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Review / Pregledni znanstveni članak

Transformation between Geodetic Datums Utilising Thin Plate Spline (TPS) Method

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ABSTRACT. The thin plate spline (TPS) method offers significant potential to reduce local geometric distortions often found in geodetic networks established through traditional methods. Despite the advantages of the TPS method, its application in coordinate transformations between local and global datums, and vice versa, remains limited. This paper investigates the flexible TPS transformation method, which effectively incorporates information about systematic distortions arising from measurement and calculation errors in triangulation and traverse networks. The findings of this paper indicate that employing this method can substantially enhance the positional accuracy of transformed points, thereby improving the quality of boundary points in digital cadastral maps. Furthermore, it was demonstrated that an adequate number of check points is crucial for accurately assessing the effectiveness of the transformation method in the specific areas where these points are located.

Keywords: coordinates, datum, thin plate spline, transformation.

1. Introduction

Many authors worldwide are actively exploring optimal solutions for transforming coordinates from inherited, primarily local geodetic datums into modern geocentric datums. This need arises particularly from the use of GNSS (Global Navigation Satellite System) positioning methods. GNSS observations provide coordinates in a global coordinate system (X, Y, Z) based on a geocentric ellipsoid, while everyday applications typically rely on 2D Gauß-Krüger coordinates (y, x) and orthometric heights (H), which are referred to local ellip-

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soids. Since local coordinates cannot be obtained directly from GNSS measurements, it is essential to determine transformation parameters using common points (identical points or ground control points – GCPs). These parameters are then applied to convert GNSS-derived coordinates into the local coordinate system. Usual methods for this transformation include the Helmert (similarity) transformation (Lin et al. 2019) and GRID transformation methods (Fazilova 2022). Additionally, other, less conventional methods have been explored, such as least-squares collocation (You and Hwang 2006), artificial neural networks (Abbas et al. 2022), and the Msplit method (Janicka and Rapinski 2013).

The thin plate spline (TPS) is a type of radial basis function that minimizes the bending energy of a thin plate through a variational formulation. As an interpolation method, TPS is often used in image warping tasks (Chen and Geman 2014). Therefore, the TPS approach has been successfully used as a non-rigid transformation method in various applications, including the photogrammetric matching of UAV (unmanned aerial vehicle) images (Atik et al. 2020), the registration of satellite images (Chan et al. 2010), and other research areas. This transformation method has also demonstrated strong performance in enhancing the quality of graphical cadastral datasets (Siriba et al. 2012, Tuno et al. 2015, Tuno et al. 2017a, Tuno et al. 2017b, Tuno et al. 2017c, Tuno et al. 2023), while the spherical TPS formulation proposed by Keller and Borkowski (2019) proved to be highly effective for GRACE (Gravity Recovery and Climate Experiment) data interpolation as well as for modelling the vertical total electron content. According to Ježek (2009), TPS yielded the best interpolation results among the tested methods within a grid-based transformation between national and global reference systems in the Czech Republic. Despite the significant potential of the TPS method for addressing local geometric distortions, it is surprising that in-depth discussions of this transformation in the context of datum transformations have been limited to just two papers: Magna Júnior et al. (2012) and Magna Júnior et al. (2014). Basic statistical indicators regarding the application of the TPS method to the Brazilian Geodetic System network are presented in these papers, along with a comparison to the results of GRID transformation. The standard deviation of the 2D positions of points after the TPS transformation in Brazil, based on check points, was found to be 11 cm. However, these papers did not compare the TPS transformation results with those of the Helmert (similarity) transformation, nor did they examine the impact of common point density and arrangement on transformation quality.

This paper investigates these issues by focusing on the transformation of the geodetic network in the city of Sarajevo. Additionally, it analyses the effectiveness of the TPS transformation for traverse networks established during different periods and using various measurement methods, including:

- optical theodolites for angle measurements combined with optical distance measurements using subtense bars and double image distance meters,
- optical theodolites for angle measurements paired with electronic distance measurements using electro-optical distance meters, and
- electronic measurement of angles and distances using total stations.

2. Materials and Methods

2.1. TPS Transformation

The thin plate spline, a radial basis function initially introduced in the 1970s, gained prominence in digital image processing through early applications in image registration during the late 1980s. Its theoretical framework focuses on minimizing the bending energy of a thin plate (Tuno et al. 2017b).

The equations used in this paper are derived from previously published works by Harder and Desmarais (1972), Duchon (1976), Goshtasby (1988), Bookstein (1989), Fitzpatrick (2007), Siriba et al. (2012) and Tuno et al. (2024).

For two sets of corresponding points, each containing n points, where (x,y) represent the source coordinates and (x',y') the target coordinates, a 2D thin plate spline transformation is defined as follows:

$$x' = a_0 + a_1 x + a_2 y + \sum_{i=1}^{n} F_i U(r_i)$$

$$y' = b_0 + b_1 x + b_2 y + \sum_{i=1}^{n} G_i U(r_i),$$

where:

a_0, a_1, a_2 and b_0, b_1, b_2 – coefficients of the global affine transformation,

$F_1, F_2, \ldots, F_n$ and $G_1, G_2, \ldots, G_n$ – TPS coefficients at point i,

$U(r_i) = r_i^2 \ln r_i^2$ – radial basis functions,

$r_i = \sqrt{(x-x_i)^2 + (y-y_i)^2 + d^2}$ – distances from the observed point (x,y) to all other points, where d is parameter of stiffness.

If the coefficients $a_0, a_1, a_2, F_1, F_2, \ldots, F_n$ and $b_0, b_1, b_2, G_1, G_2, \ldots, G_n$ are known, as well as the coordinates $x_1, y_1; x_2, y_2; \ldots; x_n, y_n$, then the x' and y' can be calculated for any x, y, i.e. $x_i, y_i \rightarrow x_i', y_i'$.

Given that there are $n+3$ unknown parameters arising from n equations, three additional orthogonal conditions are introduced to effectively determine the coefficients $a_0, a_1, a_2, F_1, F_2, \ldots, F_n$:

$$x_1' = a_0 + a_1 x_1 + a_2 y_1 + \sum_{i=1}^{n} F_i r_{1,i}^2 \ln r_{1,i}^2,$$

$$x_2' = a_0 + a_1 x_2 + a_2 y_2 + \sum_{i=1}^{n} F_i r_{2,i}^2 \ln r_{2,i}^2,$$

$$x_n' = a_0 + a_1 x_n + a_2 y_n + \sum_{i=1}^{n} F_i r_{n,i}^2 \ln r_{n,i}^2,$$

$$\sum_{i=1}^{n} F_i = 0,$$
$$\sum_{i=1}^{n} x_i F_i = 0,$$
$$\sum_{i=1}^{n} y_i F_i = 0.$$

The final three equations serve as "boundary conditions", ensuring that the transformation approaches a purely affine form as $x, y \to \infty$.

This discussion focuses on the case where $n = 5$. If the column vector $\mathbf{x}'$ and the column vector of coefficients $\mathbf{c}^{(\mathbf{x})}$ are:

$$\mathbf{x}' = \begin{bmatrix} x'_1 \\ x'_2 \\ x'_3 \\ x'_4 \\ x'_5 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{c}^{(\mathbf{x})} = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \end{bmatrix},$$

as well as the matrix $\mathbf{A}$:

$$\mathbf{A} = \begin{bmatrix} 1 & x_1 & y_1 & r_{1,1}^2 \ln r_{1,1}^2 & r_{1,2}^2 \ln r_{1,2}^2 & r_{1,3}^2 \ln r_{1,3}^2 & r_{1,4}^2 \ln r_{1,4}^2 & r_{1,5}^2 \ln r_{1,5}^2 \\ 1 & x_2 & y_2 & r_{2,1}^2 \ln r_{2,1}^2 & r_{2,2}^2 \ln r_{2,2}^2 & r_{2,3}^2 \ln r_{2,3}^2 & r_{2,4}^2 \ln r_{2,4}^2 & r_{2,5}^2 \ln r_{2,5}^2 \\ 1 & x_3 & y_3 & r_{3,1}^2 \ln r_{3,1}^2 & r_{3,2}^2 \ln r_{3,2}^2 & r_{3,3}^2 \ln r_{3,3}^2 & r_{3,4}^2 \ln r_{3,4}^2 & r_{3,5}^2 \ln r_{3,5}^2 \\ 1 & x_4 & y_4 & r_{4,1}^2 \ln r_{4,1}^2 & r_{4,2}^2 \ln r_{4,2}^2 & r_{4,3}^2 \ln r_{4,3}^2 & r_{4,4}^2 \ln r_{4,4}^2 & r_{4,5}^2 \ln r_{4,5}^2 \\ 1 & x_5 & y_5 & r_{5,1}^2 \ln r_{5,1}^2 & r_{5,2}^2 \ln r_{5,2}^2 & r_{5,3}^2 \ln r_{5,3}^2 & r_{5,4}^2 \ln r_{5,4}^2 & r_{5,5}^2 \ln r_{5,5}^2 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & x_1 & x_2 & x_3 & x_4 & x_5 \\ 0 & 0 & 0 & y_1 & y_2 & y_3 & y_4 & y_5 \end{bmatrix},$$

then the equations can be expressed in the following form:

$$\begin{bmatrix} x'_1 \\ x'_2 \\ x'_3 \\ x'_4 \\ x'_5 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 & r_{1,1}^2 \ln r_{1,1}^2 & r_{1,2}^2 \ln r_{1,2}^2 & r_{1,3}^2 \ln r_{1,3}^2 & r_{1,4}^2 \ln r_{1,4}^2 & r_{1,5}^2 \ln r_{1,5}^2 \\ 1 & x_2 & y_2 & r_{2,1}^2 \ln r_{2,1}^2 & r_{2,2}^2 \ln r_{2,2}^2 & r_{2,3}^2 \ln r_{2,3}^2 & r_{2,4}^2 \ln r_{2,4}^2 & r_{2,5}^2 \ln r_{2,5}^2 \\ 1 & x_3 & y_3 & r_{3,1}^2 \ln r_{3,1}^2 & r_{3,2}^2 \ln r_{3,2}^2 & r_{3,3}^2 \ln r_{3,3}^2 & r_{3,4}^2 \ln r_{3,4}^2 & r_{3,5}^2 \ln r_{3,5}^2 \\ 1 & x_4 & y_4 & r_{4,1}^2 \ln r_{4,1}^2 & r_{4,2}^2 \ln r_{4,2}^2 & r_{4,3}^2 \ln r_{4,3}^2 & r_{4,4}^2 \ln r_{4,4}^2 & r_{4,5}^2 \ln r_{4,5}^2 \\ 1 & x_5 & y_5 & r_{5,1}^2 \ln r_{5,1}^2 & r_{5,2}^2 \ln r_{5,2}^2 & r_{5,3}^2 \ln r_{5,3}^2 & r_{5,4}^2 \ln r_{5,4}^2 & r_{5,5}^2 \ln r_{5,5}^2 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & x_1 & x_2 & x_3 & x_4 & x_5 \\ 0 & 0 & 0 & y_1 & y_2 & y_3 & y_4 & y_5 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \end{bmatrix},$$

or simply $\mathbf{x}' = \mathbf{A}\mathbf{c}^{(\mathbf{x})}$.

The corresponding equations for $\mathbf{y}'$ can be expressed as follows:

$$\begin{bmatrix} y'_1 \\ y'_2 \\ y'_3 \\ y'_4 \\ y'_5 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 & r_{1,1}^2 \ln r_{1,1}^2 & r_{1,2}^2 \ln r_{1,2}^2 & r_{1,3}^2 \ln r_{1,3}^2 & r_{1,4}^2 \ln r_{1,4}^2 & r_{1,5}^2 \ln r_{1,5}^2 \\ 1 & x_2 & y_2 & r_{2,1}^2 \ln r_{2,1}^2 & r_{2,2}^2 \ln r_{2,2}^2 & r_{2,3}^2 \ln r_{2,3}^2 & r_{2,4}^2 \ln r_{2,4}^2 & r_{2,5}^2 \ln r_{2,5}^2 \\ 1 & x_3 & y_3 & r_{3,1}^2 \ln r_{3,1}^2 & r_{3,2}^2 \ln r_{3,2}^2 & r_{3,3}^2 \ln r_{3,3}^2 & r_{3,4}^2 \ln r_{3,4}^2 & r_{3,5}^2 \ln r_{3,5}^2 \\ 1 & x_4 & y_4 & r_{4,1}^2 \ln r_{4,1}^2 & r_{4,2}^2 \ln r_{4,2}^2 & r_{4,3}^2 \ln r_{4,3}^2 & r_{4,4}^2 \ln r_{4,4}^2 & r_{4,5}^2 \ln r_{4,5}^2 \\ 1 & x_5 & y_5 & r_{5,1}^2 \ln r_{5,1}^2 & r_{5,2}^2 \ln r_{5,2}^2 & r_{5,3}^2 \ln r_{5,3}^2 & r_{5,4}^2 \ln r_{5,4}^2 & r_{5,5}^2 \ln r_{5,5}^2 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & x_1 & x_2 & x_3 & x_4 & x_5 \\ 0 & 0 & 0 & y_1 & y_2 & y_3 & y_4 & y_5 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ G_1 \\ G_2 \\ G_3 \\ G_4 \\ G_5 \end{bmatrix},$$

or simply $\mathbf{y}' = \mathbf{A}\mathbf{c}^{(\mathbf{y})}$.

At this stage, there are two distinct problems: the x problem and the y problem. Each one is addressed separately. Since the number of equations matches the number of unknowns, there is no need to resort to a least-squares approach. The solutions are:

$$\mathbf{c}^{(\mathbf{x})} = \mathbf{A}^{-1}\mathbf{x}'$$

$$\mathbf{c}^{(\mathbf{y})} = \mathbf{A}^{-1}\mathbf{y}'$$

2.2. Determination of Geodetic Point Coordinates in the ETRS89

To define the transformation model for the Sarajevo region, spatial ETRS89 coordinates for 18 existing geodetic points were determined during the SARA-REF 06 GPS campaign in 2006 and 2007. This allowed for the establishment of a relationship between the global system (European Terrestrial Reference System 1989 – ETRS89) and the local system (Bosnia and Herzegovina State Coordinate System – BHSCS), which is based on the Hermannskögel 1871 datum (Mugnier 2023). The BHSCS utilises the Bessel 1841 ellipsoid and employs the Gauß-Krüger map projection of 3 degrees meridian zones with a linear scale factor of 0.9999 along the central meridian (Tuno et al. 2017a). Additionally, ETRS89 coordinates for three existing geodetic points in the area were already known prior to this campaign (Bilajbegović 2007). This resulted in a total of 21 points being used to calculate the transformation model. For these points, coordinates in the Gauß-Krüger projection and orthometric heights are known, alongside the ETRS89 geocentric Cartesian coordinates. Of the 21 points, 18 were treated as fixed in horizontal position (Figure 1, left), while 16 were fixed in height. This approach achieved an average density of common points of 1 point per 14 km^2 across the entire transformation area. The parameters of a 3D similarity transformation were calculated in 2007 and continue to be widely used, although users of the BiHPOS (Bosnia and Herzegovina Positioning System) network of permanent stations have had access to a new GRID transformation service since 2019, which allows for the transformation of coordinates from the ETRS89 to the BHSCS (Zimić and Đonlagić 2017, Tabučić 2019). In order to explore the possibilities of transforming coordinates between BHSCS and ETRS89 using the TPS method, this paper analysed points from the city geodetic network that serve as the basis for the cadastral survey. A sample of traverse points located in the cadastral municipality (CM) of Sarajevo II was measured for this purpose.

CM Sarajevo II (Figure 1, right), situated in the Municipality of Stari Grad, spans an area of 146 hectares. Its central region is a densely built-up area comprising several city quarters that represent some of the oldest parts of Sarajevo, dating back to the early 16th century. This area is also home to numerous national monuments of Bosnia and Herzegovina. The geodetic survey of the cadastral municipality of Sarajevo II was conducted by the Geodetic Institute of Sarajevo in 1968, utilising a combined method that included orthogonal detailed surveying, detailed levelling, and aerial photogrammetry. At the time, this cadastral municipality encompassed 3,453 land parcels and was represented on 12 map sheets at a scale of 1:1000. The base for the state survey consisted of approximately 450 geodetic points set by traverses of the 1st, 2nd and 3rd order. From 1968 to 1992, an additional 330 traverse points were established in this cadastral municipality, primarily to support a survey conducted in 1986. After 1995, the geodetic network was further expanded with 215 new traverse points, most of which were established by the Geoprof Sarajevo company during a survey in 2004 (Tuno et al. 2021).

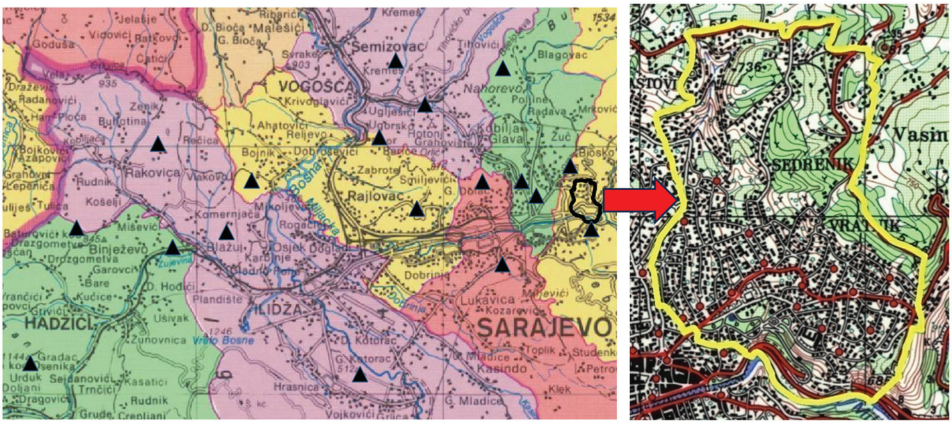

Figure 1. *Common points, indicated by black triangles, determined during the SARAREF 06 campaign (left) and CM Sarajevo II (right).*

During the SARAREF 06 GPS campaign, ETRS89 coordinates were determined for three existing traverse points in the cadastral municipality of Sarajevo II. However, this number was insufficient for a comprehensive analysis, leading to extensive efforts to establish ETRS89 coordinates for all available traverse points in the area. Upon inspecting the location, it was discovered that 88% of the points had been destroyed, leaving 80 points suitable for GNSS observations. These 80 points were then measured using a fast static GNSS method with dual-frequency instruments (Tuno et al. 2021). After adjusting the coordinates, an accuracy assessment showed that the average standard deviation of the horizontal positions was 1 cm, which is quite acceptable for the purposes of this research.

3. Results and Discussion

In order to explore the potential for the TPS transformation of coordinates between the BHSCS and ETRS89, this paper analysed the traverse points that form the basis of the cadastral survey and its maintenance. A sample of check points from the cadastral municipality of Sarajevo II was used for this purpose, with analyses conducted using Gauß-Krüger coordinates. The effectiveness of the transformation procedures was assessed by examining the displacements of points that have known coordinates in both reference systems.

According to the National Standard for Spatial Data Accuracy (NSSDA) (Tuno et al. 2022), the horizontal positional accuracy of the TPS transformation was evaluated using coordinates (y_i, x_i) obtained from traversing, alongside the transformed coordinates $(y_i^{tr.}, x_i^{tr.})$ determined by GNSS method. The quality of the transformation was examined based on the coordinate differences along the y and x axes, defined as $d_{y_i} = y_i - y_i^{tr.}$ and $d_{x_i} = x_i - x_i^{tr.}$. The resulting positional distortions were calculated as $d_{yx_i} = \sqrt{{d_{y_i}}^2 + {d_{x_i}}^2}$. In the analyses, the root mean square error (*RMSE*) served as the main statistical indicator of accuracy assessment:

$$RMSE = \sqrt{\frac{1}{n}\sum_{1}^{n} {d_{y_i}}^2 + \frac{1}{n}\sum_{1}^{n} {d_{x_i}}^2} = \sqrt{{RMSE_y}^2 + {RMSE_x}^2}.$$

3.1. TPS Transformation Based on the Common Points from the SARAREF 06 Campaign

The TPS transformation method, based on 18 common geodetic points from the SARAREF 06 campaign, was tested. The same points were also used to define the official 2D Helmert (similarity) transformation. The quality of the transformation was assessed by comparing the transformed coordinates of all 80 check points from CM Sarajevo II to their BHSCS (true) values, as illustrated by the statistical indicators in Table 1 (“All points” column). Notable horizontal positional deviations were observed, with some reaching up to 25 cm. Additionally, the relatively small standard deviations suggest poor absolute orientation of the transformation parameters. This issue is particularly evident in the north-western part of the network (Figure 2), where a pronounced local systematic error remains uncorrected by the transformation.

Table 1. *Statistics of the remaining residuals after application of the TPS transformation using the SARAREF 06 GCPs.*

Statistical indicator		All points			Points from 1968			Points from 1968–1992			Points after 1995		
		d_y	d_x	d_{yx}	d_y	d_x	d_{yx}	d_y	d_x	d_{yx}	d_y	d_x	d_{yx}
Minimum [m]		-0.093	-0.102	0.008	-0.035	-0.037	0.010	-0.093	-0.102	0.010	-0.078	-0.053	0.008
Average [m]		0.032	0.023	0.064	0.029	0.008	0.046	0.024	0.021	0.063	0.043	0.037	0.076
Maximum [m]		0.157	0.200	0.254	0.143	0.062	0.143	0.107	0.109	0.125	0.157	0.200	0.254
Range [m]		0.250	0.302	0.246	0.178	0.099	0.133	0.199	0.211	0.115	0.236	0.253	0.246
***RMSE* [m]**		0.057	0.055	0.079	0.051	0.026	0.057	0.051	0.049	0.071	0.066	0.072	0.098
Distribution of distortion residuals [%]	**|0–5 cm|**	66.3	73.8	46.3	77.8	94.4	72.2	67.6	70.6	41.2	57.1	64.3	35.7
	|5–10 cm|	23.8	18.8	38.8	16.7	5.6	16.7	26.5	23.5	44.1	25.0	21.4	46.4
	|10–15 cm|	8.8	5.0	10.0	5.6	0.0	11.1	5.9	5.9	14.7	14.3	7.1	3.6
	|15–20 cm|	1.3	2.5	2.5	0.0	0.0	0.0	0.0	0.0	0.0	3.6	7.1	7.1
	|20–25 cm|	0.0	0.0	1.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.6
	>|25 cm|	0.0	0.0	1.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.6

The analysis of the means values for d_y and d_x further indicates that systematic errors persist in the transformed coordinates. This finding is supported by the histogram of remaining distortions along the *y*-axis and the corresponding normal distribution curve (Figure 3), both generated using the 'fitmethis' MATLAB function (De Castro 2025). The shift in the distribution curve, caused by the remaining systematic errors, is clearly evident.

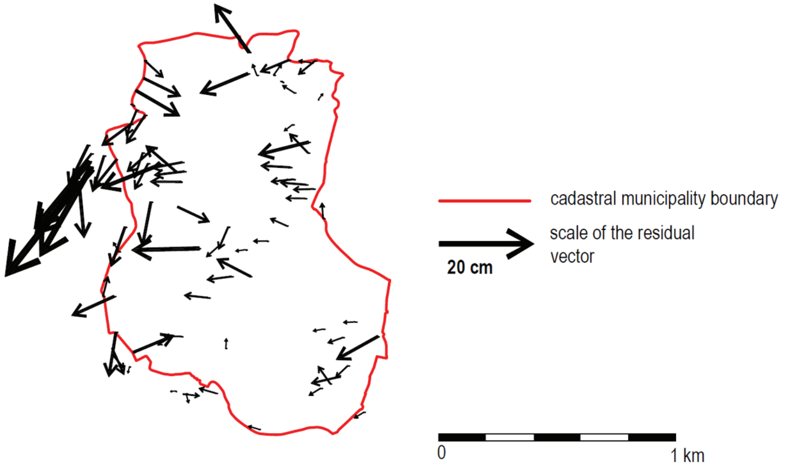

Figure 2. *Remaining distortions of check points after performing the TPS transformation based on the SARAREF 06 GCPs.*

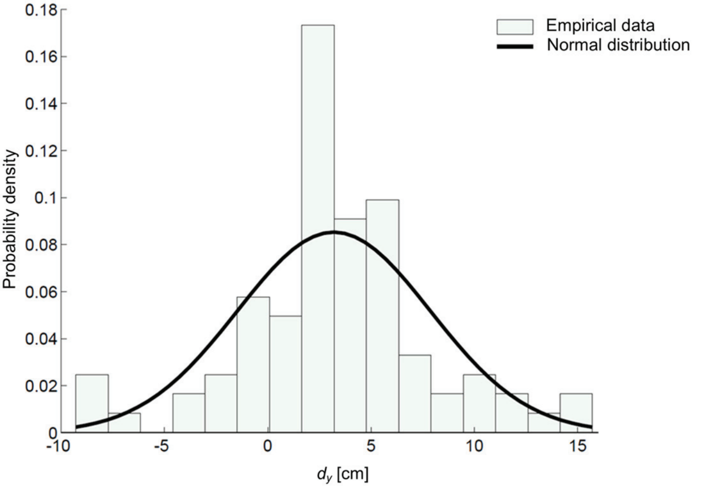

Figure 3. *Histogram and normal distribution curve of differences between theoretical y coordinates and those obtained after applying the TPS transformation based on the SARAREF 06 GCPs.*

The differences in the coordinates of the check points were analysed based on the periods in which their BHSCS coordinates were determined: 18 points from 1968 (optical theodolites for angle measurements combined with optical distance measurements using subtense bars and double image distance meters), 34 points from the period 1968–1992 (optical theodolites for angle measurements paired with electronic distance measurements using electro-optical distance meters), and 29 points determined after 1995 (electronic measurement of angles and distances using total stations). This analysis is presented in the corresponding columns of Table 1. It was found that the smallest deviations occur among the points from the 1968 traverse network, indicating the high quality of the original survey of Sarajevo and its alignment with the common points from the SARAREF 06 campaign. The 1968 traverse network was based on the closest points from the city's triangulation network and was calculated using group adjustment. In contrast, the situation is significantly worse for points determined after 1968. The largest deviations are found among points established after 1995, highlighting their heterogeneity in relation to the triangulation network on which the transformation method from the SARAREF 06 campaign is based. By comparing the transformation indicators, it is evident that the *RMSE* of the points determined after 1995 is twice that of the points from 1968, while the maximum deviation is nearly double. Almost one-fifth of the points established after 1995 exhibit positional deviations greater than 10 cm, with 7% showing significantly large deviations (over 20 cm). Since these points were intensely used in maintaining cadastral records, large distortions will persist in the coordinates of boundary points for land parcels and buildings after the transformation.

The results of the TPS transformation were compared with those of other 2D transformation methods, with the key statistical indicators presented in Table 2. As previously noted, the inhomogeneity of the transformed traverse points persists after transformation using the triangulation network, revealing areas with both greater and lesser deviations. This observation also applies to the official GRID transformation method, which utilises a distortion grid with a resolution of 1 km. Interestingly, the results of the TPS and Helmert (similarity) transformations are nearly identical, while the results of the GRID method are approximately 10% worse.

Table 2. *Overview of distortion statistics for different transformation methods.*

Statistical indicator		Helmert (SARAREF 06 based)			GRID			TPS (SARAREF 06 based)		
		d_y	d_x	d_{yx}	d_y	d_x	d_{yx}	d_y	d_x	d_{yx}
Minimum [m]		-0.093	-0.087	0.007	-0.075	-0.075	0.007	-0.093	-0.102	0.008
Average [m]		0.035	0.024	0.067	0.035	0.042	0.070	0.032	0.023	0.064
Maximum [m]		0.158	0.203	0.258	0.177	0.215	0.279	0.157	0.200	0.254
Range [m]		0.251	0.290	0.251	0.252	0.290	0.272	0.250	0.302	0.246
***RMSE* [m]**		0.058	0.057	0.081	0.059	0.064	0.088	0.057	0.055	0.079
Distribution of distortion residuals [%]	**|0–5 cm|**	62.5	70.0	41.3	65.4	61.7	48.1	66.3	73.8	46.3
	|5–10 cm|	27.5	23.8	42.5	23.5	28.4	28.4	23.8	18.8	38.8
	|10–15 cm|	8.8	3.8	11.3	9.9	4.9	17.3	8.8	5.0	10.0
	|15–20 cm|	1.3	1.3	2.5	1.2	2.5	1.2	1.3	2.5	2.5
	|20–25 cm|	0.0	1.3	1.3	0.0	2.5	3.7	0.0	0.0	1.3
	>|25 cm|	0.0	0.0	1.3	0.0	0.0	1.2	0.0	0.0	1.3

3.2. TPS Transformation Based on the Geodetic Points from CM Sarajevo II

The previous analysis indicates that a density of 1 common point per 14 km² is insufficient for the successful transformation of cadastral data between different reference systems. Research conducted by Tuno et al. (2017b) and Tuno et al. (2023) revealed that the accuracy of geometric transformations is significantly influenced by the number and arrangement (density) of GCPs. To examine how the density of these points affects the horizontal accuracy of the TPS transformation, four different densities were selected for this research: 0.1 GCP per km², 1 GCP per km², 2 GCPs per km², and 10 GCPs per km². For this analysis, a subset of traverse points within CM Sarajevo II was selected from a total of 80 observed points to serve as GCPs. The traverse points not used as common points were designated as check points to evaluate the quality of the transformation according to the NSSDA. This approach enabled a comparison of transformation results using different densities of common points. The traverse points selected as GCPs were incorporated into the previously generated TPS model, which consisted of triangulation points.

Table 3 illustrates the impact of the TPS transformation on the coordinates of check points in the CM Sarajevo II area. It is evident that increasing the density of common points enhances the quality of the transformation. Table 3 shows that the maximum remaining horizontal positional deviation in the model based on 10 GCPs per km² is 13 cm, which is half of the deviation observed in the transformation based on the SARAREF 06 points, while the *RMSE* value improves by 20%. The mean values for the coordinate components along the *y* and *x* axes, which are nearly zero, indicate good absolute orientation of the transformation method. Although systematic errors are not completely eliminated, the range of deviations has been reduced by more than half. A histogram was created to show the remaining coordinate distortions after applying the TPS transformation based on the traverse points, along with the corresponding normal distribution curve that aligns with the empirical data (Fig. 4). Figure 4 clearly shows a minimal (negligible) shift in the mean value of d_y, which cannot be attributed to significant systematic errors.

The trends in data improvement resulting from the transformation methods, defined by traverse points within the cadastral municipality, can be further observed by comparing the graphical representations in Figure 5 with those in Figure 2, and the numerical results presented in Table 3. The magnitudes of the vectors in Figure 5 are significantly smaller than those resulting from the transformation using the SARAREF 06 points, and their directions exhibit a random distribution.

Table 3. *Statistics of the remaining residuals after application of the TPS transformation with different densities of GCPs.*

Statistical indicator		TPS 0.1 GCP/km²			TPS 1 GCP/km²			TPS 2 GCPs/km²			TPS 10 GCPs/km²		
		d_y	d_x	d_{yx}	d_y	d_x	d_{yx}	d_y	d_x	d_{yx}	d_y	d_x	d_{yx}
Minimum [m]		-0.093	-0.102	0.008	-0.103	-0.105	0.000	-0.144	-0.101	0.000	-0.112	-0.121	0.005
Average [m]		0.032	0.023	0.064	0.018	0.021	0.056	-0.004	0.015	0.055	0.007	0.001	0.054
Maximum [m]		0.157	0.200	0.254	0.148	0.198	0.248	0.125	0.187	0.225	0.122	0.125	0.132
Range [m]		0.250	0.302	0.246	0.251	0.303	0.248	0.269	0.288	0.225	0.233	0.246	0.127
***RMSE* [m]**		0.057	0.055	0.079	0.049	0.054	0.073	0.048	0.051	0.070	0.042	0.048	0.062
Distribution of distortion residuals [%]	**|0–5 cm|**	66.3	73.8	46.3	76.3	75.0	56.3	77.5	78.8	53.8	78.5	76.9	55.4
	|5–10 cm|	23.8	18.8	38.8	16.3	17.5	31.3	16.3	15.0	33.8	18.5	15.4	32.3
	|10–15 cm|	8.8	5.0	10.0	7.5	5.0	7.5	6.3	3.8	6.3	3.1	7.7	12.3
	|15–20 cm|	1.3	2.5	2.5	0.0	2.5	2.5	0.0	2.5	5.0	0.0	0.0	0.0
	|20–25 cm|	0.0	0.0	1.3	0.0	0.0	2.5	0.0	0.0	1.3	0.0	0.0	0.0
	>|25 cm|	0.0	0.0	1.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Clearly, a transformation approach that uses a higher density of common points provides more homogeneous geometric data for the cadastral municipality. This underscores the importance of selecting a larger number of common points. The quality indicators achieved through this transformation are representative and can be expected in most other urban areas as well.

Using a transformation model that incorporates traverse points from the specific cadastral municipality significantly improves results compared to the official GRID method and the official Helmert (similarity) transformation, which rely on triangulation points. This approach allows for satisfactory transformation accuracy even in smaller areas.

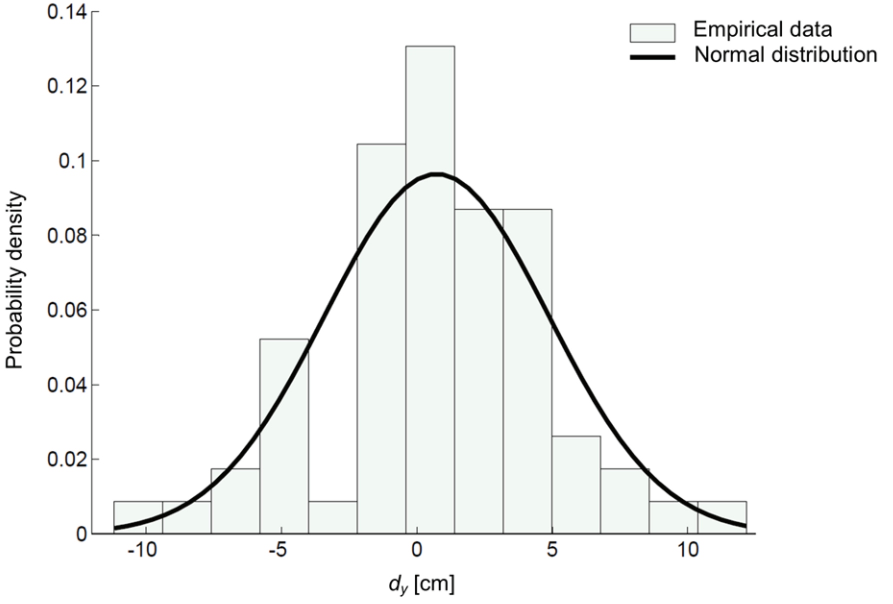

Figure 4. *Histogram and normal distribution curve of differences between theoretical y coordinates and those obtained after applying the TPS transformation based on the CM Sarajevo II traverse points.*

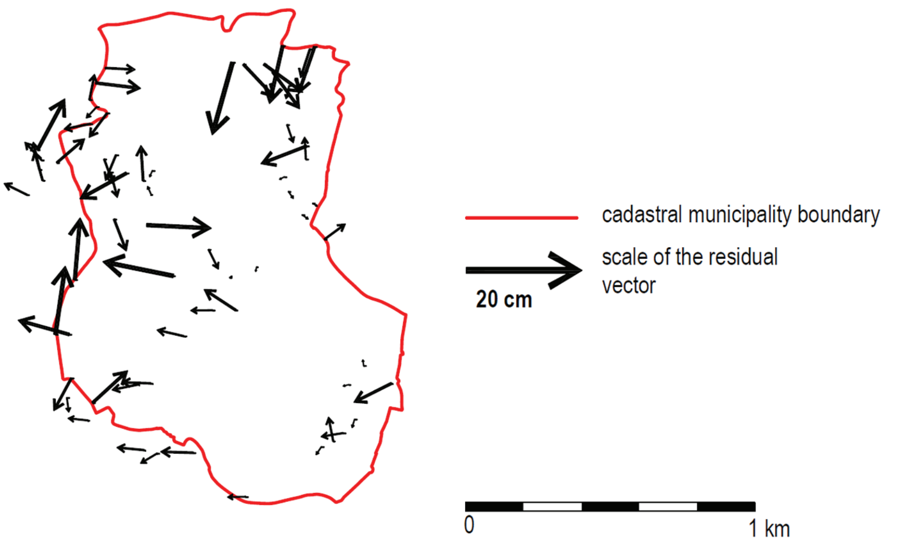

Figure 5. *Remaining distortions of check points after performing the TPS transformation based on the CM Sarajevo II traverse points.*

4. Conclusion

The assessment of the geometric quality of traverse points obtained by applying the TPS procedure based on triangulation points revealed significant positional errors in the transformed coordinates. It is evident that a transformation with a density of 0.1 GCP/km² has limited ability to model and eliminate the non-uniform and unevenly distributed distortions typically present in traverse point coordinates. These distortions often stem from errors during the measurement and computation of the traverse network, and as a result, they are propagated to the coordinates of the transformed points. Consequently, it is not possible to eliminate these errors through this transformation procedure. The approach that provides more homogeneous data for the transformed points relies on incorporating traverse points from the tested cadastral municipality into the local TPS transformation model.

This investigation demonstrated that with an adequate number of GCPs, satisfactory results can be achieved using the TPS method. The accuracy assessment revealed that the *RMSE* decreased as the number of GCPs increased. Notably, the TPS method with a density of 10 GCPs per km² provided the best results, resulting in an *RMSE* of 6 cm, with all positional deviations remaining below 13 cm, which is quite satisfactory. Therefore, this paper confirms that the TPS method can be successfully employed in local to global coordinate transformation tasks, but only when adequate common points are used as the basis for the transformation. The selection of GCPs clearly plays a key role in the quality of the results when transforming coordinates from one datum to another.

This paper clearly demonstrates that when transforming geodetic point coordinates using the TPS method, it is essential to ensure an adequate number of check points to accurately assess the transformation model's effectiveness in the area where these points are located. If the discrepancies between the theoretical and transformed coordinates of the check points are significant, the transformation model can be updated by incorporating additional points.

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Transformacija između geodetskih datuma primjenom metode tankoslojnog splajna

SAŽETAK. Metoda tankoslojnog splajna (engl. Thin Plate Spline – TPS) pruža značajan potencijal za smanjenje lokalnih geometrijskih distorzija koje se često javljaju u geodetskim mrežama uspostavljenim tradicionalnim metodama. Unatoč prednostima metode TPS, njezina je primjena u transformacijama koordinata između lokalnih i globalnih datuma, i obratno, još uvijek ograničena. U ovom radu istražuje se fleksibilna metoda transformacije TPS, koja učinkovito uključuje informacije o sustavnim distorzijama proizašlim iz pogrešaka mjerenja i računanja triangulacijskih i poligonskih mreža. Rezultati rada ukazuju da se primjenom ove metode znatno povećava položajna točnost transformiranih tačaka, što pozitivno utječe na kvalitetu međnih točaka na digitalnim katastarskim planovima. Dodatno, pokazano je da je dovoljan broj kontrolnih točaka ključan za točnu procjenu učinkovitosti metode transformacije na specifičnom području gdje se te točke nalaze.

Ključne riječi: koordinate, datum, tankoslojni splajn, transformacija.

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