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Two great mathematicians of the 18$^{\text{th}}$ century: Bošković and Gelenbevî

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Abstract

In this paper, we describe and compare the lives and mathematical contributions of the greatest Croatian and the greatest Turkish mathematician of the 18$^{\text{th}}$ century: Ruđer Josip Bošković and Gelenbevî İsmail Efendi.

Keywords: *mathematics in the* 18$^{\text{th}}$ *century, mathematics education in the* 18$^{\text{th}}$ *century, Ruđer Josip Bošković, Gelenbevî İsmail Efendi*

Dva velika matematičara 18. stoljeća: Bošković i Gelenbevî

Sažetak

U ovom članku opisujemo i uspoređujemo životopise i matematičke doprinose najvećeg hrvatskog i najvećeg turskog matematičara 18. stoljeća, Ruđera Josipa Boškovića i Gelenbevî İsmail Efendija.

Ključne riječi: *matematika u 18. stoljeću, nastava matematike u 18. stoljeću, Ruđer Josip Bošković, Gelenbevî İsmail Efendi*

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1 Introduction: Historical context

It is general knowledge, at least here in Croatia, that during the 18th century practically all of modern Croatia was liberated from the Ottoman rule, even if almost all of it was conquered in the period of the "Hundred Years' Croatian–Ottoman War" (1493–1593). The parts lost when Croatian lands were reduced to the *reliquiae reliquiarum* from the end of the 16th century were in most part restored by 1699, when the Treaty of Karlowitz was signed. These parts (Kingdom of Croatia) remained in the greater part under the rule of the Habsburgs, whom the nobility of the earlier Hungarian-Croatian kingdom chose as their rulers in 1527, and in a smaller part (southern Croatia) under Venetian rule. An exception was Dubrovnik, an independent republic in continuity since the 15th century, with a predominantly Croatian population. The Dubrovnik Republic remained in peaceful relations with the Ottoman Empire, to which they paid a yearly tribute from 1458 until 1808 (when the Dubrovnik Republic lost their independence to Napoleon), and in return received protection and trade benefits. They had an even longer standing similar agreement with the Venetian Republic, and they hugely profited from the interruption of direct trading between Venetians and Ottomans. Dubrovnik's neighbours being Ottoman and Venetian territories, the Habsburg monarchy had comparatively little impact on Dubrovnik's trade, culture, and politics before end of 18th century [12]. Thus, in the 18th century, most of Croatia was again experiencing, like originally from its earliest times, predominantly "western" (either Habsburg monarchy or Venetian Republic) influence, in political, economical, sociological, cultural, educational and scientific sense.

With respect to politics, the governments of European states became more centralized, and monarchs in most cases had an absolute, unrestrained power, so this time is usually referred to as the "age of absolutism". A particularly important factor was the influence of the philosophy of enlightenment. The philosophy of enlightenment, accompanied by the rationalistic approach, resulted also in an increase of scientific research, in particular, empirical methods became more common than before. Under its influence, the idea that knowledge and education form the basis of progress, European states began to reform their educational systems towards a more accessible education separated from the previous predominantly church-conducted schools [4, 11, 15, 20].

However, one should not forget that in Croatia some influences of the Ottoman rule remain even today, and in the 18th century it was more prominent in the border regions (Vojna Krajina) and small parts of modern Croatia that were not yet liberated. At that time, the Ottoman Empire still ruled a big part of Europe, more precisely, most of the Balkans. It enjoyed a period of peace, but also of stagnation. In the periodization of Ottoman history, the era between the Treaty of Karlowitz (1699) and the Treaty of Jassy (1792) is generally described as the "Period of Decline" [8]. However, this perspective is misleading; when the political, cultural, artistic, and social developments of the time are examined, it becomes evident that defining this century as a "period of disintegration and reorganization" is far more consistent with historical realities.

During the 18^{th} century, after the Ottoman Empire experienced its first territorial loss with the Treaty of Karlowitz, and a series of political events followed that undermined public confidence in the state. The sense of despair and distrust toward the administration grew significantly, and as a reflection of this atmosphere of pessimism, for the first time, alternative ideas concerning the legitimacy of the Ottoman dynasty began to emerge. With the accession of Sultan Ahmed III. (1703–1730), the political climate temporarily stabilized, and state policy focused on regaining the territories that had been lost. For this reason, the 18^{th} century became a period marked by conflicts in the East and West of the Empire. In the aftermath of these conflicts, the Empire initiated a series of reform movements aiming to repair the devastation and to implement adjustments in accordance with the political, social, economic, and military needs of the age [8]. The so-called "Tulip Era" (1718–1730), a period of restoration, began under the patronage of Sultan Ahmed III. and his Grand Vizier, Nevşehirli Damad İbrahim Pasha. For the first time in Ottoman history, an ambassador, 28 Çelebi Mehmed Efendi, was sent to Paris, thus enabling closer observation of major European centers [8]. In 1727, the first imperial printing press was established; new libraries were opened, and both literary and scientific works were translated from Eastern and Western sources. Paper manufacturing was initiated in Yalova, while textile and ceramic workshops were founded in Istanbul. The earliest form of a fire brigade, known as the *Tulumbacı* corps, was organized; the smallpox vaccine began to spread; and new approaches emerged in civil architecture and spatial design. The arts and literature also witnessed remarkable advancements during this period [8]. However, the discomfort provoked by such deviations from tradition, combined with deliberate incitement, resulted in the Patrona Halil Revolt (1730), which brought the Tulip Era to an abrupt end. In response, the new ruling elite, having grasped the underlying message of the rebellion, redirected reform efforts toward more militarized fields, such as the training of officers, engineering education, and the establishment of the Imperial Naval Engineering School (1775) [8].

In this article we shall describe the lives and works of two great scientists of that time, the Croatian mathematician Ruđer Josip Bošković, surely well-known to our readership, and the Turkish mathematician Gelenbevî İsmail Efendi, probably unknown to most of the public here in Croatia. In order to better understand their contributions, we shall first give a short account of the most important characteristics and events of mathematics in 18$^{\text{th}}$ century Croatia and Türkiye, both with respect to research and education.

2 Mathematics in 18$^{\text{th}}$ century Croatia and Türkiye

The 18$^{\text{th}}$ century in Europe is well known to have been highly productive regarding mathematics research (it is enough to just mention Leonhard Euler as an example), and also brought many educational reforms in various parts of Europe. Before these reforms earlier forms of schooling persisted: Organised schooling in the form of secondary and higher education, in most parts of Europe (in particular in the Habsburg monarchy) attached to the Catholic church. A particularly important form of secondary schools were Jesuit gymnasia, where by the 18$^{\text{th}}$ century mathematics became a standard even if minor part of curricula, focusing mostly on basic arithmetic and some geometry based on Euclid's *Elements* for the purpose of teaching astronomy. Another noteworthy development in the 18$^{\text{th}}$ century for mathematics education in greater parts of Europe even before the educational reforms was that mathematical textbooks started to appear in various local languages [5, 15]. Croatia was no exception to this trend, and the first arithmetic textbooks in Croatian were published, both intended for a general public, and not as school textbooks. These two were *Aritmetika Horvatszka* by Mijo Šilobod Bolšić (1758), and *Aritmetika u slavni jezik ilirički sastavljena* (1766) by Mate Zoričić [9].

The empress Maria Theresa of the Habsburg monarchy introduced several school reforms to the monarch, of which the most important and comprehensive was the *Allgemeine Schulordnung für die deutschen Normal-, Haupt- und Trivialschulen in sämmtlichen Kayserl. Königl. Erbländern* (1774), which concerned the Austrian part of the Monarchy. Among many important novelties, this document introduced obligatory schooling for children of age 6 to 12, specified both the subjects, including arithmetic, to be taught and the corresponding textbooks, defined the sorts of primary schools, prescribed that the method of teaching in public schools should not be individual, that exams should be held at end of each semester, and much more [4, 6, 7]. An earlier reform of 1752 introduced arithmetic and geometry into the gymnasial curricula in the Austrian part of the Habsburg Empire, and a fundamental law (known as *Anonymischer Plan*) which was in effect 1775–1804 defined the age of 10 to be the age of enrollment into gymnasia, the duration of gymnasial schooling to be five years, and promoted teaching of non-linguistic topics, including arithmetic and geometry. These reforms were not automatically binding for the Hungarian part of the Empire, because of the terms under which the Hungarian-Croatian Kingdom and Austria were unified in 1527, so the reforms were introduced to the Hungarian part separately in 1777 (*Ratio educationis totiusque rei literariae per regnum Hungariae et provincias eidem adnexas*). This document is very similar to the *Allgemeine Schulordnung*, with some important differences. For example, while the *Allgemeine Schulordnung* promoted the German language to be used in primary education, here the official language of education remained Latin [4].

This implied that in Croatia, of which the greater part at that time was included in the Hungarian part of the Monarchy, educational reforms were also introduced by the end of the 18^{th} century [4]. Consequently, by the end of the 18^{th} century, the primary education in Croatia regularly included at least some basic arithmetic (arithmetical operations with positive integers), and in gymnasial education this was extended to fractions, proportions, rule of three and applications of arithmetic, as well as some basic geometry.

In the Ottoman Empire, mathematical education was carried out primarily in madrasas until the mid-18th century, and the curriculum was based on the classical Islamic mathematical tradition. The syllabus included courses in arithmetic (*hisâb*), geometry (*hendese*), and astronomy (*hey'et*), which were primarily taught at a theoretical level. During this period, the influence of newly emerging mathematical fields in Europe, such as analytic geometry and calculus, on Ottoman education remained quite limited.

Following the Tulip Era (*Lâle Devri*, 1718–1730), the reform movements that were undertaken led to an increased interest in engineering fields, an interest which gradually affected the nature of educational institutions [8]. In the last quarter of the 18th century, institutions that can be considered the first examples of Western-style education in the Ottoman Empire emerged. Among these, the Mühendishâne-i Bahrî-i Hümâyun (1775) and the Mühendishâne-i Berrî-i Hümâyun (1795) stand out [1]. These institutions not only provided military engineering education but also laid the institutional foundations of modern mathematical instruction. All these developments deeply influenced the mathematicians of the period, whose works increasingly focused on applied mathematics within the military and engineering sciences.

These developments were largely parallel in time to the *Ratio Educationis* reforms implemented in the Habsburg lands. Thus, by the late 18th century, the Ottoman Empire experienced an innovative transitional period in which the traditional Islamic scientific tradition and the modern European educational approach converged. This period can be regarded as the beginning of modern mathematical education in the Ottoman context. Gelenbevî İsmail Efendi is considered one of the representatives of this era [19].

At the same time, this century saw also the life and works of the probably greatest Croatian scientist so far, Ruđer Josip Bošković (1711–1787), who was born in the independent Dubrovnik Republic. So, in totally different circumstances, at the same time, Croatia a "western" type great mathematician arose, and the first great Turkish mathematician: Bošković and Gelenbevî. Very different their lives were, and they most probably never met (even if Bošković spent some time in İstanbul), but hugely influenced mathematics not only in their respective countries.

3 Bošković and Gelenbevî: Biographies

3.1 The life of Ruđer Josip Bošković

Ruđer Josip Bošković was born in Dubrovnik on May 18$^{\text{th}}$, 1711. He was schooled first in the Jesuit school *Collegium Ragusinum*, then continued to higher education at the also Jesuit *Collegium Romanum* in Rome, Italy. He studied rhetoric, then philosohy and finally theology. During his studies he became interested in mathematics, in particular Newton's works. In 1744, he was ordained a priest and became the head of the mathematics department until 1760 (but he was already teaching mathematics at the *Collegium Romanum* since 1740). During that time, and later, he contributed to statics and engineering. He was interested in problems related to the planet Earth (shape, size, gravity) and contributed to their studies by several treatises. Particularly famous are his geodetic-cartographic measurements (1750–52) of meridian degrees, conducted between Rome and Rimini with fellow Jesuit Christopher Le Maire. To improve the reliability of the results he developed a forerunner of the least squares method — he used minimization of the sum of absoulute values of the errors [13, 17].

Bošković became involved in diplomacy, and travelled quite a bit both for diplomatic and scientific reasons. His major work *Philosophiae naturalis theoria redacta ad unicam legem virium in natura existentium* (1758) was finished and printed while he was in Vienna. In this work he presented a new and unique theory of forces and structure of matter, that was essentially a visionary forerunner of modern physical theories on this topic. In 1761, after a writing a poem *De Solis ac Lunae defectibus* on eclipses of Sun and Moon that he dedicated to *Royal Society*, he was elected their member. They encouraged him to visit Constantinople (modern İstanbul) to observe the passing of Venus between Earth and Sun. For this, he arrived too late, so instead he researched the remains of ancient Troy. In contrast to his contemporaries, he advocated the thesis that these remains are not on the coast, but more to the inland of Anatolia, which was later confirmed by excavations by H. Schliemann [13, 14].

In the period 1764–69, he was a professor of mathematics at the University of Pavia, and continued to actively contribute not only to mathematics, but also to astronomy and physics. Being a Jesuit, the dissolution of the Jesuit order in 1773 affected him, and he moved to Paris and became a French citizen and employee of the French ministry of Foreign Affairs. Here, he actively contributed to optics and astronomy, but his health began to deteriorate. In 1782, he returned to Italy, and on February 13, 1787 he died of pneumonia in Milano. One should, however, emphasize that even if most of Bošković's life was connected to Italy and France, he always kept connections to Dubrovnik, even if as a grown-up he only visited it once. Bošković repeatedly used his connections to friends from European nobility and clergy to help the Dubrovnik Republic in various political issues, for example, his friend the Polish king Stanislav Poniatowski helped prevent a Russian siege during the Russian-Turkish war (1768–75), as the king intervened in Dubrovnik's behalf with the Russian empress Catherine II. [14, 17].

3.2 The life of Gelenbevî İsmail Efendi

Gelenbevî İsmail Efendi was among the last distinguished scholars of the Ottoman Empire's so-called "Golden Age," possessing a mathematical intellect that served as an inspiration for subsequent generations. He was born in 1731 CE (1143 AH) in the town of Gelenbe, located within the Saruhan Sanjak of Aydın Province—an area corresponding to modern-day Manisa in western Anatolia [19]. His birth name was İsmail, but due to the absence of surname laws in the 18th-century Ottoman Empire, he became known by the epithet "Gelenbevî," denoting his birthplace.

He hailed from an established scholarly family. His father was Mustafa bin Mahmud Efendi, and thus, in some works, he is identified as "İsmail bin Mustafa bin Mahmud al-Gelenbevî" [2]. According to Ottoman naming conventions, individuals typically included their father's or grandfather's name to indicate lineage. Both his father and grandfather served as *muftis*—jurists authorized to issue Islamic legal opinions (*fatāwā*)—and as *müderris* (professors in the madrasa system). Despite his family's scholarly heritage, Gelenbevî could not benefit from their guidance, having lost his father at a young age and been orphaned. Consequently, lacking a guardian to oversee his education, he spent much of his childhood playing in the streets without formal schooling until age thirteen or fourteen. This situation changed dramatically following a pivotal encounter. One day, while playing with friends, a former colleague of his father admonished him: "For shame! While your father and grandfather were renowned for their scholarship, you squander your life on games and amusement!" [10]. These words profoundly affected İsmail, who thereafter abandoned frivolity and devoted himself entirely to learning.

He commenced elementary education in his hometown of Gelenbe, spending his youth immersed in study. Over time, however, Gelenbe's limited scholarly resources proved insufficient. Seeking advanced education, he moved to Istanbul and continued his studies at the Fatih Complex (*Külliye*) [2]. In the imperial capital, he studied Arabic and Islamic religious sciences under Yâsîncizâde Osman Efendi, and logic, physics, and mathematics under Müftîzâde Mehmed Emîn Efendi—renowned for his encyclopedic knowledge and nicknamed the "Walking Library" (*Ayaklı Kütüphane*). The rigorous instruction he received during this period nurtured his intellectual development and laid the foundation for his later mathematical achievements.

Although Ottoman madrasa curricula of the era included minimal positive sciences, Gelenbevî's perseverance and intellectual brilliance earned him recognition extending beyond Ottoman borders into European scholarly circles [10]. His contributions to mathematics and philosophy were acknowledged during his lifetime, yet his prominence in philosophy was overshadowed by his teacher Mehmed Emîn Efendi, while his mathematical achievements were eclipsed by Muğlalı Mehmed Emîn Efendi. Only posthumously, through the study of his extant works, was his superior scholarly stature fully appreciated. As the statesman-historian Cevdet Pasha famously noted, were it not for Hoca Gelenbevî, no scholarly record would remain to represent that generation [10].

In 1763 (1177 AH), Gelenbevî passed the *müderrislik* examination, formally earning the right to teach at the madrasa level [19]. During the reign of Sultan Abdulhamid I (1774–1789), he was appointed professor of mathematics at the Imperial School of Naval Engineering (*Mühendishâne-i Bahrî-i Hümâyûn*) with strong support from the Grand Vizier and the Kapudan Pasha (Grand Admiral).

A notable episode underscoring his scholarly stature occurred in 1787, when a French engineer visiting Istanbul presented Ottoman officials with logarithmic tables and inquired which branch of science they belonged to. Dissatisfied with their responses, the engineer mockingly declared that "the peoples of the East no longer comprehend mathematics, hence their decline." Officials then brought him to Gelenbevî's residence, requesting an explanation within a limited timeframe. Historical records indicate that Gelenbevî composed a Turkish treatise in a single night, entitled *Şerh-i Cedâvil-i Ensâb* (also known as *Logaritma Şerhi*), detailing the use and function of logarithmic tables. Upon reviewing the work, the French engineer was reportedly astounded by its depth, exclaiming, "Had this man lived in Europe, he would be worth his weight in gold" [2]. Some accounts even suggest Gelenbevî was honored with a sable robe and a portrait commission. This incident demonstrates that Gelenbevî, though not the inventor of logarithms, was well-informed of contemporary European scientific developments when logarithmic knowledge remained scarce in the Ottoman context.

During Sultan Selim III's reign (1789–1807), Gelenbevî reemerged in the public eye. During military drills in Kağıthane, imperial artillery (*humbara*) repeatedly missed targets, angering the Sultan. When asked whether anyone could compute the correct trajectories, Gelenbevî was summoned [2]. Through precise mathematical calculations, he adjusted the range and angles, enabling successful hits. As a reward, he was granted a lifelong daily stipend of four *okka* (approximately 5.2 kg) of rice.

In 1790, he was appointed Chief Judge (*Qadi*) of Yenişehir in the Peloponnese (then Ottoman Morea), serving for about a year. During this tenure, a controversy arose over the commencement of Ramadan. Islamic tradition requires visual sighting of the crescent moon to determine the month's start. While some witnesses claimed to have seen the new moon, Gelenbevî contested their testimonies based on astronomical calculations, asserting sighting was impossible. In response, Sheikh al-Islam Hamidizade Mustafa Efendi issued an official letter harshly criticizing Gelenbevî. Deeply distressed by this public censure, Gelenbevî suffered a stroke and died shortly thereafter in Yenişehir [2].

4 Bošković's and Gelenbevî's mathematical contributions

4.1 The mathematics of Ruđer Josip Bošković

Bošković is well known for his milestone contributions to physics, optics, geophysics, astronomy, and — mathematics. He contributed to spherical and differential trigonometry, theory of conic sections and geometric transformations, applied mathematics, and more. He wrote several original works, from which one discerns his original and systematic approach, with a preference for "pure" geometric methods, even if many of his results were proven both geometrically and using calculus.

While still studying theology in Rome, Bošković became interested in mathematics. His first published mathematical treatise was *Trigonometriae sphaericae constructio* (1737), in which he described methods for solving spherical triangles. To this effect he used geometric constructions in the plane, even if he comments that in practice one can use trigonometric tables for results of arbitrary precision. He remained interested in trigonometry all his life, and in a few later works he systematically described spherical trigonometry. His most important contribution to trigonometry is his *De formules différentielles de trigonométrie* (1785), where he showed that there are four fundamental equations of differential trigonometry from which all other can be deduced [17, 21].

Being interested in the physical sciences, it is surely of no surprise that Bošković also contributed to mathematical physics. Here his most famous result is the solution of the problem of the body of maximal attraction (1743). The problem consists in determining the shape of the rotational solid that exerts maximal attraction force to a poin on its axis. Bošković solved this problem both geometrically and analytically, and his solutions are determined depending on the type of the attraction force [17, 21].

As mentioned in his biography, Bošković also developed a method for error minimization. For a given set of pairs of measurement data, he estimates the relationship of the variables by assuming the existence of unknown errors of measurement and minimizing the absolute deviations (*De Litteraria expeditione per pontificam ditionem ad dimetiendos duos meridiani gradus a P.P. Maire et Boscovich*, 1755, and *Philosophiae recentioris versibus traditae libri decem*, 1765). This was the first such method in history, and it was later, in 1789, described by P. S. Laplace. This method became the forerunner of the method of least squares which was developed shortly after by A.-M. Legendre and C. F. Gauß [17, 21].

The probably most important of all Bošković's mathematical contributions is his theory of conic sections: In *Sectionum Conicarum Elementa* (1754) he develops the first purely planar theory of conic sections in history. For definition, Bošković uses what Pappus of Alexandria showed to be a common property of conic sections, and that is that all of them have the property of a fixed ratio of distances of their point to a given line and a given point. Depending on if this ratio, which Bošković calles *ratio determinans* (modern numerical eccentricity), is greater, equal or less than 1 one obtains hyperbolas, parabolas, and ellipses. Before Bošković it was only Newton, with whose works Bošković was well acquainted, who noticed and highlighted the importance of this property. Bošković's theory of conics is particularly original and systematic, developed by purely geometric arguments, without use of coordinates or calculus. The most famous term from this theory is what is known as the (eccentric) circle of Bošković. This is a circle for which the ratio of the radius and the distance from the centre to the plane of the conic is equal to the *ratio determinans* of the conic section. Although it is the fundamental and main novelty of his theory, Bošković did not name this circle. Using it, he proved a number of well-known and many new results about conic sections [17, 16, 18].

As a supplement to *Sectionum Conicarum Elementa*, Bošković added a text named *De transformatione locorom geometricorum*. Here he discusses transformations of geometric loci, suggests a corresponding axiomatization and describes how various curves transform. His transformations are a type of what is now known as projective transformations, and it was suggested by various authors that this supplement is a particularly simple introduction to this topic. Here Bošković also discussed the notions of infinity and continuity, two topics that he repeatedly discussed in various texts. His earliest discussion of infinitely small and infinitely large quantities was written as early as 1741 (*De natura & usu infinitorum & infinite parvorum*). Here and in some other works Bošković argumented that actual geometric infinity is impossible. His paradoxes belong to a class of many other that were resolved by Bolzano and Cantor in the 19th century by showing that infinite sets can be in one-to-one correspondence with their proper subsets. Another topic that greatly interested Bošković is the concept of continuity, and his most important text on this topic is *De continuitatis lege* (1754). Here he not only discusses geometric continuity, but also in the set of (real) numbers. His approach to continuity is essentially Aristotelian, as he says that the essence of the continuity of a curve consists in the existence of a common boundary that connects its two parts, the *continuum praecedens* and *continuum sequens*. This is is *de facto* an early form the modern definition of the continuum of real numbers by Dedekind cuts. As one can notice, his ideas on continuity and infinity are predecessors of the more than 100 years younger modern concepts of real numbers and infinite sets [3, 17, 21].

Bošković also considered the possibility of geometry in dimensions higher than 3, and was one of the early authors who allowed the possibility of non-Euclidean geometries, as he considered the Euclidean parallel postulate unprovable, i.e., a proper axiom. He also described the geometric form of a honeycomb cell, discussed logarithms of negative numbers, and the binomial theorem [17, 21].

4.2 The mathematics of Gelenbevî İsmail Efendi

Gelenbevî İsmail Efendi served as a bridge between the classical mathematical tradition of the eighteenth-century Ottoman Empire and the emerging modern approach to science. As a scholar well-versed in both the *naqlī* (religious) and *'aqlī* (rational, including natural sciences and philosophy) disciplines, Gelenbevî demonstrated particular mastery in logic, mathematics, astronomy, physics, and *kalām* (Islamic theology) [2]. Although the exact number of his works remains uncertain, it is reported that he authored more than thirty treatises, written in both Turkish and Arabic. This section primarily focuses on his mathematical works.

In his treatise *Hesâbu'l-Küsûr* ("The Arithmetic of Fractions"), Gelenbevî examined topics in arithmetic and algebra. Written in Turkish, the work consists of five sections, discussing calculations with fractions, arithmetic rules, and the determination of unknowns through algebraic methods. In the fifth section, he introduced and solved problems involving equations up to the second degree. Where necessary, he also used the extraction of cube roots and, in such cases, demonstrated how to obtain the roots of equations with the help of logarithmic tables [19]. In this context, he briefly introduced the concept of logarithms and referred to his earlier work *Şerhu Cedâvili Ensâb* ("Commentary on the Tables of Proportions").

In his treatise *Risâle-i Azlâ el-Müsellesât*, Gelenbevî studied the elements of triangles, analyzing the relationships between their sides and angles. He provided explanations concerning the types of angles and the foundations of trigonometry, and he described the relevant computational methods. According to Gelenbevî, in order to precisely determine the sides and angles of triangles—essential in both plane and solid geometry—mathematicians employ three principal methods [2]: the Pythagorean theorem, the tangent rules, and the sine rules.

His already mentioned work *Şerh-i Cedâvil-i Ensâb* ("Commentary on the Tables of Proportions") is a brief treatise concerning the formation and usage of logarithmic tables. It can be regarded as one of the first Turkish works in the field of logarithms [2].

In his Arabic treatise *Risâletü'l-Ceyyib ve'l-Mukantara*, Gelenbevî combined mathematical and astronomical knowledge to discuss the calculation of prayer times and other acts of worship, such as fasting and pilgrimage. Another of his Arabic works, *Risâletü'l-Kıble*, also focuses on astronomy and trigonometry; therein he demonstrated how trigonometric calculations were carried out and supported his explanations with astronomical observations. He emphasized that in Islam, facing the *qibla* (the Ka'ba in Mecca) is an essential condition for valid prayer; otherwise, the prayer is invalid. However, he added that those outside Mecca need only face the imaginary straight line extending from their location toward the Ka'ba [2]. Through fixed-star observations, latitude–longitude calculations, and the application of trigonometric methods to geography, Gelenbevî showed—using figures and examples—how a location's position relative to a central point can be expressed in degrees and minutes. He also provided the coordinates (in degrees and minutes) of several key locations relative to Mecca. His application of mathematical and astronomical principles to Islamic practice further strengthened his intellectual and social standing.

For Gelenbevî, mathematics was not merely a computational tool but also a discipline for the refinement of the intellect. This perspective explains why he employed mathematics across various fields of inquiry. The same approach can be observed in the aforementioned works, wherein he treated mathematics as both a theoretical and applied science—making it teachable, practical, and accessible.

In his geometric works, particularly in *Usûl el-Hendese* ("Principles of Geometry"), Gelenbevî adopted a simplified, didactic approach based on Euclidean geometry. He emphasized reasoning in geometric proofs, a characteristic that reinforced the theoretical foundations of engineering education in the Ottoman Empire. His lectures at the *Mühendishâne-i Berrî-i Hümâyun* (Imperial School of Military Engineering) were among the pioneering efforts in modern engineering and topography education.

In summary, Gelenbevî systematized mathematical teaching and thought during a period in which the eighteenth-century Ottoman scientific milieu began to open toward the West. His works continued to be used as textbooks in madrasas and military schools throughout the nineteenth century. Today, numerous copies of his manuscripts are preserved in the Süleymaniye, Topkapı, and Millet libraries (in İstanbul and Ankara). Through his students and contemporaries, Gelenbevî İsmail Efendi opened the door to modern mathematics and exerted an indirect yet lasting influence through his methods [2].

5 Conclusion

Even if Bošković and Gelenbevî lived at approximately the same time and hugely influenced mathematics in their respective homelands, their lives were significantly different. However, one cannot but notice some similarities: their religious education background, their public services, their originality. While the social, educational and scientific conditions remained quite different in the western European and Ottoman lands of their times, they were both born into the age of reforms and new awakenings. Both were honoured already in life, even if Gelenbevî achieved more fame posthumously. Their mathematical contributions differ, but they shared a common interest in trigonometry, astronomy, and applications of mathematics. Curiously enough, they could have met: When Bošković came to Constantinople in 1761, Gelenbevî was there for his higher studies. Even if we as of now have no definite answer to the question if they ever met, just the possibility of the meeting of the two great minds of the two red-and-white countries of the authors of this paper is a cool thought. We hope to investigate this conjecture in the future, and bid farewell to our readership — *güle güle*, *zdravi i veseli bili*!

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