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Design of Modified Express Certificates on One Underlying Asset

Abstract

The aim of this paper is to focus on new, innovative financial instruments, specifically modifications of express certificates, namely Step-down and Memory express certificates. The creation techniques are introduced, employing a combination of zero-coupon bonds and vanilla put and/or barrier put options that play a significant role in financial engineering. The analyzed creation of express certificates is examined from the perspectives of issuers and potential investors. The methodology of the paper is based on European-style vanilla put and barrier put options. Due to the lack of real-traded European vanilla options and barrier options, the calculations of option premiums are conducted using the statistical program R. The evaluation of the analyzed express certificates is scrutinized with an emphasis on costs and profits for the issuer and potential investor. The theoretical evaluation of the introduced modifications of the express certificates is applied to the stocks of the European company Bayer AG. Additionally, an analysis of profitability for the issuer and investor is performed at maturity. Specific characteristics of the proposed modifications of the express certificates are highlighted and compared with one another. The minimum profit is presented for the issuer and the investor in an ideal situation. Proposed certificates can be part of a personal investment portfolio. The main objective is to demonstrate the nature of the creation of Step-down and Memory express certificates while enhancing the knowledge of all potential investors. The research has one limitation, which is the unavailability of European vanilla put and barrier put options for European companies. Therefore, the theoretical prices are calculated.

Keywords: Investment certificate, express certificate, vanilla option, barrier option, option pricing.

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1. INTRODUCTION

The development of capital markets is linked to the creation of new products and approaches. The structured investment products are one alternative tailored directly for the investor.

According to the Swiss Structured Products Association (SSPA), structured products contain two main components, namely the underlying asset (referred to as UA henceforth) and the derivative tool, primarily an option component (plain-vanilla and/or barrier option). The advantages of these products include the flexibility of aspects such as the asset class itself, their maturity, currency, or the level of capital protection. Several studies by different authors address structured products from various perspectives. Bertrand and Prigent (2019) examined the suitability of several standard financial structured products, demonstrating that standardized products are not optimal and identifying significant monetary losses due to their non-optimality. Das and Statman (2013) illustrated that structured products play a role in behavioral portfolios and explored the roles of option collars, capital guaranteed notes, and barrier range notes within behavioral portfolios.

Investment certificates represent the largest portion of structured products. Therefore, issuers (mainly banks) continuously offer new types of these products with varying levels of risk-return profiles. They serve as an alternative to direct investments in equity, bond, commodity, and currency assets. An example is a debt security, which consists of two components: a fixed-income security and an option. The fixed-income security is usually a zero-coupon bond that protects either part or all of the invested capital's value. The option can be utilized to generate returns based on the behavior of the UA. Several authors address the topic of investment certificates. For instance, Hernandez, Lee, Liu & Dai (2013) examined outperformance certificates, pricing them as a combination of the UA, a short position in zero-coupon bonds, and call options. Younis and Rusnáková (2014) developed bonus certificates as a combination of purchasing of an UA and acquiring down and knock-out put options on the same UA. Timková and Bobriková (2023) proposed Twin-Win Outperformance certificates and their modification with cap based on the combination of the UA and European call options and down and knock-out put options. Rossetto and Bommel (2009) analyzed endless leverage certificates using the Monte Carlo method. Entrop, McKenzie, Wilkens & Winkler (2016) studied the performance of individual investors in portfolio holdings of discount and bonus certificates.

The paper proposes new modifications of express certificates, specifically Step-down and Memory express certificates, with different levels of payouts using vanilla and barrier options. The proposition includes an analysis of potential profits from both the investor's and issuer's perspectives. These certificates can be created by combining a zero-coupon bond and an option. Step-down express certificate contains the vanilla put option and Memory express certificate contains the barrier down and knock-out put option. The contribution of the paper is the introduction of the creation of Step-down and Memory express certificates with a practical application to the shares of the German company Bayer AG.

In the beginning, the paper presents a brief overview of classic express certificates and their modifications. The next section outlines the methodology based on the papers that cover the topic of vanilla and barrier options. This is followed by an analysis of new modifications of the express certificate using these options. The final section contains the application to shares of Bayer AG, along with its results and discussion.

2. THEORETICAL BACKGROUND

The following section focuses on the theoretical background of express investment certificates, identifies key authors and researchers who have addressed this topic, and introduces newly proposed structures of Step-down and Memory express certificates.

The ***classic express certificate*** was introduced by Harčariková and Bánociová (2015) as a combination of the UA, down and knock-out put option and vanilla call option of European type. Hernandez, Tobler & Brusa (2010) focus on the express certificates' valuation on the index, based on zero-coupon bonds, and cash-or-nothing call options, along with put options. Additionally, Šoltés, Timková & Gičová (2019) examined modifications of express certificates on two assets using correlation and barrier options.

Step-down express certificates represent a specific type of express certificate in which the early redemption level decreases annually as the certificate's maturity is extended. The product has a fixed duration and is redeemed early if the UA reaches or exceeds the declining early maturity level on any valuation date. Otherwise, maturity is extended to the next valuation date, up to the final one. On the final date, if the UA is above the barrier (typically equal to the final maturity level), the nominal value (referred to as NV henceforth) and the agreed yield are paid. If the UA falls below the barrier, investors receive a predefined number of shares (NV/initial UA value). The structure is based on European-style options.

Other modifications of express certificates include ***Memory express certificates***, that incorporate a memory mechanism, allowing coupon payments even without early redemption. On each valuation date, the UA price is compared with the early maturity level and the barrier. If the UA price is at or above the early maturity level, the certificate is redeemed early at its NV with the coupon. If the price is below the early maturity level but above the barrier, the coupon is paid, and the maturity is extended by one year. If the price falls below the barrier, the coupon is not paid, but the memory mechanism stores the missed payment, which is later paid out once the UA price exceeds the barrier or early maturity level on a future valuation date. At final maturity, only the barrier is assessed: if the UA is above it, the NV and all accumulated coupons are paid; if not, the investor receives a predefined number of shares (NV/initial UA value). Even in this case, the conditions for redemption depend on the value of UA on the valuation dates; therefore, the issuer will use European - type options.

3. RESEARCH METHODOLOGY

The valuation of structured products is a complex process. Many authors focus on the pricing of these products with applications to concrete examples. Henderson and Pearson (2011) focused on the pricing of 64 issues of a retail structured equity product using option pricing methods where they found that, on average, investors pay an 8% in a US sample premium above the estimated fair market value. Madan, Pistorius & Schoutens (2013) investigated the valuation of structured products using Markov chain models, while Choi, Doshi, Jacobs & Turnbull (2020) introduced a top-down no-arbitrage model for pricing structured products, where losses are described by Cox processes. Schertler (2016) studied how the overpricing of discount certificates in Germany changed when competing products were issued, indicating that retail investors' failure to recognize the value implications of issuers' credit risk can undermine product competition.

The options are the main tool forming the express certificate. Options form a significant part of structured products because they provide their specific profile. They represent a risky part of the product, as they can expire worthless and lose their value. Step-down express certificate are created by the vanilla put options and the Memory express certificate by barrier down and knock-out put options, all of the European type. The barrier level is monitored only on the final valuation date of the certificate. Hull (2018) defines a vanilla option as a financial instrument that the option holder has the right, but not the obligation, to buy (call option) or sell (put option) an agreed amount of the UA at an agreed strike price at a precise time in the future (European option type) or during maturity (American option type). For this right, the option holder must pay an option premium to the seller (option issuer). Modifications to vanilla options are barrier options. According to Zhang (1998), barrier options can be divided into buying (call) options and selling (put) options and categorized as down or up depending on whether the barrier condition is below or above the strike price. If the barrier is reached or exceeded, the option is activated (knock-in) or deactivated (knock-out). Down and knock-out put options are activated if the barrier is not breached and, at the same time, the price of the UA is less than the strike price. In this case, the option allows us to sell the UA at a higher strike price while simultaneously buying on the market for a lower price.

The valuation of the options is important for using of express certificates' creation. Black and

Scholes (1973) introduced theoretical pricing for European vanilla call and put options on stocks without dividends. The modification of the Black-Scholes formula is introduced by Merton (1973), where he derived the pricing for vanilla call and put options on stocks with dividends. The pricing of barrier options is influenced by the barrier level. Additionally, Merton (1973) derived the first relationship for the European down and knock-out call option price. Rubinstein and Reiner (1991) applied the Black-Scholes-Merton formula to eight basic types of barrier options, and Haug (2007) applied it to all sixteen types of standard European barrier options.

Calculations of the theoretical prices of vanilla and barrier options were carried out using the available data for the selected underlying asset from the financial portal Yahoo Finance. The calculations are performed in the RStudio program, where the functions are used from the installed packages (fOptions, RQuantLib), following the pricing model of Haug (2007). The price of a European-type vanilla put option is defined as:

GBSOption(TypeFlag = "p", S = S_0, X = K,
Time = T, r = r, b = r-q, sigma = sigma), (1)

where p represents the put option, S_0 the current price of the UA, K the exercise price, T the time to maturity, r the risk-free interest rate, b the cost of carry of the UA, q the dividend yield of the UA, and σ the volatility of the UA.

The price of the down and knock-out put option is calculated as:

BarrierOption(barrType="downout",
type="put", underlying, strike, dividendYield,
riskFreeRate, maturity, volatility, barrier) (2)

4. RESEARCH RESULTS

The following section focuses on the analysis of the structure of Step-down and Memory express certificates using vanilla and barrier options, as well as on the identification of all possible price scenarios of the UA until the certificates' maturity.

The authors Harčariková and Bánociová (2015) analyzed the creation of classic express certificates for one UA. Šoltés et al. (2019) designed modifications of express certificates involving two assets based on correlation and barrier options. Based on two analyses mentioned above, the design of two modified express certificates is being proceeded for one UA, namely a Step-down express certificate and a Memory express certificate.

In the case of a ***Step-down express certificate***, the investor invests EUR 1,000 during the subscription period (this amount is equal to the NV of the certificate) until the purchase of the express certificate. The issuer pays an entry fee f for the acquisition, set at a certain percentage of the NV of the certificate (in most cases, 2% of the NV). Therefore, he invests a total of EUR 1,020, i.e. $(1{,}000 + 0.02 \cdot 1{,}000)$ and after that, he can then observe how the price of the UA will develop over the lifetime of the certificate.

The issuer buys a zero-coupon bond with t-year maturity for the funds, acquired from the investor (there is considered a total maturity of 3 years) equal to the maximum maturity of the certificate, discount rate r, and the nominal value NV = EUR 1,000, for the present value of D, based on relationship (3). After purchasing the bond, the issuer will be left with $1{,}020 - D$ funds that he will invest in the purchase of the options on the UA.

$$D = \frac{1{,}000}{(1 + r)^t} \tag{3}$$

The issuer buys a vanilla put option with a strike price $X_1 = S_0$ and expiration in 1 year (the 1st valuation date) for the price p_1, where S_0 represents the initial value of the given UA on the opening day of the valuation (representing the closing price). After the purchase of the option, the issuer will be left with $1{,}020 - D - p_1$. He will keep this balance in the bank for 1 year at the interest rate r_1 and current taxation d, and after a year (the 1st valuation date), he will have:

$$M_1 = (1{,}020 - D - p_1) \cdot [1 + r_1 \cdot (1 - d)] \tag{4}$$

If after a year (the 1st valuation date for the determination of early maturity), the situation will be:

a) $S_1 \geq S_0$, the issuer sells the bond at a price D_1, he will have a total $M_1 + D_1$. He will pay the investor the agreed amount of (1,000 + $R \cdot$ 1,000), where R represents the agreed annual yield (% of NV certificate), and the rest $[(M_1 + D_1) - (1{,}000 + R \cdot 1{,}000)]$ is his profit after the first year. The issuer will let the option expire, and the certificate will end prematurely.

b) $S_1 < S_0$, the issuer realizes the option, i.e., buys the UA on the market for S_1 and sells it for S_0. After a year, he will have a total of $M_1 + S_0 - S_1$ and the certificate continues.

If the certificate has not been repaid early, the issuer will repurchase the vanilla put option with the exercise price $X_2 = 0.9 \cdot S_0$ and expiration in 1 year (2nd valuation date) for the price p_2. After the purchase of the option, the issuer will be left with a balance $M_1 + S_0 - S_1 - p_2$. He will leave this balance in the bank for 1 year at the interest rate r_2 and the current taxation d. After the second year (2nd valuation date), he will have:

$$M_2 = (M_1 + S_0 - S_1 - p_2) \cdot [1 + r_2 \cdot (1 - d)] \qquad (5)$$

If after the second year (2nd valuation date for determination of early maturity) the situation will be as follows:

a) $S_2 \geq 0.9 \cdot S_0$, the issuer sells the bond at a price D_2, he will have the amount of $M_2 + D_2$. The amount $(1{,}000 + 2R \cdot 1{,}000)$ will be paid to the investor, and the rest of $[(M_2 + D_2) - (1{,}000 + 2R \cdot 1{,}000)]$ is his profit after the second year. The issuer will let the option expire, and the certificate will end prematurely.

b) $S_2 < 0.9 \cdot S_0$, the issuer realizes the option, i.e. buys the UA on the market for the S_2 and uses the option to sell it for amount of $0.9 \cdot S_0$. He will have after two years of $M_2 + 0.9 \cdot S_0 - S_2$, and the certificate continues.

If the total maturity is 3 years and the certificate has not been repaid early, the issuer buys a barrier down and knock-out put option with a strike price $X_3 = 0.8 \cdot S_0$, barrier $B = 0.6 \cdot S_0$ and expiration in 1 year (3rd valuation date) for the price p_3. After the purchase of the option, the issuer will be left with $M_2 + 0.9 \cdot S_0 - S_2 - p_3$. He will let this balance in the bank for 1 year at the interest rate r_3 and current taxation d, and after the third year (3rd valuation date) he will have:

$$M_3 = (M_2 + 0.9 \cdot S_0 - S_2 - p_3) \cdot [1 + r_3 \cdot (1 - d)] \qquad (6)$$

If after the third year (3rd valuation date for final maturity) it will be:

a) $S_3 \geq 0.8 \cdot S_0$, the issuer will sell the bond for EUR 1,000, therefore together he will have M_3 + 1,000. He will pay the investor the amount of $(1{,}000 + 3R \cdot 1{,}000)$ and the rest of $[(M_3 + 1{,}000) - (1{,}000 + 3R \cdot 1{,}000)]$ is his profit. The issuer will let the option expire.

b) $B < S_3 < 0.8 \cdot S_0$, the issuer sells the bond for EUR 1,000 and at the same time exercises the option, i.e. buys the UA on the market for S_3 and sells for $0.8 \cdot S_0$. He will have together $M_3 + 0.8 \cdot S_0 - S_3 + 1{,}000$ out of which he pays the investor EUR 1,000 and the rest is his profit.

c) $S_3 < B$, the issuer sells the bond for EUR 1,000 while buying on the market n (i.e. NV/S_0) pieces of the UA for S_3, hands them over to the investor and the remainder of $M_3 + 1{,}000 - nS_3$ is his profit. In this case, the investor incurs a loss.

We would proceed in the same way even if the total maturity of the certificate were several years, i.e., 5 years, while the exercise price of the vanilla put option at maturity would be at the barrier level.

Design of ***Memory express certificates*** is proceeded in the same way as in the creation of step-down express certificates. The investor, during the subscription period, invests the NV certificate (1,000 EUR) in the purchase of a memory express certificate, paying the issuer an entry fee f set at a certain % of the NV certificate, i.e., 2% *of NV*. The investor invests $1{,}020 = (1{,}000 + 0.02 \cdot 1{,}000)$ and he can observe the development of the UA price in relation to the level of early maturity and the barrier throughout the entire duration of the certificate.

The issuer buys a t-year bond (t = 3 years) with a zero-coupon, a nominal value of NV = EUR 1,000 and a discount rate of r, for the present value D, calculated according to the relationship (3). After purchasing the bond, the issuer will be left with an amount 1,020 – D, which it invests in the purchase of options on the UA.

The issuer buys a barrier down and knock-out put option with a strike price $X_1 = S_0$, a barrier $B = 0.6 \cdot S_0$, and an expiration of 1 year (1st valuation date) for the price p_1. After buying the option, he will have a remaining amount of 1,020 – D – p_1. He will then leave this balance in the bank for 1 year at an interest rate r_1 and the current tax rate d and after a year (1st valuation date), he will have:

$$M_1 = (1{,}020 - D - p_1) \cdot [1 + r_1 \cdot (1 - d)] \qquad (7)$$

If, after the first year (1st valuation date to determine early maturity), the situation will be:

a) $S_1 \geq S_0$, the issuer lets the option expire, sells the bond prematurely at the price D_1 and pays the investor a sum of $(1{,}000 + R \cdot 1000)$, where R represents the agreed annual revenue (% of NV certificate). The remainder of $[(M_1 + D_1) - (1{,}000 + R \cdot 1{,}000)]$ is his profit after the first year, and the certificate ends prematurely.

b) $B < S_3 < 0.8 \cdot S_0$, the issuer exercises the option, i.e. buys the UA on the market for S_1 and sells it for S_0. The profit from the option will be in the amount of $S_0 - S_1$ and the certificate continues. The issuer will pay the investor a coupon in the amount of $R \cdot 1{,}000$ and $M_{1b} = M_1 + S_0 - S_1 - R \cdot 1{,}000$ is his balance after the first year.

c) $S_3 < B$, the option is valueless, the investor will not receive the coupon and the issuer will be left with $M_{1c} = M_1$ after the first year and the certificate continues.

The issuer again buys a barrier down and knock-out put option with a strike price $X_2 = S_0$, barrier $B = 0.6 \cdot S_0$ and expiration in 1 year (2nd valuation date) for the price p_2. After the purchase of the option, he will be left with $M_{1b} - p_2$ (if the coupon was paid for the first year) or $M_{1c} - p_2$ (if the coupon was not paid for the first year). This remainder he will leave this balance in the bank for 1 year at an interest rate r_2 and the current tax rate d, and after the second year (2nd valuation date), he will have:

$$M_{2b} = (M_{1b} - p_2) \cdot [1 + r_2 \cdot (1 - d)] \qquad (8)$$

$$M_{2c} = (M_{1c} - p_2) \cdot [1 + r_2 \cdot (1 - d)] \qquad (9)$$

If after the first year (1st valuation date to determine early maturity) the situation will be:

a) $S_2 \geq S_0$, the issuer lets the option expire, sells the bond prematurely at the price D_2, and pays off to the investor $(1{,}000 + R \cdot 1{,}000)$ (if the coupon was paid for the first year) or $(1{,}000 + 2R \cdot 1{,}000)$ (if the coupon was not paid for the first year). The remainder of $[(M_{2b} + D_2) - (1{,}000 + R \cdot 1{,}000)]$ or $[(M_{2c} + D_2) - (1{,}000 + 2R \cdot 1{,}000)]$ is his profit after the second year and the certificate ends prematurely.

b) $B < S_3 < 0.8 \cdot S_0$, the issuer exercises the option, i.e. buys the UA on the market for S_2 and sells it for S_0. The profit from the option will be in the amount of $S_0 - S_2$ and the certificate continues. The issuer will pay the investor a coupon in the amount of $R \cdot 1{,}000$ (if the coupon was paid for the first year) or $2R \cdot 1{,}000$ (if the coupon was not paid for the first year) and $M_{3b} = M_{2b} + S_0 - S_2 - R \cdot 1{,}000$ or $M_{4b} = M_{2c} + S_0 - S_2 - 2R \cdot 1{,}000$ is his balance after the second year.

c) $S_3 < B$, the option is valueless, the investor will not receive the coupon, and after the second year, the issuer will be left with $M_{3c} = M_{2b}$ or $M_{4c} = M_{2c}$, and the certificate continues.

If the total maturity of the t certificate is equal to 3 years, then the issuer again buys a barrier down and knock-out put option at the strike price $X_3 = S_0$, barrier $B = 0.6 \cdot S_0$ and expiration in 1 year (3rd valuation date) at price p_3. After purchasing of the option, he will be left with $M_{3b} - p_2$ (if the coupon was paid annually) or $M_{4b} - p_3$ (if the coupon was paid for both years in the second year) or $M_{3c} - p_3$ (if the coupon was paid for the first year but not for the second year) or $M_{4c} - p_3$ (if the coupon has not been paid in either year). He will keep this balance in the bank for 1 year at interest rate r_3 and the current tax rate d, and after the third year (3rd valuation date), he will have:

$$M_{5b} = (M_{3b} - p_3) \cdot [1 + r_3 \cdot (1 - d)] \qquad (10)$$

$$M_{6b} = (M_{4b} - p_3) \cdot [1 + r_3 \cdot (1 - d)] \qquad (11)$$

$$M_{5c} = (M_{3c} - p_3) \cdot [1 + r_3 \cdot (1 - d)] \qquad (12)$$

$$M_{6c} = (M_{4c} - p_3) \cdot [1 + r_3 \cdot (1 - d)] \qquad (13)$$

Table 1: Overview of the Proposed Products

Name	„Bayer Step-Down Express“ „Bayer Memory Express“
Nominal value:	1,000
Currency:	EUR
Investment horizon:	5 years
Initial valuation date:	05.11.2019
Valuation dates for determining early maturity:	05.11.2020 / 05.11.2021 / 04.11.2022 / 06.11.2023
Final valuation date:	05.11.2024
Early maturity levels such as % initial valuation value (valid for Step-Down Express certificate)	2020 (100%), 2021 (90%), 2022 (80%), 2023 (70%), 2024 (barrier 60%)
Early maturity levels such as % initial valuation value (valid for Memory Express certificate)	2020 (100%), 2021 (100%), 2022 (100%), 2023 (100%), 2024 (100% and barrier 60%)
Settlement method:	financial settlement or delivery of shares
Yield	6% p.a.
Barrier:	observation on the final valuation date
Underlying asset	Bayer AG
Initial value:	S_0 = EUR 71.64
Barrier (60 %):	B = EUR 42.98
Annualized volatility:	σ = 27.23%
Dividends:	δ = 0.11%
Risk - free interest rate:	r = 2.07%

Source: own research based on data from Yahoo Finance (2024)

There would proceed in the same way even if the total maturity of the certificate were several years, i.e., 5 years, while there would consider the purchase of the barrier down and knock-out put option every year with a strike price of S_0 and a barrier *B*.

5. RESEARCH RESULTS - DESIGN OF MODIFIED EXPRESS CERTIFICATES FOR BAYER SHARES

The paper deals with the design of two modified express certificates, namely Step-down express and Memory express certificates, for shares of the German company Bayer AG. Company Bayer AG is a German multinational pharmaceutical and life sciences company, as well as one of the largest pharmaceutical companies in the world. The proposal involves analyzing potential profits from both the investor's and the issuer's perspectives. The creation of both types of certificates consists of two main components: zero-coupon bonds and vanilla put options (applicable to step-down certificates), as well as barrier down and knock-out put options (applicable to memory certificates).

5.1. Data

The provided certificates are proposed and valued as of the fixation date on November 5, 2019 using actual market data. The barrier is set at the level of 60%, which allows the value of assets to fall by up to 40%, whereas the investor assumes that no major decline will occur. It is important to note that the barrier does not follow continuously, but only on the last day of observation (November 5, 2024). The products are issued with a NV of EUR 1,000 with a 5-year expiration date and a maturity date of November 5, 2024. During the subscription period, an entry fee is also set on the level of 2% of the invested NV of the certificate. During the term, there may be an early repayment, when the investor, under certain conditions, receives a predetermined return of 6% of the NV of the certificate. An overview of the proposed products is shown in Tab. 1.

The construction assumes that the transaction costs associated with purchasing individual components of the proposed certificates are neglected, and that there is a possibility on the market to carry out such operations. As of the initial valuation date of November 5, 2019, the risk-free interest rate is at 2.07%, historical volatility at 27.23%, and the dividend yield at 0.11%. During the lifetime of the certificates, there are considered the given parameters to be constant, and they form the basis for the theoretical valuation of individual options, based on which the certificates are created. In the case of a deposit of funds, we consider an annual interest rate of 1% and taxation of 19%.

5.2. Results - Step-Down Express Certificate

At the time of the issue (November 5, 2019), the investor purchased the Bayer Step-Down Express certificate for EUR 1,020 (*EUR* 1,000 + 2% *fee*) and was subsequently able to monitor the development of the Bayer AG share price throughout the lifetime of the certificate.

The issuer buys a zero-coupon bond with a 5-year maturity and a discount rate of 5% for the present value of EUR 783.53 (based on relationship (3)). After purchasing the bond, the issuer will have EUR 236.47 (1,020 – 783.53), which he will invest in the purchase of 13 options (i.e., 1000/71.64) on Bayer AG shares. The issuer buys 13 vanilla put options with the strike price X_1 = 71.64 and the expiration in 1 year (1st valuation date) for the price of EUR 7.61 each (totaling EUR 98.96), based on relation (1). After the purchase of the options, the issuer will have EUR 137.51. He will leave this balance in the bank for 1 year at an annual interest rate of 1% and current taxation of 19% and after a year (November 5, 2020), he will have EUR 138.63 based on relation (4).

On the 1st valuation date (November 5, 2020), the shares of Bayer AG are quoted at the level S_1 = 43.79, i.e., [S_1(43.79) < S_0(71.64)]. The issuer exercises the options, he buys 13 shares of Bayer AG for EUR 43.79 each and sells using options for EUR 71.64 each, resulting in a profit of EUR 362.05 from 13 options. After a year, the issuer will have EUR 500.68, and the certificate continues. Since the certificate was not early redeemed, the issuer buys 13 vanilla put options on the 1st valuation date with a strike price X_2 = 64.48 and an expiration of 1 year (2nd valuation date) at a price of EUR 20.80 each (totaling EUR 270.34). After purchasing of the options, the issuer will have EUR 230.34. He will leave this amount in the bank for 1 year and after one year (November 5, 2021), he will have EUR 232.20.

In the 2nd valuation date (November 5, 2021), the shares of Bayer AG are quoted at the level S_2 = 49. 72, i.e. [S_2(49.72) < 0.9 ∗ S_0(64.48)] the issuer exercises options, he buys 13 shares of Bayer AG for EUR 49.72 each and sells for EUR 64.48 each, (a profit of EUR 191.83 from the 13 options). After a year, he will have a total of EUR 424.03 and the certificate continues. Since the certificate was not early redeemed, the issuer buys 13 vanilla put options in the 2nd valuation date with the strike price X_3 = 57.31 and an expiration of 1 year (3rd valuation date) at EUR 10.16 each (i.e. EUR 132.13). After purchasing of the options, the issuer will have EUR 291.90. He will leave this amount in the bank for 1 year and after one year (November 4, 2022), he will have EUR 294.26, and the certificate will continue.

In the 3rd valuation date (November 4, 2022), the shares of Bayer AG are quoted at the level S_3 = 53.88, i.e. [S_3(53.88) < 0.8 ∗ S_0(57.31)], the issuer exercises options and buys 13 shares of Bayer AG for EUR 53.88 each and sells using options for EUR 57.31 each, the profit from 13 options is EUR 44.62. After a year, it will have a total of EUR 338.88 and the certificate continues. As the certificate was not early redeemed, the issuer will buy 13 vanilla put options in the 3rd valuation date with the strike price X_1 = 42.06 and the expiration of 1 year (4th valuation date) at EUR 3.89 each (i.e. EUR 50.51). After the purchase of options, the issuer will have EUR 288.37. He will leave this amount in the bank for 1 year and after one year (November 06, 2023) it will have EUR 290.71, and the certificate will continue.

On the 4th valuation date (November 6, 2023), the shares of Bayer AG are quoted at the level S_4 = 42.06, i.e. [S_4(42.06) < 0.7 ∗ S_0(50.15)]. The issuer exercises options and buys 13 shares of Bayer AG for EUR 42.06 each and sells for EUR 50.15 each, resulting in a total profit of EUR 105.14. After a year, it will have a total of EUR 395.85, and the certificate continues. As the certificate was not redeemed early, the issuer will buy 13 vanilla put options on the 4th valuation date with the strike price equal to the barrier level, i.e. X_5 = 42.98 and an expiration of 1 year (5th final valuation date) at EUR 4.98 each (i.e. EUR 64.79). After the purchase of options, the issuer will have EUR 331.06. He will leave this amount in the bank for 1 year, and after one year (November 5, 2024), he will have EUR 333.74.

On the 5th final valuation date (November 5, 2024), the shares of Bayer AG are quoted at the level S_5 = 24.58, i.e. [S_5(24.58) < 0.6 ∗ S_0(42.98)]. The issuer will sell the bond for EUR 1,000, exercise options, he will buy 13 shares of Bayer AG at EUR 24.58 each, totaling EUR 319.54, which he will deliver to the investor with a cash surcharge of EUR 0.9587. At the same time, he will sell 13 shares of Bayer AG at EUR 42.98 each using the options, amounting to EUR 558.79. The remainder, EUR 1,252.49, i.e. (333.74 + 1,000 – 319.54 – 0.9587 + 558.79), represents the issuer's profit at maturity. In this case, the investor incurs a loss.

5.3. Results - Memory Express Certificate

At the time of the issue (November 5, 2019), the investor purchased the Bayer Memory Express certificate for EUR 1,020 (*EUR* 1.000 + 2% *fee*) and was subsequently able to monitor the development of the Bayer AG share price throughout the lifetime of the certificate.

The issuer buys a zero-coupon bond with a 5-year maturity and a discount rate of 5% for the present value of EUR 783.53 (based on relation (3)). After purchasing the bond, the issuer will have EUR 236.47 (1,020 – 783.53), which he will invest in the purchase of 13 barrier options (i.e., 1,000/71.64 on Bayer AG shares. The issuer bought 13 barrier down and knock-out put options with a strike price of X_1 = 71.64, a barrier level of 60% (B = 42.98), and an expiration in 1 year (1st valuation date) for the price of EUR 5.10 each. He will leave this balance in the bank for 1 year at an annual interest rate of 1%, and a current tax rate of 19%, and after a year (November 5, 2020), he will have EUR 171.57 based on relation (7).

On the 1st valuation date (November 5, 2020), the shares of Bayer AG are quoted at the level S_1 = *EUR* 43.79, i.e., [B(42.98) < S_1(43.79) < S_0(71.64)]. The issuer exercises the options, he buys 13 shares of Bayer AG for EUR 43.79 each and sells them for EUR 71.64 each. The profit from 13 options is EUR 362.05, and the certificate continues. The issuer will pay the investor a coupon of EUR 60, and his balance will be EUR 473.62. Since the certificate was not early redeemed, the issuer buys 13 barrier down and knock-out put options on the 1st valuation date with a strike price of X_2 = 71.64, a barrier level B = 42.98, and an expiration of 1 year (2nd valuation date) at a price of EUR 0.60 each (i.e., EUR 7.83). After one year (November 5, 2021), the bank balance will be EUR 469.56 after purchasing of the options.

On the 2nd valuation date (November 5, 2021), the shares of Bayer AG are quoted at the level S_2 = *EUR* 49.72, i.e., [B(42.98) < S_2(49.72) < S_0(71.64)]. The issuer exercises options, he buys 13 shares of Bayer AG for EUR 49.72 each and sells them for EUR 71.64 each. The profit from 13 options is EUR 284.96, and the certificate continues. The issuer will pay the investor a coupon of EUR 60 and his balance will be EUR 637.78. Since the certificate was not early redeemed, the issuer buys 13 barrier down and knock-out put options on the 2nd valuation date with a strike price X_3 = 71.64, a barrier level B = 42.98, and an expiration of 1 year (3rd valuation date) at EUR 4.37 each. After purchasing the options, the issuer will have EUR 637.78. He will leave this amount in the bank for 1 year and after one year (November 4, 2022), he will have EUR 642.94.

On the 3rd valuation date (November 4, 2022), the shares of Bayer AG are quoted at the level S_3 = *EUR* 53.88, i.e., [B(42.98) < S_3(53.88) < S_0(71.64)]. The issuer exercises options and buys 13 shares of Bayer AG for EUR 53.88 each, and sells them for EUR 71.64 each. The profit from 13 options is EUR 230.88, and the certificate continues. The issuer will pay the investor a coupon of EUR 60 and his balance will be EUR 813.82. Since the certificate was not early redeemed early, the issuer will buy 13 barrier down and knock-out put options on the 3rd valuation date with a strike price X_4 = 71.64, a barrier level B = 42.98, and an expiration of 1 year (4th valuation date) at EUR 5.95 each (i.e., EUR 77.39). After purchasing the options, the issuer will be have EUR 736.43. He will leave this amount in the bank for 1 year and after one year (November 6, 2023), he will have EUR 742.39, and the certificate will continue.

On the 4th valuation date (November 6, 2023), the shares of Bayer AG are quoted at the level S_4 = 42.06, i.e., [S_4(42.06) < B(42.98)]. In this case, the options are worthless, the investor does not receive the coupon, and the certificate continues until the final valuation date. The issuer's balance is at EUR 742.39. He will leave this amount in the bank for 1 year. After one year (November 5, 2024), he will have EUR 748.41. As the shares on November 6, 2023, are quoted below the barrier level, it is not possible to buy barrier down and knock-out put options in the given year, because $S_4 < B$, and the given options do not exist. The issuer has created a financial reserve from previous periods in the case of the investor is paid out at maturity. Due to this fact, the issuer does not have to purchase barrier options.

On the 5th final valuation date (November 5, 2024), the shares of Bayer AG are quoted at the level S_5 = 24.58, i.e., [S_5(24.58) < B(42.98)]. The issuer will sell the bond for EUR 1,000, buy 13 shares of Bayer AG at EUR 24.58 each, totaling EUR 319.54, which he will transfer to the investor along with a cash surcharge of EUR 0.9587. The investor does not receive the coupon. The remainder, EUR 1,427.91, i.e., (748.41 + 1,000 – 319.54 – 0.9587), constitutes the issuer's profit at maturity. In this scenario, the investor incurs a loss.

5.4. Results - Discussion

The analysis of the Step-down Express and Memory Express certificates applied to Bayer AG shares confirms the theoretical feasibility of these products through a combination of zero-coupon bonds and option strategies. Issuers realised profits in all scenarios, whereas investors faced potential losses when the UA declined. This asymmetry in payoffs aligns with findings by Henderson and Pearson (2011) in U.S. retail equity structured products.

Unlike traditional express certificates (Harčariková & Bánociová, 2015; Hernandez et al., 2010) and two-asset variants (Šoltés et al., 2019), the Step-down design employs progressively lower early-redemption barriers, and the Memory version accrues unpaid coupons until barrier conditions are satisfied. However, these features also reduce the likelihood of investor gains, corroborating Bertrand and Prigent (2019) on the suboptimality of standardized structured products from the investor's standpoint.

A limitation of this study is the absence of actively traded European options, which may affect the practical applicability of the results in terms of actual market costs and liquidity. The complexity of these instruments, as documented by Entrop et al. (2016), further underscores the high level of financial literacy required. Retail investors face a clear disadvantage in these products' design and distribution, making it vital they fully grasp the complex pay-off mechanisms. Regulators and advisors must boost transparency and education to close the information gap and protect investors.

6. CONCLUSIONS

Structured products are a group of investment instruments that offer investors several benefits, such as the ability to invest even with a relatively low minimum input. This makes them an interesting and useful tool, especially for retail investors, who are seeking easy access to derivatives. They provide a suitable investment alternative for risk-averse investors, as many of them provide at least partial protection for the invested amount. These investment instruments are relatively new, which often leads to a lack of relevant information about them.

The main benefit of the paper is the combination of theoretical knowledge about the functioning of investment certificates as a group of structured products, along with an understanding of the use of vanilla and barrier options, aimed at contributing to a better understanding of their creation. Another benefit is the presentation of two proposed express certificates (step-down express and memory express certificates) and the possibility of increasing their attractiveness to the investor. The creation of these products is demonstrated using the right combination of vanilla and barrier options. An empirical approach is applied to Bayer AG shares, with an issue date of November 6, 2019, and a maturity date of November 6, 2024. Due to the lack of European vanilla put and barrier put options, there are calculated their prices according to Haug's option pricing models in the statistical program R.

The main contribution of this paper is to prove the nature of these certificates' creation using vanilla put and/or barrier put options, with the aim of enhancing their understanding for all investors. From the methodological aspect, our research can serve as inspiration for the creation and valuation of further types of investment certificates.

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Dizajn modificiranih ekspres investicijskih certifikata na jednoj referentnoj vrijednosnici

Sažetak

Cilj rada je analizirati nove, inovativne strukturirane financijske instrumente, odnosno modifikacije ekspres investicijskih certifikata s postupnim snižavanjem praga (Step-down) i s akumulacijom prava na isplatu (Memory). U radu se prikazuju tehnike njihova strukturiranja primjenom kombinacije bezkuponskih obveznica te standardnih europskih put opcija i/ili barijernih put opcija, koje imaju važnu ulogu u financijskom inženjeringu. Proces strukturiranja ekspres certifikata razmatra se iz perspektive izdavatelja i potencijalnih ulagatelja. Metodološki pristup temelji se na modeliranju europskih standardnih i barijernih put opcija. Budući da takve opcije na promatrane dionice nisu predmet organiziranog tržišnog trgovanja, njihove su teorijske premije procijenjene primjenom statističkog programa R. Vrednovanje analiziranih ekspres certifikata provodi se s naglaskom na strukturu troškova i potencijalnu profitabilnost za izdavatelja i ulagatelja. Teorijska analiza primijenjena je na dionice europske kompanije Bayer AG. Također je provedena analiza ishoda po dospijeću s aspekta minimalne i očekivane dobiti za obje strane. U radu se ističu specifične značajke predloženih modifikacija te se provodi njihova usporedna analiza. Predloženi certifikati mogu predstavljati sastavni dio individualnog investicijskog portfelja. Ograničenje istraživanja odnosi se na nedostupnost stvarno tržišno kotiranih europskih standardnih i barijernih put opcija za europske kompanije, zbog čega su njihove vrijednosti procijenjene teorijskim modelima.

Ključne riječi: strukturirani proizvodi, ekspres investicijski certifikat, standardna put opcija, barijerna put opcija, modeliranje cijena opcija.