistics (Table 1), we can claim that students in the experimental group achieved statistically significantly better results in comparison with students in the control group. The obtained results indicate that solving combinatorial problems within the teaching and learning mathematics content can affect the development of logical-combinatorial thinking elements in younger students.

Also, after the final test, we compared the scores of both groups in terms of task types. The findings show that the experimental group scored more points than the control group in solving both task types (typical combinatorial tasks and context-based tasks), which we believe can be ascribed to the effect of the experimental programme (Table 4). Apart from examining the significance of mean differences, we also wanted to examine the level of the mean difference between the groups in terms of task types, by performing *Cohen's d* effect size.

The obtained results indicate an extremely large effect size in both task types (logical-combinatorial and real-world problems), which is a clear confirmation of a strong positive effect of the experimental programme on student achievement in both task types.

Table 4
*Comparison of the experimental and the control group's scores in terms of task types in final testing (Cohen's d)*

| Task type | Experimental group (M) | Control group (M) | Mean difference | *Cohen's d* | *p* |
|---|---|---|---|---|---|
| Logical-combinatorial tasks | 17.49 | 10.08 | 7.41 | 2.19 | $< 0.001$ |
| Real-world problems | 15.30 | 7.32 | 7.98 | 2.62 | $< 0.001$ |

## Discussion

The findings of this research indicate that the implemented experimental programme based on the specially designed approach to algebraic and geometrical content and which enables a range of combinatorial situations in the teaching process (solving typical logical-combinatorial tasks and context-based examples) can significantly contribute to the development of logical-combinatorial thinking elements in the first cycle of mathematical education. As evidenced by our research findings, students in the experimental group, who were involved in the implementation of the specially designed programme for solving combinatorial tasks, have shown significantly better developed elements of logical-combinatorial thinking in the final test in comparison with the students who were not involved in the programme. The initial test scores were very similar, which proves the efficiency of the implemented programme. Apart from that, another proof testifying to the efficiency of the programme designed for developing logical-combinatorial thinking elements in students is the fact that the experimental group's final test score was much higher than its score in the initial test.

The findings obtained in this research make a significant contribution to studying the factors which have an impact on the formation of logical-combinatorial thinking in lower grades of primary school, and as such they are a segment of a picture created with the existent research findings which have also pointed to the importance of solving combinatorics for development of higher and more complex thinking patterns at an early age. Some of the more stressed segments are the formation of formal thinking, the ability to generalize and to systemize (English, 2005; Hidayati et al., 2019; Rezaie & Gooya, 2011; Zapata-Cardona, 2018). Bearing in mind the importance of promoting logical-combinatorial thinking in the context of the holistic cognitive development and progress in childhood, it is clear how important it is to determine all the individual and environmental factors, especially the teaching process factors, which contribute to it (Melusova & Vidermanova, 2015; Nunes & Csapó, 2011).

Our research findings are in line with the findings obtained in some previous studies described in the introductory part of this paper. They also show that solving problem tasks improves students' cognitive abilities, especially in the domain of promoting logical-combinatorial thinking. On the other hand, there is a reversible relationship - well-developed logical-combinatorial thinking contributes to successful solving of various problem tasks (Aini et al., 2020), which paves the way for future research on this reversible relationship in early childhood.

The obtained results are significant as they reveal that such tasks can also be very useful for the cognitive development of students in lower grades of primary school, not only of older students, as has frequently been assumed. These tasks enable students to develop their problem-solving techniques and to use not only combinatorics but also their intuitiveness and creativity. In that way students' thinking processes become more flexible, more open to new possibilities and ready to go beyond the standard problem-solving framework. This is a skill that should be promoted and developed in various ways, even at a young developmental stage, i.e., in early school age because that is when thinking activities develop intensely and can be shaped by diverse educational content and work methods (Maylisa et al., 2020; NCTM, 2000).

Due to the confirmed importance of the impact combinatorial problems have on the development of logical-combinatorial thinking in students in lower grades of primary school, a practical implication of this research is to expand the mathematics instruction teaching content by adding tasks of this type, which not only contribute to the development of students' cognitive capacities but also their creative potential and the ability to face every-day, practical tasks. For that reason, starting from the lowest educational level, it is necessary to promote continuously and systematically the development of students' cognitive potential, by providing them with various types of complex tasks which will contribute to their active participation in the learning process in an interesting and creative way. This will also develop their general cognitive capacities and more complex thinking activities, as well as encourage them to actively engage in various situations.

## Conclusion

The propaedeutics of logic and combinatorics as concepts has an important role in the initial mathematics teaching: it is reflected in understanding various everyday life phenomena in nature, society and play, in mathematics, in the immediate development of logical-combinatorial thinking and free, creative mathematical thinking, and combination of play and learning, which strongly motivates students in initial mathematics instruction. One of the ways to contribute to the benefits mentioned above is an intuitive way of solving simple combinatorial problems.

Bearing in mind the undisputable significance of mathematical thinking in the overall educational potential and a wide range of its development possibilities, we believe that it is necessary to provide adequate support and to promote primarily students' logical-combinatorial abilities in an appropriate way in the first cycle of primary mathematics education. That could be achieved by enriching mathematics teaching with content which promotes logical-combinatorial thinking, and by providing concrete models for the implementation of the teaching process.

By introducing combinatorics elements in the initial mathematics teaching, the necessary conditions for an early development of ideas about probability and combinatorial reasoning would be ensured, which is especially important for recognizing and understanding the essence of combinatorial problems. At the same time, students exhibit a high level of motivation for solving such structured, simple combinatorial tasks and possible ways of arranging, grouping and detecting possible solutions. This entails the improvement of mathematical competences where logical-combinatorial thinking has special importance.

The findings of this research show that real-life combinatorial tasks have a significant influence on the development of logical-combinatorial thinking and that they can be implemented in mathematics teaching in lower grades of primary school. By carefully choosing and enriching the content which includes logic and combinatorics, with proper guidance, children learn how to recognize and solve combinatorial problems while discovering similarities in various problems. Finding as many possible solutions to a problem comes after finding all possible solutions, that is, determining the number of those solutions. This way of solving tasks also implies various combinations or variations of the applied elementary mathematical knowledge that students already possess, as well as related algorithmic steps, which speaks about the essence of logical-combinatorial thinking during problem-solving process.

The approach described above helps set the primary goals of contemporary mathematics teaching, the development of critical thinking and functional knowledge, and the concept of creating the conditions in which students acquire complex knowledge necessary for functioning in the 21st century (Ravitz et al., 2012). Bearing in mind the necessity of ensuring students' constant mental activity in initial mathematics teaching, increasing the level of their motivation for learning about mathematics topics and facilitating the process of solving practical real-life problems, we emphasized the role and importance of promoting logical-combinatorial abilities at an early age. The obtained results support the assumption that such content is not only suitable for older students but can also be beneficial to students in lower grades of primary school. In addition, gaining certain experience, knowledge, skills and habits in games, stories, and combinatorial tasks makes a significantly enriches assumptions while solving problems both in mathematical and interdisciplinary context. It also prepares students well for successful further study of these important topics at higher levels of education.

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# Logičko-kombinatorno mišljenje u početnoj nastavi matematike – učinci i mogućnosti poticanja


## Sažetak

*U radu se ukazuje na važnost ranoga poticanja i razvoja logičko-kombinatornoga mišljenja učenika u početnoj nastavi matematike te se ističu određene specifičnosti ovoga modela nastavnoga rada. S tim u vezi, autori se najprije usmjeravaju na definiciju pojma logičko-kombinatornoga mišljenja i potrebu ranoga uvođenja elemenata logike i kombinatorike u nastavni proces. Na temelju specifičnosti početne nastave matematike i dobi učenika, osmišljen je poseban pristup sadržajima koji predviđa niz različitih kombinatornih situacija u nastavi. Provedeno je eksperimentalno istraživanje na uzorku od 276 učenika četvrtih razreda osnovnih škola u Republici Srbiji, s ciljem da se ispita može li se odabirom primjerenih sadržaja (zadataka) koji uključuju primjenu i rješavanje tipičnih logičko-kombinatornih zadataka, kao i kontekstualnih primjera, razviti logičko-kombinatorno mišljenje učenika u nastavi matematike? Rezultati istraživanja pokazuju da se primjerenim izborom sadržaja može značajno utjecati na optimalan razvoj logičko-kombinatornoga mišljenja učenika.*


**Ključne riječi:** *kombinatorika; logika; matematika; rasuđivanje; zadatak*

## Uvod

Učinkovito i društveno prihvatljivo osnovno matematičko obrazovanje, odnosno povećanje ukupnoga obrazovnog potencijala učenika, zahtijeva veliku i bogatu strukturu odgojnoga i obrazovnoga procesa. Povećane potrebe suvremenoga čovjeka za razumijevanjem stohastičkih pojava u stvarnom životu nameću razvoj neophodnih funkcionalnih znanja i logičko-kombinatornih sposobnosti kao jednih od primarnih ciljeva matematičkoga obrazovanja. Osjetljivost za problem, bogatstvo ideja, originalnost i variranje osnovnih znanja pri rješavanju problema, kao i fluentnost i fleksibilnost matematičkoga mišljenja željene su karakteristike matematički obrazovnoga pojedinca. U tom pravcu, kombinatorno mišljenje, sustavno iscrpljivanje mogućih rješenja problema, sposobnost kombiniranja poznatih pojmova, stavova i pravila za dobivanje novih iskaza opravdani je imperativ ishoda učenja već u početnoj nastavi matematike. Osim toga, shvaćanje i određivanje statističkih zakonitosti vjerojatnoće u sličnim pojavama, kojih je najviše u svakodnevnom životu, prepoznati su kao izuzetno značajni sadržaji.

Polazeći od činjenice da su logičko-kombinatorno mišljenje i zaključivanje u fokusu ciljeva nastave matematike tijekom čitavoga školovanja, opravdano je pretpostaviti da postoje pedagoško-psihološke i didaktičko-metodičke potrebe i mogućnosti ranoga poticanja njegova razvoja već u početnoj nastavi. Propedeutičkim uvođenjem pojedinih sadržaja vezanih uz temeljne ideje i metode kombinatorike u osnovnoškolskoj matematici, moguće je utjecati na razvoj mentalnih, logičko-kombinatornih sposobnosti i funkcionalnih znanja učenika, potrebnih za rješavanje različitih problema u svakodnevnom životu. Imajući u vidu važnost ranoga poticanja matematičkoga mišljenja učenika, ovo istraživanje usmjerili smo na teorijsko i empirijsko razmatranje razvoja elemenata logičko-kombinatornoga mišljenja u nastavi matematike u primarnom obrazovanju. U radu se ispituje može li se obogaćivanje nastave matematike sadržajima koji potiču elemente logičko-kombinatornoga mišljenja poboljšati.

## Teorijske osnove rada

Nastavu matematike karakterizira sve usmjereniji razvoj misaonih sposobnosti u svladavanju odgovarajućih sadržaja. Velika se pažnja posvećuje tome da učenici steknu odgovarajuća znanja te da ih u najvećoj mjeri mogu uočiti, adekvatno vrednovati i logički primijeniti. Brojna teorijska i empirijska istraživanja u svijetu ističu značajnu ulogu logičko-kombinatornoga mišljenja u primarnom obrazovanju kojim su prožeti različiti sadržaji nastave matematike.

Rano uvođenje pojma i elemenata logike i kombinatorike u početnu nastavu matematike opravdano je, prije svega, jer je logičko-kombinatorno mišljenje važna sastavnica postizanja ishoda učenja matematike. To znači da ne treba čekati formalno učenje kombinatorike kao posebne teme u srednjoj školi, već učenicima treba što ranije ponuditi zadatke iz zanimljivih, poznatih situacija kako bi se razvila sposobnost logičko-kombinatornoga mišljenja (Zapata-Cardona, 2018).

Uvođenjem elemenata kombinatorike u početnu nastavu osiguravaju se potrebni uvjeti za kreativno opažanje, mišljenje i ponašanje, kao i za racionalno-kauzalno objašnjenje stohastičkih pojava, odnosno za rano razvijanje ideja vjerojatnosti (English, 2005). Aktivno sudjelovanje učenika, prihvaćanje rizika, pretpostavki i variranje mogućih rješenja ključna su obilježja nastave s elementima kombinatorike koja pridonosi kvaliteti nastavnoga procesa (Pinter, Krekić, 2006). Takvu okolinu trebaju karakterizirati prikladne kombinatorne problemske situacije koje će poticati misaonu aktivnost učenika u smjeru poticanja elastičnosti mišljenja, osjetljivosti na problem, pronalaženja nestandardnih odgovora, otkrivanja novih kombinacija i originalnih ideja (Pinter, 1997) te logičnoga objašnjenja istih (Syafitri i sur., 2020).

Incikabi i sur. (2013) definiraju logičko-kombinatorno mišljenje kao zaključivanje korištenjem konzistentnoga zaključivanja (prema Lazić i sur., 2022), kada se na temelju uvjeta zadatka traže različite opcije, permutacije, kombinacije i odnosi koji se utvrđuju (Savenkov i Romanov, 2021). Lockwood (2012) pod osnovama logičko-kombinatornoga mišljenja podrazumijeva pojmovno logičku analizu i misaonu aktivnost učenika usmjerenu na pronalaženje rješenja poticajnih kombinatornih problema, temeljenih na postupcima i aktivnostima kombinatornoga nabrajanja i razumijevanja istih.

O važnosti ove kategorije mišljenja, kao i o potrebi poticanja njegova razvoja tijekom cijeloga školovanja, govore i međunarodni programi učeničkih postignuća u matematici u okviru TIMSS (Trends in International Mathematics and Science Study) i PISA (Programme for International Student Assessment) istraživanja. Bitne razlike između Srbije i zemalja koje su postigle bolje rezultate odnose se upravo na tematske sadržaje nastave matematike u mlađim razredima osnovne škole, prvenstveno za funkcionalna znanja koja uključuju brzinu mišljenja i zaključivanja (Milinković i Lazić, 2018).

Kombinatorika, kao područje matematike, usko vezano uz algebru, teoriju vjerojatnosti i geometriju, našla je široku primjenu u različitim područjima života i rada suvremenoga pojedinca. Prepoznavanje i razumijevanje suštine kombinatornih problema prepoznato je kao iznimno važan sadržaj za razvoj matematičkoga mišljenja i multiplikativnoga zaključivanja te se nalazi u programima temeljnoga obrazovanja nekoliko europskih zemalja (Slovačka, Mađarska, Njemačka, Slovenija). Nacionalni centar za matematičko obrazovanje (NCTM, 2000.) također je istaknuo važnost ranoga uvođenja ovih sadržaja, s preporukom za uvođenje diskretne matematike u kurikul početnoga učenja i nastave matematike.

S druge strane, gledajući programske sadržaje nastave matematike u Republici Srbiji, uočljiva je tek neznatna zastupljenost ovih sadržaja, posebno u prvom ciklusu osnovnoga obrazovanja. Područje logike i kombinatorike ovdje se smatra presloženim za doba razredne nastave, iako ima potrebe i mogućnosti za njihovo propedeutičko uvođenje jer povoljno utječu na formiranje mentalnih slika i matematičkih pojmova kod/u djece. To dovodi do razvoja logičkoga mišljenja i zaključivanja, kao nužnih sposobnosti u stvarnim životnim situacijama (Milinković i Lazić, 2018).

U doba industrijske revolucije nužan je svestrani razvoj pojedinca koji uključuje poticanje logičko-kombinatornoga mišljenja formuliranjem, primjenom i tumačenjem matematike u različitim kontekstima. Kako je logičko-kombinatorno mišljenje i zaključivanje bitan element, neodvojiv od učenja matematike (Hidayati i sur., 2019), evidentno je da je ono u fokusu ciljeva nastave matematike na svim razinama obrazovanja te da je predmet istraživanja široke znanstvene i obrazovne zajednice.

Suvremena škola treba razvijati logičko-kombinatorno mišljenje tako što će učenicima omogućiti dovoljno različitih kombinatornih situacija u nastavi (Melusova i Vidermanova, 2015). Da bi uspješno razvijali logičko-kombinatorno mišljenje, učitelji moraju eksplicitno ugraditi sposobnosti i vještine logičkog amišljenja u cjelokupan proces stjecanja znanja (Maričić i sur., 2016). S tim u vezi, potrebno je početnu nastavu povezati sa situacijama iz stvarnoga života i tako da učenici stvaraju sustav međusobno povezanih misaonih operacija i cjelinu koja uključuje logiku kao ključnu sposobnost misaone aktivnosti. Napredak u razvoju matematičkih sposobnosti, a time i logičko-kombinatornoga mišljenja, kroz rješavanje problemskih zadataka potvrđuju rezultati nekoliko sličnih istraživanja (Anić i Pavlović Babić, 2015; Kadum, 2005; Mandak i Pavličić, 2016). Ističu da rješavanje problema, kao proces, predstavlja niz metodičkih i logičkih operacija koje se provode radi rješavanja problemskoga zadatka i ne može se promatrati odvojeno od mišljenja i učenja. Takvo temeljeno učenje karakterizira aktivan mentalni proces koji potiče tumačenje, prikupljanje informacija i prepoznavanje mogućih rješenja problema. Učenjem temeljenim na problemima poboljšavaju se logičko razmišljanje i matematičke komunikacijske vještine (Anwarudin i sur., 2020).

Skrb za rani razvoj logičko-kombinatornoga mišljenja započinje poticanjem matematičkoga obrazovanja dobro strukturiranim vježbama i zadatcima koji se mogu povezati s drugim nastavnim predmetima (Nunes i Csapó, 2011).

Neki istraživači matematičkoga obrazovanja, poput Graumanna i Hackinga (prema: Rezaie i Gooya, 2011), ističu da učenje pojmova kombinatorike zahtijeva poseban način razmišljanja. Stoga kombinatorika ima posebnu ulogu u nastavi matematike, kao važan čimbenik razvoja misaonih metoda višega reda. Smatraju da je kombinatorika poseban aspekt matematičkoga mišljenja te da predstavlja neophodan „alat” za rješavanje kombinatornih problema u geometriji, pri čemu se učenici koriste sposobnostima kombinatorike i, razmatrajući sve mogućnosti, pronalaze sustavno rješenje.

Rezultati istraživanja (Batanero i sur., 1997) pokazuju da je „sposobnost kombiniranja” učenika važna komponenta formalnoga mišljenja. Prema ovoj studiji, čak i mala djeca mogu koristiti eksperimentalne ili empirijske vjerojatnosti. Višestrukim provođenjem pokusa utvrđuju na koji se način događaj dogodio, a uspoređujući ih s ukupnim brojem pokusa dobivaju eksperimentalnu procjenu vjerojatnosti. Navodi se da bi učenici osnovnih škola trebali razviti stvarno razumijevanje ograničenja eksperimentiranja uspoređujući eksperimentalne rezultate s matematički izvedenim vjerojatnostima, koje često zahtijevaju kombinatoričko zaključivanje (Batanero i sur., 1997).

Aini i sur. (2020) ističu da je kombinatoričko zaključivanje osnovna kompetencija koju svaki učenik mora imati kako bi rješavao matematičke probleme određenom strategijom. Tijekom istraživanja podijelili su zadatak koji je sadržavao nekoliko zadataka sličnoga koncepta, a zatim organizirali intervju kako bi ispitali kombinatorno razmišljanje učenika pri rješavanju zadanih problema. Došli su do zaključka da kombinatorno mišljenje igra važnu ulogu u rješavanju matematičkih problema jer predstavlja kognitivni sustav koji sadrži strategiju za donošenje zaključaka. Takav stav nadalje implicira da su rješavanje problema i logičko-kombinatorno zaključivanje dva ključna aspekta ciljeva učenja matematike (Aini i sur., 2020).

Vještine koje učenici razvijaju kombinatornim razmišljanjem u matematici odnose se na procjenu, generalizaciju i sustavno razmišljanje (Maylisa i sur., 2020). U studiji Pinter-Krekić i sur. (2015), istaknute su strategije rješavanja kombinatornih problema (permutacije, kombinacije i varijacije), s posebnim naglaskom na važnost metodičke transformacije sadržaja kombinatorike u nastavi matematike. Rezultati istraživanja (Krpec, 2014), koje je obuhvatilo kombinatorne sposobnosti učenika nižih razreda osnovne škole, pokazali su da učenici imaju poteškoća u rješavanju kombinatornih zadataka. Oparnica i sur. (2016) prikazali su određene modele kombinatornih zadataka koji se lako mogu integrirati u postojeće programske sadržaje u nižim razredima osnovne škole, dok Gordić i sur. (2019) ukazuju na motivaciju učenika za rješavanje jednostavnih kombinatornih zadataka u vezi s mogućim rasporedom, grupiranjem ili izborom predmeta u početnoj nastavi matematike. Rakić i sur. (2021) ukazali su da pojedini programski sadržaji početne nastave matematike omogućuju poticanje logičko-kombinatornoga mišljenja učenika primjenom različitih tipova diferenciranih zadataka. Ovakvi rezultati daju potrebnu osnovu za daljnje tumačenje i ispitivanje ovoga aktualnog istraživačkog problema.

## Metodologija

Glavni je cilj istraživanja ispitati mogućnost razvijanja elemenata logičko-kombinatornoga mišljenja u četvrtom razredu osnovne škole osmišljavanjem i odabirom sadržaja. Istraživanje se temeljilo na pretpostavci da se elementi logičko-kombinatornog mišljenja mogu razvijati adekvatnim oblikovanjem matematičkih sadržaja i stavljanjem učenika u problemske situacije u kojima do izražaja dolaze sposobnosti logičko-kombinatornoga mišljenja.

Istraživanje je provedeno na uzorku učenika (N = 276) koji je odabran iz populacije učenika četvrtih razreda osnovnih škola u Republici Srbiji. Uzorak ispitanika bio je slučajan, odabrani su razredi unutar škola i svi učenici unutar njih. Budući da se istraživanje temelji na eksperimentu s paralelnim skupinama, formirane su dvije ravnopravne skupine učenika – eksperimentalna (E) i kontrolna (K). Eksperimentalnu skupinu činilo je šest odjeljenja četvrtih razreda dviju osnovnih škola (N = 138), a kontrolnu skupinu činilo je isto toliko učenika (N = 138), šest odjeljenja četvrtih razreda dviju osnovnih škola (grad Novi Sad). Za izjednačavanje istraživačkih skupina koristili smo se općim uspjehom učenika i uspjehom učenika iz matematike na kraju trećega razreda, kao i rezultatima inicijalnoga testa (IT) u kojem do izražaja dolazi logičko razmišljanje i kombinatorika.

U istraživanju je korištena eksperimentalna metoda u modalitetu eksperimenta s paralelnim skupinama. Eksperimentalni program bio je usmjeren na rano poticanje i unaprjeđivanje razvoja logičko-kombinatornoga mišljenja učenika, kroz posebno osmišljen pristup algebarskim i geometrijskim sadržajima. Program se temeljio na kreiranju, primjeni i rješavanju različitih tipičnih logičko-kombinatornih zadataka danih u matematičkom konktestu te realističnih kombinatornih problema. Obje skupine ovih tipova zadataka bile su podjednako zastupljene tijekom istraživanja. Eksperimentalni program obuhvatio je sve programske sadržaje nastave matematike iz područja algebre i geometrije za 4. razred osnovne škole. Provodio se tijekom drugog polugodišta u sklopu dvadeset i tri sata matematike te je uklopljen u redoviti nastavni program po kojem učenici rade. Prema uputama autora eksperimentalnog programa, program su provodile učiteljice koje predaju u četvrtom razredu osnovne škole. Autorica je bila u stalnoj komunikaciji s učiteljima, pratila tijek realizacije i pružala pomoć učiteljima. Za to vrijeme kontrolna skupina radila je po redovnom programu. Ilustracije radi, navest ćemo nekoliko primjera korištenih zadataka iz eksperimentalnoga programa.

Primjer 1: *Napiši sve troznamenkaste brojeve čiji je zbroj znamenki 3. Koliko ih ima?*

Prikazani primjer 1 pripada varijacijama. Pri rješavanju navedenoga primjera učenik prije svega treba pronaći sve moguće kombinacije triju brojeva koji zbrojem daju 3, pokazujući tečnost u razmišljanju. U tu svrhu sastavlja kombinacije: (3, 0, 0), (2, 1, 0) i (1, 1, 1). Sljedeći korak su permutacije, pri čemu učenik za svaku kombinaciju brojeva zapisuje troznamenkaste brojeve: (3, 0, 0) – 300; (2, 1, 0) – 210, 201, 102, 120; (1, 1, 1) – 111. Navedeni primjer potiče logičko-kombinatorno mišljenje, analizu misaonih operacija, sintezu i usporedbu, kreativno opažanje i pronalaženje mogućih rješenja.

Osim takvih primjera, danih u čisto matematičkom kontekstu, učenici su rješavali i kontekstualno utemeljene zadatke (Primjer 2).

Primjer 2: *Od Sombora do Novog Sada postoje 2 rute, a od Novog Sada do Niša 3 rute. Na koliko načina možemo putovati od Sombora do Niša preko Novog Sada?*

U ovom primjeru do izražaja dolazi vještina rješavanja kombinatornoga problema, odnosno pokazivanje elemenata kombinatornoga mišljenja, sposobnost procjene, generalizacije, interpretacije i vizualizacije zadanih podataka (*optical processing*), variranje i sistematizacija mogućih rješenja, logičko rezoniranje, kombinatorno rasuđivanje i zaključivanje.

Navest ćemo i jedan primjer iz područja geometrije (Primjer 3).

Primjer 3. *Petra je u postolje složila figure različitih oblika: kvadrat, pravokutnik, trokut i krug. Ukupno je složila 6 figura jednu do druge. Svaka figura ima točno jedan oblik kao bazu.*

Petra nam je otkrila sljedeće: Sve figure koje imaju isti broj stranica stoje jedna do druge.

- Dvije figure imaju u osnovi kvadrat.
- Likovi koji imaju više od 3, a manje od 5 stranica nalaze se između kvadrata i trokuta.
- Likovi koji se temelje na krugu ne dodiruju kvadrate.
- Figure u sredini (3. i 4. u nizu) u osnovi imaju trokut i pravokutnik, ali ne tim redom.

Na temelju zadanih podataka napiši redoslijed figura prema osnovama kako je Petra posložila.

Rješenje: kvadrat - kvadrat - pravokutnik - trokut - trokut - krug.

Učinci eksperimentalnoga programa utvrđeni su tehnikom ispitivanja. Provedena su dva testa: inicijalni, prije uvođenja eksperimentalnoga programa, i završni (FT), nakon implementacije eksperimentalnoga programa. Za potrebe istraživanja sastavili smo testove znanja s po 10 zadataka, kako bismo utvrdili razinu logičko-kombinatornoga mišljenja učenika tijekom rada na problemskim zadatcima: inicijalni test (IT), završni test (FT). Završni test (FT) obuhvatio je sadržaje algebre i geometrije za 4. razred osnovne škole. Osim tipičnih kombinatornih zadataka (permutacija, kombinacija i varijacija), testovi uključuju i različite, kontekstualno utemeljene, realistične primjere u kojima do izražaja dolazi logično-kombinatorno razmišljanje. Ove dvije skupine zadataka na oba testa bila su brojčano ujednačene sa po 5 zadataka.

Najprije je obavljeno preliminarno testiranje na uzorku od 45 učenika, nakon čega su izvršene korekcije instrumenta i izrađena konačna forma testova. Tijekom testiranja svi su učenici bili stavljeni u približno istu situaciju testiranja, a zadatci su pregledani na isti način prema jedinstvenim uputama temeljenim na ključu kako bi se osigurala objektivnost testiranja. Provedena je logička i sadržajna validacija testova, utvrđivanje usklađenosti testova sa zahtjevima nastavnoga plana i programa i sadržajima na koje se odnose. Pouzdanost instrumenta određena je izračunom Cronbachova alfa koeficijenta ($\alpha$ = ,88) koji ukazuje na visoku pouzdanost instrumenta. Diskriminativnost testa određena je analizom ajtema. Koeficijent diskriminativne vrijednosti zadataka kreće se od 0,16 do 0,25. Zadatci čija je diskriminativna vrednost bila manja od 0,20 zamijenjeni su novim, nakon čega je diskriminativna vrijednost svakog zadatka bila veća od 0,20.

Podatci dobiveni istraživanjem obrađeni su pomoću statističkoga programskog paketa IBM Statistics SPSS23. Jednofaktorska analiza varijance (ANOVA) korištena je za praćenje učinaka eksperimentalnoga programa u eksperimentalnoj skupini, kao i za statističku kontrolu ujednačenosti skupina u istraživanju.

## Rezultati

Rezultati deskriptivne statistike (Tablica 1) pokazuju da su eksperimentalna i kontrolna skupina približno jednake prema aritmetičkim sredinama rezultata na inicijalnom testiranju: eksperimentalna skupina (M = 10,65; SD = 3,614), kontrolna skupina (M = 10,17; SD = 2,816). U inicijalnom mjerenju stupnja razvijenosti logičko-kombinatornoga mišljenja učenici obiju skupina ostvarili su samo polovicu od maksimalnoga broja bodova (20).

Tablica 1

Nakon inicijalnoga testiranja, u eksperimentalnu skupinu uveden je eksperimentalni program koji se temelji na rješavanju kombinatornih zadataka, odnosno poticanju elemenata logičko-kombinatornoga mišljenja, dok je kontrolna skupina radila prema predviđenom programu nastave i učenja i odabranom udžbeniku (isti kao u E skupini). Nakon realiziranoga eksperimentalnog programa obavljena su završna ispitivanja. Usporedbom rezultata (Tablica 1) pokazalo se da je eksperimentalna skupina nakon provedenoga eksperimentalnog programa ostvarila veći broj bodova na finalnom testu od kontrolne skupine, što se opet promatra kroz prikazane aritmetičke sredine: eksperimentalna skupina (M = 15,73; SD = 2,746), kontrolna skupina (M = 10,09; SD = 2,438). Također, vidimo da su rezultati eksperimentalne skupine značajno viši u finalnom nego u inicijalnom testiranju, što pripisujemo utjecaju eksperimentalnoga programa. Pritom je kontrolna skupina postigla približno ujednačene rezultate, kako na inicijalnom tako i na finalnom testiranju.

Na ilustriranim prikazima distribucije uspješnosti eksperimentalne i kontrolne skupine, unutar plot dijagrama (Slika 1), vidljivo je da je vidljiva približna ujednačenost te da eksperimentalna skupina ima osjetno veći raspon rezultata u odnosu na rezultate kontrolne skupine, koji su grupiraniji. Vidimo da su rezultati na finalnom testu za obje skupine simetrično raspoređeni, s približno jednakim rasponom, ali da je eksperimentalna skupina postigla statistički značajno viši rezultat, kako u odnosu na rezultat kontrolne skupine, tako i u odnosu na rezultat koji je postigla u inicijalnom testu. Istodobno, u kontrolnoj skupini dvoje učenika postiglo je značajno bolje rezultate na završnom testu od ostalih učenika u skupini.

Slika 1

Tablica 2

Vrijednost Leveneova testa kod inicijalne ($F(1,274) = 8,586$; $p = 0,004$) pokazuje početnu neujednačenost skupina, dok kod finalnoga mjerenja ($F(1,274) = 3,127$; $p = 0,078$) pretpostavka o homogenosti varijance nije narušena, skupine su ujednačene te se rezultat dobiven analizom varijance može smatrati vjerodostojnim (Tablica 2).

Tablica 3

Proveden je statistički postupak analize varijance (ANOVA) kako bi se ispitalo postojanje značajnih razlika između eksperimentalne i kontrolne skupine na inicijalnom i finalnom testu. Rezultati ANOVA (Tablica 3) također pokazuju da su eksperimentalna i kontrolna skupina bile izjednačene prema aritmetičkim sredinama rezultata na inicijalnom testiranju, što se vidi iz činjenice da nema statistički značajnih razlika između eksperimentalne i kontrolne skupine na inicijalnom testiranju. Vrijednost izračunate varijance na inicijalnom mjerenju ($F(1,274) = 1,527$; $p = 0,218$) pokazuje da ne postoji statistički značajna razlika između skupina. S obzirom na neujednačenost varijance pri inicijalnom mjerenju, koristili smo Welchov t-test ($t(260) = 1,236$, $p = 0,218$) čija vrijednost ukazuje da ne postoji statistički značajna razlika između skupina u inicijalnom testiranju.

Međutim, kako je potvrđeno ANOVA-om, konačni test je pokazao da je eksperimentalna skupina postigla statistički značajno bolje rezultate od kontrolne skupine. Rezultat analize varijance na finalnom mjerenju ($F(1,274) = 324,830$; $p = 0,000$) pokazuje da postoje statistički značajne razlike između eksperimentalne skupine učenika koji su radili po eksperimentalnom programu i učenika kontrolne skupine koji su radili po utvrđenom načinu rada. Uzmemo li u obzir rezultate deskriptivne statistike (Tablica 1), možemo reći da su učenici eksperimentalne skupine postigli statistički značajno bolje rezultate u odnosu na učenike kontrolne skupine. Dobiveni rezultati pokazuju da je rješavanjem kombinatornih problema u okviru programskih sadržaja nastave i učenja matematike moguće utjecati na razvoj elemenata logičko-kombinatornoga mišljenja učenika mlađe školske dobi.

Također, na finalnom testu usporedili smo rezultate skupina prema tipovima zadataka. Rezultati pokazuju da je eksperimentalna skupina ostvarila veći broj bodova od kontrolne skupine pri rješavanju obaju tipova zadataka (tipični kombinatorni zadatci i kontekstualno utemeljeni problemi), što pripisujemo utjecaju eksperimentalnoga programa (Tablica 4). Osim značajnosti razlike prosjeka htjeli smo ispitati i jačinu razlike prosjeka između skupina po tipovima zadataka primjenjujući Cohenovu d mjeru veličine efekta.

Tablica 4

Dobiveni rezultati pokazuju izrazito veliku veličinu efekta kod obaju tipova zadataka (logičko-kombinatorni i realni problemi), što jasno potvrđuje snažan pozitivan utjecaj eksperimentalnoga programa na uspješnost učenika u obje vrste zadataka.

## Rasprava

Rezultati ovoga istraživanja pokazali su da kreirani eksperimentalni program temeljen na posebno osmišljenom pristupu algebarskim i geometrijskim sadržajima, koji omogućuje niz različitih kombinatornih situacija u nastavi (rješavanje tipičnih logičko-kombinatornih zadataka i kontekstualno utemeljenih primjera), značajno doprinosi razvoju elemenata logičko-kombinatornoga mišljenja, u prvom ciklusu osnovnoga matematičkog obrazovanja. Kako pokazuju nalazi ovoga istraživanja, učenici eksperimentalne skupine, koji su sudjelovali u izrađenom programu za rješavanje kombinatornih zadataka, imaju značajno razvijenije elemente logičko-kombinatornoga mišljenja u odnosu na učenike koji nisu bili obuhvaćeni ovim programom, na završnom testu, iako su im početni rezultati bili približno isti, što potvrđuje učinkovitost provedenoga programa. Osim toga, u prilog učinkovitosti izrađenoga programa u poticanju razvoja elemenata logičko-kombinatornoga mišljenja kod učenika govore i značajno viši rezultati postignuti u finalnom u odnosu na inicijalno testiranje u eksperimentalnoj skupini.

Rezultati dobiveni ovim istraživanjem daju značajan doprinos proučavanju čimbenika koji utječu na formiranje logičko-kombinatornoga mišljenja u nižim razredima osnovne škole, te kao takvi predstavljaju svojevrstan segment u slici izgrađenoj nalazima dosadašnjih istraživanja, koja su ukazivala na važnost rješavanja kombinatorike za razvoj viših i složenijih obrazaca mišljenja u dječjoj dobi, pri čemu se posebno ističe formiranje formalnoga mišljenja, sposobnost generaliziranja i sistematiziranja (English, 2005; Hidayati i sur., 2019; Rezaie i Gooya, 2011; Zapata-Cardona, 2018). Imajući u vidu važnost poticanja logičko-kombinatornoga mišljenja u kontekstu ukupnoga kognitivnog razvoja i napretka u djetinjstvu, jasno je koliko je važno utvrditi sve one individualne i okolinske čimbenike, a posebice čimbenike nastavnoga procesa koji tome pridonose (Melusova i Vidermanova, 2015; Nunes i Csapó, 2011).

Nalazi ovoga istraživanja sukladni su uvodno prikazanim rezultatima ranijih istraživanja koja su pokazala kako rješavanje problemskih zadataka poboljšava kognitivne sposobnosti učenika, posebice u domeni poticanja logičko-kombinatornoga mišljenja. S druge strane, postoji i povratna veza, u smislu da razvijeno logičko-kombinatorno mišljenje dodatno pridonosi uspješnosti rješavanja različitih problemskih zadataka (Aini i sur., 2020), što otvara mogućnost da se u budućim istraživanjima dodatno ispita priroda ovoga dvosmjernog odnosa u nižoj dobi.

Posebna važnost dobivenih rezultata ogleda se u tome što se pokazalo da ovakvi zadatci mogu biti od velike koristi u kognitivnom razvoju učenika mlađih razreda, a ne samo starijih dobnih skupina, kako se obično pretpostavlja. Naime, ovakvi zadatci omogućuju učenicima da unaprijede svoje vještine rješavanja problema, pri čemu, osim kombinatorike, do izražaja dolaze intuitivnost i kreativnost. Na taj način mišljenje učenika postaje fleksibilnije, otvorenije novim mogućnostima i izlasku iz postojećih standardnih okvira rješavanja problema, što je važno poticati i razvijati na različite načine, čak i u nižoj razvojnoj dobi, točnije, upravo u ranoj školskoj dobi, jer se mišljenje u tom razdoblju vrlo intenzivno razvija i podložno je oblikovanju pod utjecajem različitih obrazovnih sadržaja i načina rada (Maylisa i sur., 2020; NCTM, 2000).

Kroz potvrđenu važnost utjecaja kombinatornih problemskih zadataka na razvoj logičko-kombinatornoga mišljenja kod učenika nižih razreda osnovne škole, kao praktična implikacija ovoga istraživanja, ukazuje se na nužnost proširenja nastavnih sadržaja matematike rješavanjem zadataka ove vrste, koji ne samo da doprinose kognitivnim kapacitetima učenika, već i njihovom kreativnom potencijalu, kao i sposobnosti suočavanja sa svakodnevnim, praktičnim zadatcima. Iz toga razloga, počevši od najranijih obrazovnih razina, potrebno je kontinuirano i sustavno poticati razvoj kognitivnih potencijala učenika, kroz različite vrste složenijih zadataka, koji će pridonijeti njihovom aktivnom uključivanju u proces učenja na zanimljiv i kreativan način, potičući razvoj općih kognitivnih sposobnosti i složenijih misaonih operacija, kao i spremnosti za praktično djelovanje u različitim situacijama.

## Zaključak

Propedeutika pojma logike i kombinatorike u početnoj nastavi matematike ima važnu ulogu u početnoj nastavi: ogleda se u razumijevanju različitih pojava u svakodnevnom životu u prirodi i društvu, u igri, ali i u samoj matematici, neposrednom razvoju logičko-kombinatornoga mišljenja, razvoju slobodnoga, kreativnoga matematičkog mišljenja, kao i povezivanjem igre i učenja, što snažno utječe na motivacijski učinak u početnoj nastavi matematike. Jedan od načina koji može doprinijeti navedenom jest intuitivno rješavanje jednostavnih kombinatornih problema.

Imajući u vidu neospornu važnost matematičkoga mišljenja u ukupnom odgojno-obrazovnom potencijalu, kao i široke mogućnosti njegova razvoja, mišljenja smo da je potrebno adekvatno podržati i unaprijediti poticanje prvenstveno logičko-kombinatornih sposobnosti učenika u prvom ciklusu osnovnoga matematičkog obrazovanja. To bi se postiglo prvenstveno obogaćivanjem nastave matematike sadržajima koji potiču elemente logičko-kombinatornoga mišljenja, davanjem konkretnih modela za realizaciju nastavne prakse.

Uvođenjem elemenata kombinatorike u početnu nastavu matematike na taj se način osiguravaju nužni uvjeti za rani razvoj ideja o vjerojatnosti i kombinatornom zaključivanju, odnosno za prepoznavanje i razumijevanje biti kombinatornih problema. Istodobno, učenici pokazuju visok stupanj motiviranosti za rješavanje tako strukturiranih, jednostavnih kombinatornih zadataka u vezi s mogućim slaganjem, grupiranjem i uočavanjem mogućih rješenja što podrazumijeva unaprjeđenje matematičkih kompetencija u kojima do izražaja dolazi logičko-kombinatorno mišljenje.

Rezultati dobiveni ovim istraživanjem pokazuju da realni kombinatorni zadatci imaju značajan utjecaj na unaprjeđivanje logičko-kombinatornoga mišljenja te da se takvi zadatci mogu implementirati u nastavu matematike u mlađim razredima osnovne škole. Pažljivim odabirom i obogaćivanjem sadržaja koji uključuju logiku i kombinatoriku, uz odgovarajuće usmjeravanje, otkrivanjem sličnosti u različitim problemima, djeca se osposobljavaju misaono prepoznavati i rješavati kombinatorne probleme. Utvrđivanje što većega broja mogućih rješenja problema slijedi pronalaženje svih mogućih rješenja, odnosno određivanje broja tih rješenja. Ovakav način rješavanja zadataka također podrazumijeva različite kombinacije ili varijacije primijenjenih elementarnih matematičkih znanja iz fonda znanja učenika, kao povezane algoritamske korake, što govori o biti logičko-kombinatornoga mišljenja u tijeku rješavanja različitih problema.

Na opisani način afirmiraju se primarni ciljevi suvremene nastave matematike, razvoj kritičkoga mišljenja i funkcionalnoga znanja, odnosno koncept stvaranja uvjeta u kojima učenici stječu kompleksna znanja neophodna za život u 21. stoljeću (Ravitz i sur., 2012). Imajući u vidu nužnost osiguranja stalne mentalne aktivacije učenika u početnoj nastavi matematike, poboljšanja njihove motivacije za matematičke sadržaje i olakšavanja rješavanja praktičnih životnih problema, u radu smo upozorili na ulogu i važnost ranoga poticanja logičkih kombinatornih sposobnosti. Dobiveni rezultati opravdavaju pretpostavku da ovi sadržaji nisu prikladni samo za starije učenike, već da se mogu učinkovito koristiti i u nižim razredima osnovne škole. Osim navedenoga, stjecanje određenih iskustava, znanja, vještina i navika u igricama, pričama i zadatcima kombinatornoga karaktera daje značajan doprinos obogaćivanju predodžbi u rješavanju problema ne samo iz nastave matematike, već i šire, interdisciplinarnih, kao i optimalnoj pripremi učenika za uspješno daljnje proučavanje ovih važnih sadržaja na višim razinama obrazovanja.