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Sixth-Grade Students' Cognitive Competencies and Difficulties in Mathematical Modelling

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Abstract

The purpose of this study was to investigate the cognitive modelling competencies of sixth-grade middle school students and to identify the difficulties they encountered during this process. The research was conducted with 50 sixth-grade students attending a public middle school located in eastern Turkey. Data were collected through the Modelling Competency Form developed by the researchers. The research was conducted as a case study, and it was based on a qualitative research approach. The obtained qualitative data were analyzed via descriptive analysis. The results revealed that students experienced cognitive deficiencies in the mathematical modelling process, particularly in the stages of validation and interpretation. Based on the results, a list of students' cognitive difficulties in the modelling process was compiled, and several recommendations were proposed to address these challenges.

Key words: *mathematical modelling; middle school students; modelling competencies*

Introduction

One of the fundamental goals of mathematics education is to develop students' mathematical literacy, which is considered a key competence in the 21st century (Doruk, 2010; Drew, 2012). Mathematically literate individuals are aware of the role of mathematics in today's world, can effectively use mathematics in real-life contexts, possess advanced thinking skills and demonstrate problem-solving competence (OECD, 2019). In this regard, mathematical modelling emerges as one of the most effective approaches for fostering the use of mathematical ideas in everyday life, as it constitutes an important interactive process through which students engage with problems derived from the real world and their immediate environment (Sabo Junger & Lipovec, 2022; Zbiek & Conner, 2006). In the process of mathematical modelling, mathematical models play a central role. The concept of a mathematical model has been defined in various ways by researchers. For example, Kaiser (1995) defined a mathematical model as a structure created to explain, predict or control a phenomenon. Lesh and Doerr (2003), on the other hand, conceptualized mathematical models as concrete or abstract structures that represent the relationships and structures of a system. Furthermore, mathematical models are also described as external representations that support the solution and interpretation of real-world problems, serving as a bridge between mathematics and everyday life (Blum et al., 2007; Borromeo Ferri, 2006; Lesh & Doerr, 2003)

In parallel with different definitions of mathematical modelling, various perspectives on the modelling process have also been proposed (Kaiser & Sriraman, 2006). For example, Blum and Leiß (2006) described the modelling process as a cycle that includes vertical and horizontal mathematization. Similarly, Berry and Houston (1995) developed a modelling approach that enabled the examination of the relationship between the real world and the mathematical world. Müller and Wittmann's approach (1984, acc. to Peter-Koop, 2004) conceptualized the modelling process in two domains—conceptual and reality-based—and highlighted the connections between these domains. Lesh and Doerr (2003) defined mathematical modelling as the process of translating real-life situations and problems into mathematics and developing models that can be applied in similar situations. In addition, some researchers have proposed approaches that incorporate not only the real world and the mathematical world but also the technological world into the process (Ang, 2010; Siller & Greefrad, 2010). From these perspectives, a mathematical model can be regarded as the product obtained at the end of the modelling process, whereas mathematical modelling can be considered a dynamic process that involves constructing mathematical models to solve real-world problems, reaching solutions and interpreting these solutions in real-life contexts (Doruk, 2023; Lesh & Doerr, 2003).

In recent years, mathematical modelling has increasingly gained importance and has become one of the key competencies included in school curricula and educational standards in many countries (Borromeo Ferri, 2018). Skills related to mathematical modelling are regarded as fundamental competencies that mathematics education aims to develop (National Council of Teachers of Mathematics [NCTM], 2000). The main reason for this is the multifaceted contribution of modelling activities to students' development. Through these activities, students enhance their mathematical thinking and cognitive skills (Blum & Ferri, 2009; English, 2006; English & Watters, 2005; Ünlü, 2023), as well as their abilities in mathematical connections, communication, problem solving and reasoning (Blum, 2011; Blum & Ferri, 2009; English, 2006; English & Watters, 2005; Kaiser, 2006; Sabo Junger et al., 2024; Swan & Burkhardt, 2012). Furthermore, modelling fosters versatile, higher-order and creative thinking skills (Doerr & English, 2003; English & Watters, 2005; Kaiser & Sriraman, 2006), alongside metacognitive skills (Borromeo Ferri, 2018; Can, 2024; Ülker & Çelik, 2025). In mathematics education, the modelling approach focuses not merely on producing solutions to singular problems but also on exploring general structures that involve connections, and on enabling the formulation of problems related to the situations under consideration (Doerr & English, 2003). In this respect, since mathematical modelling is an approach that emphasizes the process rather than only the outcome, it also provides teachers with opportunities to observe and evaluate their students' ways of thinking (Erbaş et al., 2014; Öztürk, 2025)

The process of mathematical modelling consists of six stages: understanding the problem, simplifying, mathematizing, working mathematically, interpreting the results and validating them (Blum, 2011; Borromeo Ferri, 2018; Kaiser, 2006). For students to succeed in modelling problems, they need to use the necessary modelling competencies effectively throughout this process (Erdem et al. 2015; Tekin Dede & Yılmaz, 2015). Modelling competencies refer to knowledge, skills and abilities required to solve modelling problems (Maaß, 2006). Moreover, it is important for students to be both conscious and motivated in order to overcome the difficulties encountered in each stage of the process (Blomhøj & Jensen, 2003; Jensen, 2007). Competencies required for success in mathematical modelling have been classified in different ways in the literature. For instance, based on a comprehensive review of the literature, Bukova Güzel (2019) identified four dimensions of modelling competencies: cognitive, affective, social and metacognitive. Borromeo Ferri and Blum (2010), on the other hand, explained cognitive competencies within the framework of the six steps of the modelling process: understanding the problem, simplifying, mathematizing, working mathematically, interpreting and validating. This study focuses particularly on students' situations and the difficulties they face in relation to these cognitive competencies.

There is a positive relationship between success in mathematical modelling and mathematical literacy (Özer-Demir & Bukova-Güzel, 2024). This finding suggests that in order for students to use mathematics effectively in their daily lives, they must develop strong modelling skills. Achieving success in modelling processes requires acquiring the necessary competencies first. Therefore, it is essential to determine whether students possess these competencies and, based on the findings, design and implement appropriate instructional methods to foster their development. Studies in the literature indicate that students encounter various difficulties with respect to modelling competencies. In particular, significant challenges have been reported in the stages of understanding and simplifying the problem (Kertil & Gürel, 2016), mathematizing (Barquero & Bosch, 2015), working mathematically (Stillman, 2011) as well as interpreting and validating solutions (Blum & Leiß, 2007).

Acquiring mathematical modelling competencies provides students with substantial benefits both in their everyday lives and within their mathematical learning trajectories. Therefore, developing these competencies at an early age may contribute positively to subsequent learning experiences. In this sense, it is crucial to develop younger students' modelling competencies, and instructional processes should be designed to support their development. Modelling activities are among the most frequently employed approaches that foster such competencies. Yet, despite encouraging results, previous studies have indicated that students often fall short of the expected level in particular competency domains, especially in validation and interpretation (Biccards & Wessels, 2011; Çoksöyler, 2020; Ozulu & Sağirli, 2021; Yurtsever, 2018). This finding suggests that these domains draw upon different sets of skills and, consequently, call for instructional approaches tailored to each specific competency.

Addressing this issue first requires a detailed examination of students' current status in these domains and the identification of the difficulties they encounter at each stage of competency. Following such analysis, it becomes necessary to design and implement instructional strategies that are responsive to the characteristics of these challenges. The present study is situated within this perspective. It aimed to explore sixth-grade middle school students' modelling competencies and the challenges they faced, and, based on the findings, propose instructional strategies for the domains in which students exhibited insufficiencies.

A review of the literature on middle school students' modelling competencies revealed that research in this area remains limited. Existing studies were inclined to focus on particular stages of competency and were often designed around instructional interventions involving modelling activities with the goal of fostering competency development (Duran, 2023; Pirimoğlu & Gürel, 2025; Tekin Dede, 2017; Tuna et al., 2013; Yıldız, 2024; Yılmaz & Kesebir, 2023). Thus, a comprehensive investigation that encompasses all stages of sixth-grade students' modelling competencies might contribute to the field significantly.

In this context, the study sought to address the following research questions:

1 What is the level of middle school students' mathematical modelling competencies?
2 What kind of difficulties do middle school students encounter during the mathematical modelling process?

Methodology

Research Design

This study was designed and conducted using the case study method, which is one of the qualitative research designs. Case studies are research designs that aim to investigate a particular phenomenon or event in depth within a bounded context (Yin, 2018). In line with this perspective, the present study sought to examine sixth-grade students' mathematical modelling competencies through the Bridge Problem.

Participants

The research group consisted of a total of 50 sixth-grade students enrolled in a public middle school (grades 5–8, equivalent to higher primary education) located in the Eastern Anatolia region of Turkey. In the Turkish context, middle school students are typically aged 10–14 years. In this study, the participants' ages ranged from 11 to 12 years, with a mean age of 11.5 years. Of these students, 24 were in one class, while the remaining 26 were in another. Both classes were taught by the same mathematics teacher. The study was conducted at the end of the spring semester of the 2023–2024 academic year. At this point in the school year, students had already been taught the majority of the learning outcomes prescribed by the sixth-grade mathematics curriculum.

Data Collection

The data were collected through the Modelling Competency Form (MCF), which was developed by the researchers. The MCF included one modelling problem and six open-ended questions designed to elicit students' modelling competencies in relation to the problem. To examine the modelling competencies, the Bridge Problem was employed, which had been adapted from Peter-Koop's (2004) traffic jam problem and presented by Bukova-Güzel et al. (n.d.). The form provided sufficient space for students to write their responses comfortably.

The Bridge Problem was selected because it is based on a simple division operation, which minimized the influence of factors outside the competencies under investigation. In addition, it had not been used in prior studies, thereby enhancing its novelty. In designing the questions, attention was paid to ensure that each item corresponded to a specific competency stage and clearly reflected the targeted competencies as defined in the analytic rubric. To evaluate the appropriateness of the form for the study, expert opinion was obtained from a mathematics professor experienced in mathematical modelling. The expert confirmed that the instrument was suitable for measuring mathematical modelling competencies.

Below is the Bridge Problem and the six related questions addressing each competency stage:

One of the largest bridges in the world is the bridge constructed over Hangzhou Bay in eastern China, which is approximately 36 kilometers long. If you imagine a line of vehicles along this bridge, how many vehicles might there be in total?

Please answer the following six questions in the spaces provided:

1. What information is given in this problem, and what is asked of you? How can you establish a relationship between the given information and the required result? Explain.
2. What information do you think is necessary to solve this problem? Which pieces of information are missing, and how might you make assumptions about them? Explain.
3. Using your assumptions, how would you express this problem mathematically? Show your equations or procedures and explain.
4. Use the mathematical model you constructed to perform the necessary calculations and find the result. Present your calculation steps clearly.
5. Evaluate your result in terms of real life. Do you think this result is realistic? Why or why not?
6. Check your solution. If you find any mistakes in your calculations, how can you correct them?

The questions were designed in consideration of students' cognitive levels, with each item aimed at assessing one of the following sub-competencies in sequence: understanding the problem, simplifying, mathematizing, working mathematically, interpreting and validating.

The study was carried out by the researcher in the classroom environment of the school where the study took place. After briefly informing the students about the purpose and procedure of the study, the researcher emphasized that participation was voluntary. The students who agreed to participate responded to the questions individually. During the implementation, the researcher explained that there was no strict time limit, but responses had to be given independently. The data collection instrument was then distributed to the participants, who completed the form and returned it to the researcher within approximately 60 minutes.

Data Analysis

The students' responses to the problem were analyzed using descriptive analysis. In determining students' levels of modelling competencies, the Modelling Competencies Assessment Rubric (MCAR) developed by Tekin Dede and Bukova-Güzel (2018) was employed. This rubric is based on six sub-competencies of the mathematical modelling process, with each sub-competency evaluated on a 12-point scale. For competencies with four levels, scores of 0, 4, 8 and 12 were assigned; for competencies with five levels, 0, 3, 6, 9 and 12; and for the competency with seven levels, 0, 2, 4, 6, 8, 10 and 12. Consequently, each student could receive a minimum score of 0 and a maximum score of 12 for each sub-competency, with an overall score ranging from 0 to 72.

The sub-competencies and their evaluation criteria were as follows:

- Understanding the problem: classified into levels 1–5, based on students' ability to demonstrate comprehension of the problem, identify the given and required information, and establish relationships between them.
- Simplification: categorized into levels 1–4, based on identifying relevant and irrelevant variables and making realistic assumptions.
- Mathematization: evaluated as levels 1–5, based on constructing mathematical models in line with realistic assumptions, explaining the models and relating them to one another.
- Working mathematically: classified into levels 1–5, based on constructing appropriate mathematical models and solving them accurately.
- Interpretation: evaluated as levels 1–5, based on obtaining correct mathematical results and interpreting them appropriately in real-world contexts.
- Validation: classified into levels 1–7, based on identifying and correcting errors as well as employing validation strategies.

For the data analysis procedure, the researchers collaboratively developed coding schemes by identifying exemplary student responses for each level. They then conducted independent coding and compared their results. The inter-rater agreement for all sub-competencies, except "understanding the problem," was at least 92 %. For "understanding the problem," the agreement was calculated as 54 %. This discrepancy was due to differences in interpretation: one researcher restricted the criterion of "establishing the relationship between the given and required information" to the specific answer provided under the relevant item, whereas the other considered the relationships demonstrated throughout the entire solution process. To resolve this, the researchers decided that, while priority would be given to the specific response, the overall solution would also be taken into account when information was missing or not explicitly stated. After repeated rounds of analysis, full consensus was reached, and the data analysis was concluded. The quantitative results obtained from the rubric were presented descriptively.

Results

In this section, students' modelling competencies are presented based on their responses to the Bridge Problem. The modelling competencies were examined under six sub-competencies: understanding the problem, simplifying, mathematizing, working mathematically, interpreting and validating. The total scores obtained by students for each competency, as assessed through the Modelling Competency Assessment Rubric (MCAR), are presented in Figure 1.

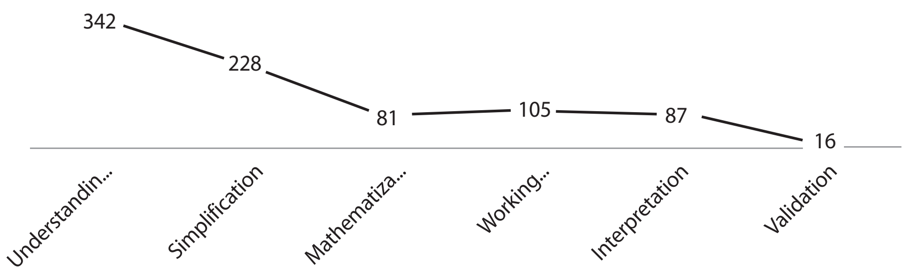


Figure 1. Students' total scores obtained from MCAR

The maximum score that could be obtained for each stage of modelling competency was 600, resulting in a maximum total score of 3600 for all competencies. The total modelling competency score of the students was 829. Based on these values, students' modelling competency in the Bridge Problem corresponded to 23 %. Accordingly, it can be stated that the students' overall modelling competency was rather low.

As shown in Figure 1, students achieved near-average performance in the competency of understanding the problem (57 %), whereas their performance in all other competencies was below average (38 %, 13 %, 17 %, 9 % and 2 %, respectively). Low level of success in mathematizing and the subsequent competencies was particularly striking. Validation emerged as the competency with the lowest achievement score. These findings indicate that students experienced the greatest difficulties in translating real-life problems into mathematical language, in connecting the results back to real-life contexts and in validating the outcomes. The following sections present a detailed analysis of each modelling competency.

Results of understanding the problem

In the modelling process, a student who demonstrates understanding the problem is expected to articulate a coherent understanding of the problem context, correctly identify the givens and the unknowns, and establish accurate relationships between them. An overview of students' levels of understanding the Bridge Problem is presented in Figure 2.

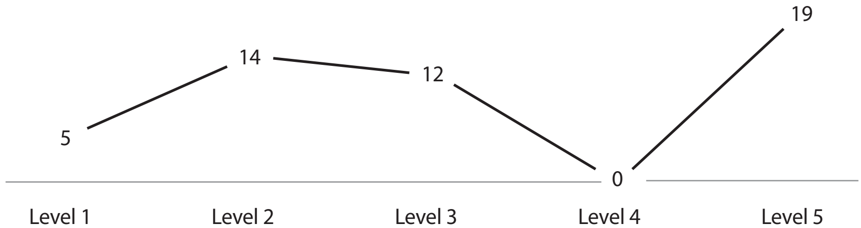
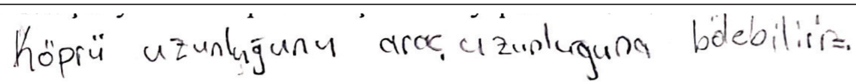


Figure 2. Levels of understanding the problem

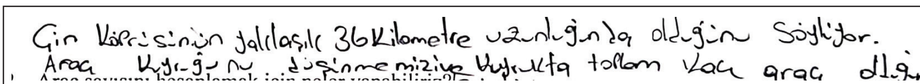
As illustrated in Figure 2, the majority of students were not at the expected level in terms of understanding the problem. This finding indicates that many students experienced difficulties in fully grasping the requirements of the task. Specifically, 19 students were able to correctly identify the givens and the unknowns of the problem and establish appropriate relationships between them, mostly based on division. An exemplary student response, which was categorized at Level 5, is presented in Figure 2a.



We can divide the length of the bridge by the length of a vehicle.

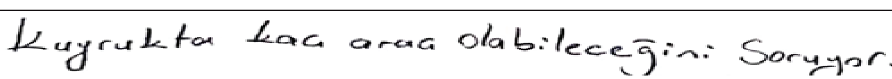
Figure 2a. Example of a level 5 student response

Twelve students identified the givens and the unknowns but failed to establish a relationship between them. Fourteen students identified only some of the givens and unknowns, while five students were unable to specify either. Figures 2b, 2c and 2d illustrate sample responses of students categorized at levels 3, 2 and 1, respectively.



It says that the Chinese Bridge is about 36 kilometers long. It asks us to think about the line of vehicles and to calculate how many vehicles are in the queue.

Figure 2b. Example of a level 3 student response



It asks how many vehicles there could be in the queue.

Figure 2c. Example of a level 2 student response

I think such a thing is not possible.

Figure 2d. Example of a level 1 student response

Results of simplifying the problem

Following the competency of understanding the problem, students' performance in simplifying the problem was examined. At this stage, students were expected to identify the necessary and unnecessary variables in the problem, reduce the problem to its essential components and make realistic assumptions. More than one simplified approach could be adopted in the solution process. The most straightforward simplification would be to consider bridge length, vehicle length and the distance between vehicles as the necessary variables. Students were expected to establish relationships among these variables based on realistic assumptions. An overview of students' performance in this competency is presented in Figure 3.

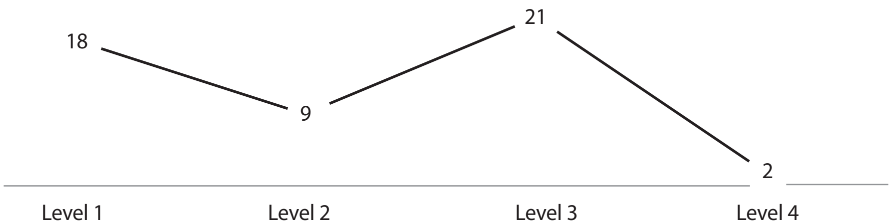
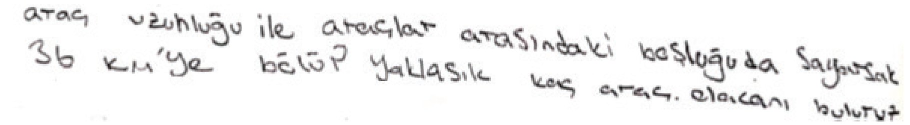


Figure 3. Levels of simplifying the problem

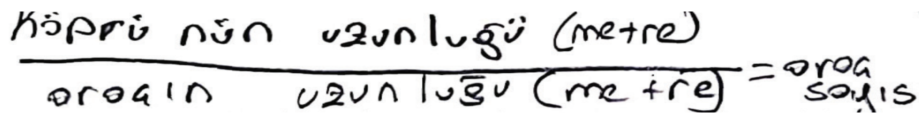
As shown in Figure 3, most students were not fully competent in simplifying the problem. Only two students successfully identified the necessary variables and established realistic assumptions about their relationships. For instance, the student response in Figure 3a illustrates that the bridge length, vehicle length and the distance between vehicles were identified as relevant variables, and the student combined vehicle length with the gap between vehicles before dividing the bridge length accordingly.



If we also count the gaps between the vehicles, we can divide the 36 km by this sum and find approximately how many vehicles there would be.

Figure 3a. Example of a level 4 student response

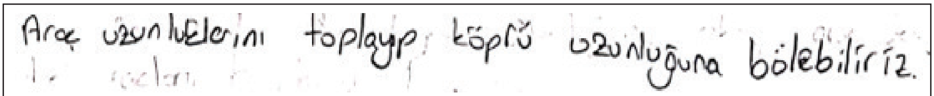
In contrast, 21 students identified the necessary variables but made only partially sufficient assumptions about their relationships. For example, the response in Figure 3b shows that the student neglected the distance between vehicles. While this could be accepted as a simplifying assumption, the student did not explicitly justify it in the written solution. Since assuming zero gaps between vehicles is highly unlikely in reality, this and similar responses were categorized as Level 3.



Bridge length (meters) / vehicle length (meters) = number of vehicles

Figure 3b. Example of a level 3 student response

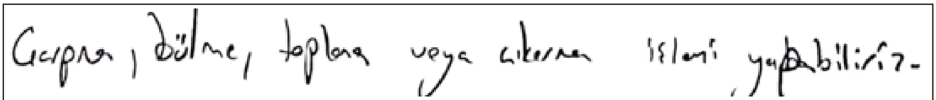
Nine students identified the relevant variables but failed to establish realistic assumptions about their interrelations. As shown in Figure 3c, the student made an incorrect assumption regarding how to combine the variables and continued with this misconception throughout the solution process.



We can add the vehicle lengths together and divide by the length of the bridge.

Figure 3c. Example of a Level 2 student response

Finally, 18 students provided irrelevant or superficial statements that were not connected to solving the problem. An example of such a response, categorized as level 1, is given in Figure 3d.



We can do multiplication, division, addition or subtraction.

Figure 3d. Example of a level 1 student response

Results of mathematizing

After understanding and simplifying the problem, students' performance in mathematizing was examined. At this stage, students were expected to construct valid mathematical models based on realistic assumptions, explain these models and establish connections among them. The distribution of students' performance levels in this competency is presented in Figure 4.

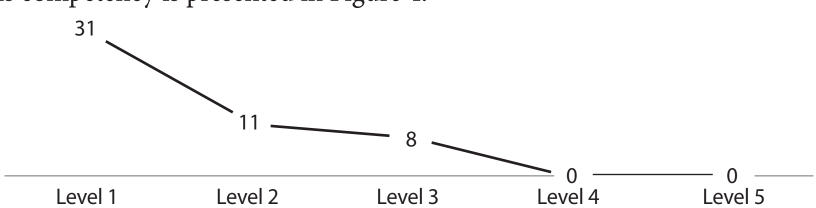
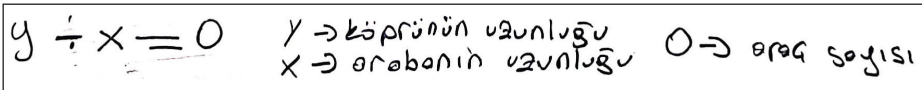


Figure 4. Levels of mathematizing competency

As seen in Figure 4, none of the students were able to construct, explain or interrelate fully valid mathematical models based on realistic assumptions. This indicates that students were largely unsuccessful in transferring the given real-world problem into the mathematical domain.

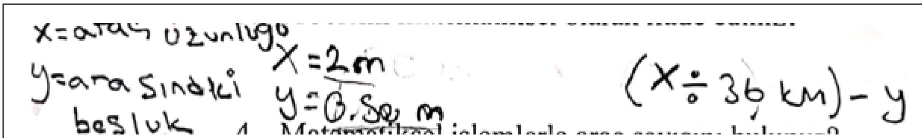
Eight students, however, constructed partially valid models based on assumptions that were somewhat acceptable. For example, the student response in Figure 4a shows that bridge length and vehicle length were considered as variables, yet the distance between vehicles was neglected. Thus, the student's model was regarded as partially valid.



y = bridge length, x = vehicle length, o = number of vehicles

Figure 4a. Example of a level 3 student response

In contrast, 11 students developed models that were invalid despite being based on partially reasonable assumptions. For instance, the student response in Figure 4b considered bridge length, vehicle length and the distance between vehicles as variables, but it failed to establish a mathematical equation to guide the solution. Furthermore, assuming a vehicle length of two meters was inconsistent with real-life conditions:



x = vehicle length, y = distance between vehicles

Figure 4b. Example of a level 2 student response

Finally, 31 students either did not construct a model or produced invalid ones. For example, the response in Figure 4c illustrates a misconception where the student attempted to relate bridge length and vehicle length through multiplication, which is mathematically inappropriate.

Figure 4c. Example of a level 1 student response

Findings on working mathematically

In the mathematical modelling process, students are expected to use the models they have constructed in order to reach correct solutions. However, as shown in the previous section, most students failed to generate valid models, which directly affected their performance in working mathematically. The distribution of students' levels in this competency is presented in Figure 5.

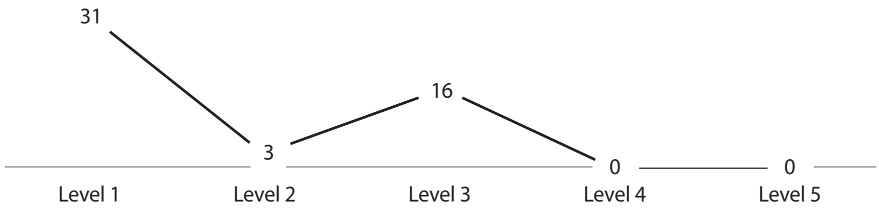
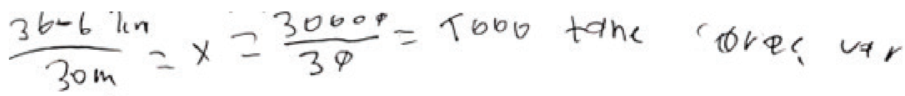


Figure 5. Levels of working mathematically

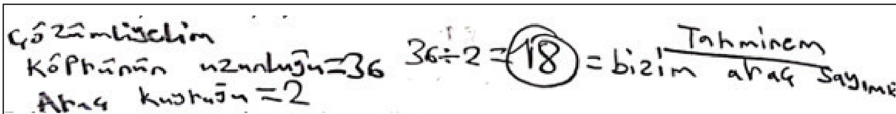
As seen in Figure 5, students were unable to reach correct solutions through valid mathematical models, mainly due to the fact that such models had not been constructed in the first place. Most students relied on flawed models. However, sixteen students correctly carried out the operations required by their flawed models. For example, in Figure 5a, the student performed the calculations consistently with the (incorrect) assumptions of the model.



(36 – 6 km) / 30 m = x = 30,000 / 30 = 1,000 cars

Figure 5a. Example of a level 3 student response

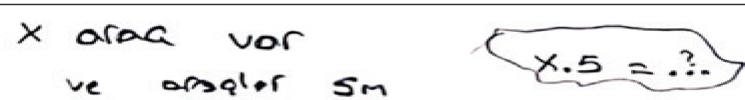
On the other hand, three students both constructed an invalid model and solved it incorrectly. An illustrative case is given in Figure 5b.



Let's solve it. Bridge length = 36. Queue = 2. 36 / 2 = 18 ≈ our number of cars.

Figure 5b. Example of a level 2 student response

Finally, 31 students did not reach any solution at all. As exemplified in Figure 5c, some responses remained at a vague or incomplete stage.



There are x cars and cars are 5 m, x·5 = ?

Figure 5c. Example of a level 1 student response

Findings on interpreting

Another sub-competency in the modelling process is interpreting. At this stage, students are expected to connect their mathematical solutions with the real-world context in a meaningful way. However, the analysis revealed that most students' models and solutions were limited, particularly regarding the variables of vehicle length and the distance between vehicles. The lane variable was not considered by any student. In this sub-competency, it is expected that students at least attempt to make realistic interpretations related to their primary variables (i.e., vehicle length and vehicle spacing). The distribution of students' performance in interpreting is presented in Figure 6.

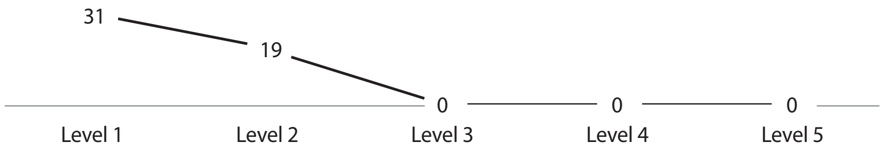
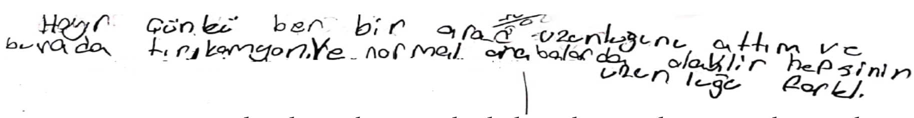


Figure 6. Levels of Interpreting

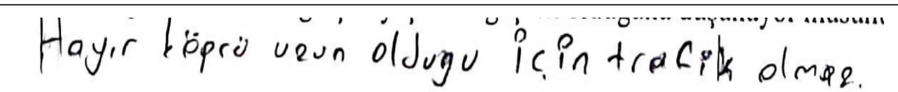
As shown in Figure 6, no student fully met the requirements of this competency, and none provided a complete real-life interpretation of a correct mathematical solution. Nineteen students partially interpreted their (incorrect or incomplete) mathematical solutions. For example, in Figure 6a, a student constructed a model using a single vehicle length but noted that vehicles in traffic vary in size (e.g., cars, trucks, lorries). However, the student ignored the variability in the distance between vehicles. Thus, the interpretation partially connected the mathematical result with real life.



No. Because I randomly took one vehicle length. But there may be trucks, lorries and cars. Their lengths are different.

Figure 6a. Example of a level 2 student response

Thirty-one students either failed to provide an interpretation of their solutions or made irrelevant real-life comments. An example is given in Figure 6b.



No, because the bridge is long, there will be no traffic.

Figure 6b. Example of a level 1 student response

Results on validating

In the modelling process, validating requires students to critically check their solutions and, when necessary, take steps to correct identified errors. Figure 7 summarises the distribution of students' performance in validating.

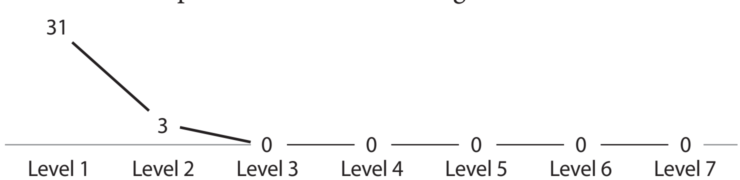


Figure 7. Levels of validating competency

As illustrated in Figure 7, students were largely unsuccessful in validation. None of the students attempted to revise or correct their work after identifying errors. Only eight students partially recognised their mistakes but did not engage in any corrective action. For example, in Figure 7a, a student acknowledged that the number of vehicles calculated (six) was unrealistic but made no attempt to revise the solution.

Yes. Because the number of cars is too few.

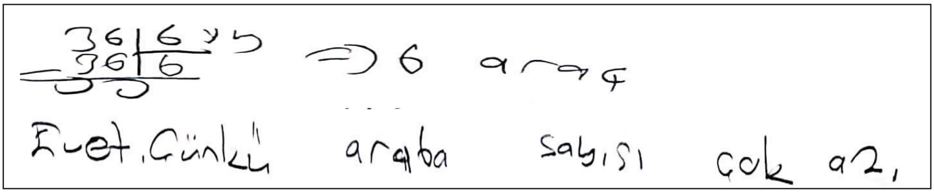


Figure 7a. Example of a level 2 student response

The remaining 42 students either failed to engage in any validation at all or provided inadequate/incorrect validation. As illustrated in Figure 7b, one student simply affirmed the solution without recognising or addressing earlier mistakes in variable selection (e.g., ignoring vehicle spacing) or computational errors.

Yes.

Figure 7b. Example of a level 1 student response

Overall, the findings suggest that the majority of students either neglected validation entirely or employed ineffective validation strategies.

Discussion and conclusion

The findings revealed that the overall level of students' modelling competencies was 23 %. This result indicates that students' modelling competencies were quite low and that they experienced difficulties in solving modelling problems. The findings are also consistent with previous research, which has reported that middle school students tend to show low performance in solving modelling problems (Blum & Leiß, 2007, Ozulu & Sağırlı, 2021; Permata et al., 2025).

One of the main reasons why students could not reach the expected competency level may be their limited experience with Fermi-type problems. Such problems require skills of estimation, logical reasoning and the construction of realistic assumptions. Although these skills are included in the elementary curricula of many countries (Burkhardt, 2006), mathematical modelling activities are rarely incorporated into classroom practices (Maaß, 2005) or textbooks (Tasarib et al., 2025). Similarly, an examination of the textbooks officially used by the participating students (Altunkaynak et al., 2024; Bektaş et al., 2019) revealed no Fermi-type problems. Therefore, to foster students' modelling competencies, it is crucial that textbooks include a greater number of modelling activities and that teachers integrate such activities into their instructional practices (Kaiser, 2020).

During the modelling process, it was observed that students' performance levels declined progressively as the competencies advanced. This trend can be attributed to the sequential and interdependent nature of the competencies. However, this downward trajectory was partially interrupted at the mathematization stage. Indeed, almost all students were able to obtain a result by correctly solving the mathematical models they had constructed. The lower scores at this stage stemmed from students' incorrect mathematical models in the preceding competency stage. This finding is consistent with previous research, and it shows that students do not generally experience major difficulties in tasks requiring procedural knowledge (Blum & Leiß, 2007; Ho, 2020).

An analysis across competency dimensions revealed that students performed above average in problem comprehension, whereas they fell below average in all other competencies. In particular, the majority of students were unsuccessful in interpretation and validation. This finding is consistent with the literature emphasizing that interpretation and validation represent the most challenging stages in mathematical modelling tasks (Blum & Leiß, 2007; Çiltaş, 2011; Çoksöyler, 2020; Kankanat, 2023; Ozulu & Sağirli, 2021; Özer Keskin, 2008).

In the literature, this lack of success has been attributed to various factors. Bukova Güzel and Uğurel (2010) emphasized that these competencies are not limited to mathematical procedures; rather, they require complex cognitive effort to assess the consistency of the model and the appropriateness of the solution in the real-world context, which leads to difficulties for students. Doruk and Cihan (2024) highlighted that insufficient feedback and guidance regarding these stages during instruction negatively affect the development of students' higher-order cognitive skills. Korkmaz (2010) noted that the demands of abstraction, cognitive complexity and inadequacies in instructional methods in the advanced stages of the modelling process contribute to students' struggles in interpretation and validation. Similarly, Blum et al. (2007) argued that traditional, artificial problem formats commonly used in education distance students from real-world contexts and thus hinder their success in validation.

Based on the results of this study, it was concluded that addressing interpretation and validation as separate competencies would be more beneficial. Students' shortcomings in the interpretation stage may stem from their limited ability to establish connections between mathematics and everyday life. Indeed, the majority of students experienced difficulties in horizontal mathematization, that is, transferring real-life problems into the mathematical world through models. A similar challenge was observed in transferring the mathematical solution back into the real-life context. Existing research highlighted that middle school students often demonstrated weak mathematical connection skills (English, 2017; Hagena et al., 2017). Likewise, deficiencies in the validation stage may be attributed to students' lack of argumentation skills, a point also supported in the literature (Eraslan & Kant, 2015).

Traditionally, instructional approaches aimed at enhancing students' interpretation and validation competencies have been grounded in modelling activities. However, numerous studies have demonstrated that such approaches only limited effects on the development of these competencies (Biccards & Wessels, 2011; Çoksöyler, 2020; Ozulu & Sağirli, 2021; Yurtsever, 2018). Therefore, instead of adopting general instructional approaches, it may be more effective to employ theoretical frameworks that directly target these specific competencies. In this regard, instructional designs that incorporate activities fostering connection-making and argumentation skills are recommended to support students' interpretation and validation competencies. In line with this perspective, mathematics educators emphasize that interdisciplinary and real-life-oriented teaching enhances students' ability to solve problems encountered in everyday contexts (Burkhardt, 2014; English, 2016; Wang et al., 2011). Learning environments that are grounded in active participation and collaboration also contribute to the development of students' metacognitive skills and help them overcome difficulties experienced during the modelling process (Lesh & Doerr, 2003). Furthermore, argumentation-based learning promotes the development of critical thinking skills, supports students' effective use of mathematical contexts and provides opportunities for more advanced, creative and meaningful thinking during problem-solving (Reichersdorfer et al., 2012; Zengin & Tapan-Broutin, 2023). Similarly, Pirimoğlu and Gürel (2025) found that argumentation-based mathematical modelling activities positively influence students' modelling competencies and inquiry skills.

One of the areas in which students performed the poorest was mathematization. A large proportion of students were unable to generate valid mathematical models to solve the problem. This finding is consistent with previous studies highlighting students' difficulties in constructing mathematical models (Blum & Leiß, 2007; Krutikhina et al. 2018). A primary reason for this challenge lies in students' inability to identify the necessary variables and develop appropriate representations that reflect the relationships among these variables. This observation also aligns with the difficulties students experience with multiple representations (English, 2018; Ledezma et al. 2022). Therefore, instructional interventions that foster representational skills may support students in expressing their ideas more effectively in mathematical language.

The study revealed that several difficulties prevented students from achieving the expected level of competency. These difficulties can be summarized as follows:

- Failure to establish a connection between the given information and the required outcome
- Inability to identify the variables necessary for solving the problem
- Producing limited and restricted assumptions (e.g., assuming that the vehicle type was always a car, vehicles were positioned without spacing, and the number of lanes was ignored)
- Inability to make realistic assumptions regarding the values of variables
- Inability to make realistic assumptions about the relationships between variables

- Failure to construct a mathematical model representing the relationships among variables
- Inability to formulate a general (prototype/model) representation
- Failure to relate the obtained results to real-life contexts
- Inability to identify shortcomings in the problem solution
- Failure to develop an approach to address the identified shortcomings

The difficulties encountered by students during the modelling process have also been reported in various ways in previous studies (Blum & Leiß, 2007; Eraslan & Kant, 2015; Tasarib et al., 2025). However, no systematic classification of difficulties encompassing the entire modelling process has yet been proposed for middle school students. Most studies have either focused on high school students, examined narrow problem types or concentrated on specific sub-competencies. In this regard, the present study is considered to provide an original contribution to the literature by offering a detailed classification of the difficulties encountered by middle school students in the modelling process. Future research may further elaborate on the underlying causes of each difficulty.

In this study, students were expected not only to solve a real-life problem but also to consider multiple possibilities and generate alternative assumptions. However, most students attempted to solve the problem by relying on a single assumption and determining only one value. This tendency restricted their creativity during the modelling process. Accordingly, classroom activities that encourage the generation of alternative assumptions and foster creative thinking skills are recommended.

This research was conducted with 50 sixth-grade students in the context of the Bridge Problem, it was grounded in the theoretical framework of modelling competencies and employed a qualitative research approach. The findings are therefore limited to the sample, the selected school, the problem situation and the theoretical framework adopted in this study. Additionally, students worked individually during the problem-solving tasks, which may have influenced the outcomes; it is possible that group work could have yielded different results. Similar research could be replicated with different student populations, data collection instruments and research designs. Furthermore, examining cognitive variables potentially related to modelling competencies (e.g., metacognitive awareness, mathematical connection-making, problem-solving strategies) could yield valuable contributions to the field.

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Kognitivne kompetencije i poteškoće učenika šestoga razreda u matematičkome modeliranju

Sažetak

Svrha ovoga istraživanja je ispitati kognitivne kompetencije modeliranja učenika šestoga razreda osnovne škole te utvrditi poteškoće s kojima se susreću tijekom toga procesa. Istraživanje je provedeno s 50 učenika šestoga razreda koji pohađaju javnu osnovnu školu smještenu u istočnom dijelu Turske. Podatci su prikupljeni pomoću obrasca za kompetencije modeliranja koji su razvili istraživači. Primjenjujući kvalitativni istraživački pristup, istraživanje je osmišljeno kao studija slučaja. Dobiveni kvalitativni podatci analizirani su metodom deskriptivne analize. Rezultati su pokazali da su učenici tijekom procesa matematičkoga modeliranja imali kognitivne nedostatke, osobito u fazama validacije i interpretacije. Na temelju dobivenih rezultata sastavljen je popis kognitivnih poteškoća učenika u procesu modeliranja te su predložene određene preporuke za njihovo prevladavanje.

Ključne riječi: *matematičko modeliranje; učenici osnovne škole; kompetencije modeliranja*

Uvod

Jedan od temeljnih ciljeva matematičkoga obrazovanja jest razvijanje matematički pismenih pojedinaca, što se smatra jednom od ključnih kompetencija koje treba razvijati u 21. stoljeću (Doruk, 2010; Drew, 2012). Matematički pismene osobe definiraju se kao pojedinci koji su svjesni uloge matematike u suvremenom svijetu, mogu učinkovito koristiti matematiku u stvarnim životnim situacijama, posjeduju razvijene sposobnosti mišljenja te pokazuju kompetenciju u rješavanju problema (OECD, 2019). U tome kontekstu matematičko modeliranje pojavljuje se kao jedan od najučinkovitijih pristupa za poticanje primjene matematičkih ideja u svakodnevnome životu, jer predstavlja važan interaktivni proces u kojem se učenici bave problemima proizašlim iz stvarnoga svijeta i vlastitoga neposrednoga okružja (Zbiek i Conner, 2006; Sabo Junger i Lipovec, 2022). U procesu matematičkoga modeliranja matematički modeli imaju središnju ulogu. Pojam matematičkoga modela različiti su istraživači definirali na različite načine. Na primjer, Kaiser (1995) definira matematički model kao strukturu stvorenu kako bi se objasnio, predvidio ili kontrolirao određeni fenomen. Lesh i Doerr (2003), s druge strane, matematičke modele konceptualiziraju kao konkretne ili apstraktne strukture koje predstavljaju odnose i strukture određenoga sustava. Nadalje, matematički modeli opisuju se i kao vanjske reprezentacije koje podupiru rješavanje i interpretaciju problema iz stvarnoga svijeta, služeći kao most između matematike i svakodnevnoga života (Blum, Galbraith, Henn i Niss, 2007; Borromeo Ferri, 2006; Lesh i Doerr, 2003).

U skladu s različitim definicijama matematičkoga modeliranja, predloženi su i različiti pristupi procesu modeliranja (Kaiser i Sriraman, 2006). Na primjer, Blum i Leiß (2006) opisali su proces modeliranja kao ciklus koji uključuje vertikalnu i horizontalnu matematizaciju. Slično tome, Berry i Houston (1995) razvili su pristup modeliranju koji omogućuje ispitivanje odnosa između stvarnoga i matematičkoga svijeta. Pristup Müllera i Wittmanna (1984, prema Peter-Koop, 2004) konceptualizira proces modeliranja u dva područja – konceptualnom i stvarnosnom – te naglašava veze između tih područja. Lesh i Doerr (2003) definiraju matematičko modeliranje kao proces prevođenja stvarnih situacija i problema u matematički jezik te razvoja modela koji se mogu primijeniti u sličnim situacijama. Osim toga, neki su istraživači predložili pristupe koji u proces uključuju ne samo stvarni i matematički svijet, nego i tehnološki svijet (Ang, 2010; Siller i Greefrad, 2010). Iz tih perspektiva matematički model može se smatrati proizvodom dobivenim na kraju procesa modeliranja, dok se matematičko modeliranje može promatrati kao dinamičan proces koji uključuje konstruiranje matematičkih modela za rješavanje problema iz stvarnoga svijeta, dolaženje do rješenja te njihovu interpretaciju u stvarnim životnim kontekstima (Doruk, 2023; Lesh i Doerr, 2003).

Posljednjih godina matematičko modeliranje dobiva sve veću važnost te je postalo jedna od ključnih kompetencija uključenih u školske kurikule i obrazovne standarde u mnogim zemljama (Borromeo Ferri, 2018). Vještine povezane s matematičkim modeliranjem smatraju se temeljnim kompetencijama koje matematičko obrazovanje nastoji razviti (National Council of Teachers of Mathematics [NCTM], 2000). Glavni razlog tome su višestruki doprinosi koje aktivnosti modeliranja pružaju učenicima. Primjenom takvih aktivnosti učenici razvijaju matematičko mišljenje i kognitivne vještine (Blum i Ferri, 2009; English, 2006; English i Watters, 2005; Ünlü, 2023), kao i sposobnosti matematičkih povezivanja, komunikacije, rješavanja problema i zaključivanja (Blum, 2011; Blum i Ferri, 2009; English, 2006; English i Watters, 2005; Kaiser, 2006; Sabo Junger, Lipovec i Ferme, 2024; Swan i Burkhardt, 2012). Nadalje, modeliranje potiče razvoj svestranoga, višega i kreativnoga mišljenja (Doerr i English, 2003; English i Watters, 2005; Kaiser i Sriraman, 2006), kao i metakognitivnih vještina (Borromeo Ferri, 2018; Can, 2024; Ülker i Çelik, 2025). U matematičkom obrazovanju pristup modeliranja ne usredotočuje se samo na pronalaženje rješenja pojedinačnih problema, nego i na istraživanje općih struktura koje uključuju međusobne veze te omogućuje formuliranje problema povezanih s promatranim situacijama (Doerr i English, 2003). U tom smislu, budući da matematičko modeliranje naglašava proces, a ne samo konačni rezultat, ono također pruža nastavnicima mogućnost promatranja i vrednovanja načina razmišljanja svojih učenika (Erbaş i sur., 2014; Öztürk, 2025).

Proces matematičkoga modeliranja sastoji se od šest faza: razumijevanje problema, pojednostavljivanje, matematizacija, matematički rad, interpretacija rezultata i njihova validacija (Blum, 2011; Borromeo Ferri, 2018; Kaiser, 2006). Kako bi bili uspješni u rješavanju modelirajućih problema, učenici moraju učinkovito koristiti potrebne kompetencije modeliranja tijekom cijeloga tog procesa (Erdem i sur., 2015; Tekin Dede i Yılmaz, 2015). Kompetencije modeliranja odnose se na znanja, vještine i sposobnosti potrebne za rješavanje problema modeliranja (Maaß, 2006). Osim toga, važno je da učenici budu i svjesni i motivirani kako bi prevladali poteškoće koje se javljaju u svakoj fazi procesa (Blomhøj i Jensen, 2003; Jensen, 2007). Kompetencije potrebne za uspjeh u matematičkome modeliranju u literaturi su klasificirane na različite načine. Na primjer, Bukova Güzel (2019), na temelju opsežnoga pregleda literature, identificirala je četiri dimenzije kompetencija modeliranja: kognitivnu, afektivnu, socijalnu i metakognitivnu. S druge strane, Borromeo Ferri i Blum (2010) objašnjavaju kognitivne kompetencije u okviru šest koraka procesa modeliranja: razumijevanje problema, pojednostavljivanje, matematizacija, matematički rad, interpretacija i validacija. Ovo istraživanje posebno se usredotočuje na situacije učenika i poteškoće s kojima se susreću u vezi s tim kognitivnim kompetencijama.

Postoji pozitivna povezanost između uspjeha u matematičkom modeliranju i matematičke pismenosti (Özer-Demir i Bukova-Güzel, 2024). Ovaj nalaz sugerira da učenici, kako bi učinkovito koristili matematiku u svakodnevnom životu, moraju razviti snažne vještine modeliranja. Postizanje uspjeha u procesima modeliranja zahtijeva prije svega stjecanje potrebnih kompetencija. Stoga je nužno utvrditi posjeduju li učenici navedene kompetencije te, na temelju dobivenih rezultata, osmisliti i primijeniti odgovarajuće nastavne metode za njihov razvoj. Istraživanja u literaturi pokazuju da se učenici susreću s različitim poteškoćama u vezi s kompetencijama modeliranja. Posebno su zabilježene značajne poteškoće u fazama razumijevanja i pojednostavljivanja problema (Kertil i Gürel, 2016), matematizacije (Barquero i Bosch, 2015), matematičkoga rada (Stillman, 2011), kao i interpretacije i validacije rješenja (Blum i Leiß, 2007).

Stjecanje kompetencija matematičkoga modeliranja donosi učenicima značajne koristi, kako u svakodnevnom životu tako i u njihovom matematičkom obrazovanju. Razvijanje tih kompetencija u ranoj dobi može pozitivno utjecati na kasnija iskustva u učenju. U tom je smislu važno da mlađi učenici razvijaju kompetencije modeliranja te da se nastavni procesi osmisle tako da podupiru njihov razvoj. Među najčešće korištenim pristupima za poticanje tih kompetencija nalaze se aktivnosti modeliranja. Međutim, unatoč ohrabrujućim rezultatima, prethodna istraživanja pokazala su da učenici često ne dosežu očekivanu razinu u određenim područjima kompetencija, osobito u validaciji i interpretaciji (Biccards i Wessels, 2011; Çoksöyler, 2020; Ozulu i Sağirli, 2021; Yurtsever, 2018). Ovaj nalaz upućuje na to da ta područja zahtijevaju različite skupove vještina, stoga zahtijevaju nastavne pristupe prilagođene svakoj pojedinoj kompetenciji.

Rješavanje ovoga pitanja najprije zahtijeva detaljno ispitivanje trenutačnoga stanja učenika u tim područjima te identifikaciju poteškoća s kojima se susreću u svakoj fazi kompetencije. Nakon takve analize potrebno je osmisliti i primijeniti nastavne strategije koje odgovaraju obilježjima tih izazova. U tom je okviru smješteno i ovo istraživanje. Cilj je istražiti kompetencije modeliranja učenika šestoga razreda osnovne škole i poteškoće s kojima se susreću te, na temelju dobivenih nalaza, predložiti nastavne strategije za područja u kojima učenici pokazuju nedostatke.

Pregled literature o kompetencijama modeliranja učenika viših razreda osnovne škole pokazuje da su istraživanja u tom području još uvijek ograničena. Postojeće studije uglavnom se usredotočuju na pojedine faze kompetencija te su često oblikovane kao nastavne intervencije koje uključuju aktivnosti modeliranja s ciljem razvoja kompetencija (Duran, 2023; Pirimoğlu i Gürel, 2025; Tekin Dede, 2017; Tuna, Biber i Yurt, 2013; Yıldız, 2024; Yılmaz i Kesebir, 2023). Stoga se može očekivati da će sveobuhvatno istraživanje koje obuhvaća sve faze kompetencija modeliranja učenika šestoga razreda značajno doprinijeti ovomu području.

U tome kontekstu istraživanjem se nastoji odgovoriti na sljedeća istraživačka pitanja:

1. Koja je razina kompetencija matematičkoga modeliranja učenika viših razreda osnovne škole?
2. S kakvim se poteškoćama učenici susreću tijekom procesa matematičkog< modeliranja?

Metodologija

Istraživački dizajn

Ovo istraživanje osmišljeno je i provedeno primjenom metode studije slučaja, koja pripada kvalitativnim istraživačkim pristupima. Studije slučaja predstavljaju istraživačke dizajne koji imaju za cilj detaljno ispitati određeni fenomen ili događaj unutar ograničenoga konteksta (Yin, 2018). U skladu s tom perspektivom, u ovome se istraživanju nastojalo otkriti kompetencije matematičkoga modeliranja učenika šestoga razreda putem zadatka s mostom.

Sudionici

Istraživačku skupinu činilo je ukupno 50 učenika šestoga razreda koji pohađaju javnu osnovnu školu (razredi 5. – 8.) u regiji Istočne Anadolije u Turskoj. U turskom obrazovnom sustavu učenici viših razreda osnovne škole obično su u dobi od 10 do 14 godina. U ovome istraživanju dob sudionika kretala se od 11 do 12 godina, s prosječnom dobi od 11,5 godina. Od tih učenika, 24 je bilo u jednom razrednom odjelu, dok je preostalih 26 bilo u drugom. Oba razreda poučavao je isti nastavnik matematike. Istraživanje je provedeno na kraju proljetnoga semestra školske godine 2023./2024. U tom razdoblju školske godine može se reći da su učenici već usvojili većinu ishoda učenja predviđenih kurikulom matematike za šesti razred.

Prikupljanje podataka

Podatci istraživanja prikupljeni su pomoću Obrasca za kompetencije modeliranja, koji su razvili istraživači. Obrazac je sadržavao jedan zadatak modeliranja i šest otvorenih pitanja osmišljenih kako bi se ispitale kompetencije modeliranja učenika u vezi s tim zadatkom. Kao zadatak modeliranja korišten je *zadatak s mostom*, koji je prilagođen na temelju problema prometne gužve koji je predstavio Peter-Koop (2004), a koji su prenijeli Bukova-Güzel i suradnici (bez godine). U obrascu je osigurano dovoljno prostora kako bi učenici mogli jasno i pregledno napisati svoje odgovore.

Zadatak s mostom odabran je zato što se temelji na jednostavnoj operaciji dijeljenja, čime se smanjuje utjecaj čimbenika izvan kompetencija koje se istražuju. Osim toga, taj zadatak nije bio korišten u prethodnim istraživanjima, što povećava njegovu novost. Pri oblikovanju pitanja vodilo se računa o tome da svako pitanje odgovara određenoj fazi kompetencije i jasno odražava ciljne kompetencije definirane u analitičkoj rubrici. Kako bi se procijenila prikladnost obrasca za istraživanje, zatraženo je stručno mišljenje profesora matematike s iskustvom u području matematičkoga modeliranja. Stručnjak je potvrdio da je instrument prikladan za mjerenje kompetencija matematičkoga modeliranja.

U nastavku je prikazan *zadatak s mostom* i šest povezanih pitanja koja se odnose na svaku fazu kompetencije:

Jedan od najvećih mostova na svijetu je most izgrađen preko zaljeva Hangzhou u istočnoj Kini, dug približno 36 kilometara. Ako zamislite niz vozila duž tog mosta, koliko bi vozila ukupno moglo biti?

Molimo odgovorite na sljedećih šest pitanja u prostorima predviđenima ispod svakog pitanja:

1. Koje su informacije dane u ovome problemu i što se od vas traži? Kako možete uspostaviti odnos između danih informacija i traženoga rezultata? Objasnite.
2. Koje su informacije, prema vašem mišljenju, potrebne za rješavanje ovoga problema? Koje informacije nedostaju i kako biste mogli pretpostaviti njihove vrijednosti? Objasnite.
3. Kako biste, koristeći svoje pretpostavke, ovaj problem izrazili matematički? Prikažite svoje jednadžbe ili postupke i objasnite ih.
4. Koristeći matematički model koji ste konstruirali, provedite potrebne izračune i pronađite rezultat. Jasno prikažite korake izračuna.
5. Procijenite svoj rezultat u odnosu na stvarni život. Smatrate li da je taj rezultat realističan? Zašto ili zašto ne?
6. Provjerite svoje rješenje. Ako pronađete pogreške u svojim izračunima, kako biste ih mogli ispraviti?

Pitanja su osmišljena uzimajući u obzir kognitivnu razinu učenika, pri čemu je svako pitanje imalo za cilj procijeniti jednu od sljedećih potkompetencija redoslijedom: razumijevanje problema, pojednostavljivanje, matematizacija, matematički rad, interpretacija i validacija.

Provedbu istraživanja u razrednom okružju škole u kojoj je istraživanje provedeno vodio je istraživač. Nakon što je učenicima ukratko objasnio svrhu i postupak istraživanja, istraživač je naglasio da je sudjelovanje dobrovoljno. Učenici koji su pristali sudjelovati odgovarali su na pitanja individualno. Tijekom provedbe istraživač je objasnio da ne postoji strogo vremensko ograničenje, ali da odgovori moraju biti dani samostalno. Instrument za prikupljanje podataka zatim je podijeljen sudionicima, koji su obrazac ispunili i vratili istraživaču u približno 60 minuta.

Analiza podataka

Odgovori učenika na zadatak analizirani su metodom deskriptivne analize. U određivanju razina kompetencija modeliranja učenika korištena je Rubrika za procjenu kompetencija modeliranja (MCAR) koju su razvili Tekin Dede i Bukova-Güzel (2018). Ova se rubrika temelji na šest potkompetencija procesa matematičkoga modeliranja, pri čemu se svaka potkompetencija procjenjuje na skali od 12 bodova. Za kompetencije s četiri razine dodjeljivani su bodovi 0, 4, 8 i 12; za kompetencije s pet razina 0, 3, 6, 9 i 12; a za kompetenciju sa sedam razina 0, 2, 4, 6, 8, 10 i 12. Prema tome, svaki je učenik za svaku potkompetenciju mogao ostvariti minimalno 0, a maksimalno 12 bodova, dok se ukupni rezultat kretao od 0 do 72 boda.

Potkompetencije i kriteriji njihove procjene bili su sljedeći:

- razumijevanje problema: razvrstano u razine 1 – 5, na temelju sposobnosti učenika da pokažu razumijevanje problema, prepoznaju zadane i tražene informacije te uspostave odnos između njih
- pojednostavljivanje: razvrstano u razine 1 – 4, na temelju prepoznavanja relevantnih i nerelevantnih varijabli te postavljanja realističnih pretpostavki
- matematizacija: procjenjivana u razinama 1 – 5, na temelju konstruiranja matematičkih modela u skladu s realističnim pretpostavkama, njihova objašnjavanja i međusobnoga povezivanja
- matematički rad: razvrstan u razine 1 – 5, na temelju konstruiranja odgovarajućih matematičkih modela i njihova točnoga rješavanja
- interpretacija: procjenjivana u razinama 1 – 5, na temelju dobivanja točnih matematičkih rezultata i njihove odgovarajuće interpretacije u kontekstu stvarnoga života
- validacija: razvrstana u razine 1 – 7, na temelju prepoznavanja i ispravljanja pogrešaka te primjene strategija provjere.

U postupku analize podataka istraživači su zajednički razvili sheme kodiranja identificiranjem primjera učeničkih odgovora za svaku razinu. Nakon toga su proveli neovisno kodiranje i usporedili rezultate. Međuprocjeniteljska suglasnost za sve potkompetencije, osim za razumijevanje problema, iznosila je najmanje 92 %. Za potkompetenciju razumijevanje problema suglasnost je iznosila 54 %. Ova razlika proizašla je iz različitih interpretacija: jedan je istraživač kriterij „uspostavljanja odnosa između zadanih i traženih informacija" ograničio samo na konkretan odgovor dan u odgovarajućoj stavci, dok je drugi istraživač uzeo u obzir odnose koji su se pojavljivali tijekom cijeloga procesa rješavanja. Kako bi se to razriješilo, istraživači su odlučili da će se prednost dati konkretnom odgovoru na pitanje, ali će se cjelokupno rješenje također uzeti u obzir u slučajevima kada informacije nedostaju ili nisu izričito navedene. Nakon ponovljenih krugova analize postignut je potpuni konsenzus te je analiza podataka završena. Kvantitativni rezultati dobiveni primjenom rubrike prikazani su deskriptivno.

Rezultati

U ovome dijelu prikazane su kompetencije modeliranja učenika temeljem njihovih odgovora na *zadatak s mostom*. Kompetencije modeliranja ispitivane su putem šest potkompetencija: razumijevanje problema, pojednostavljivanje, matematizacija, matematički rad, interpretacija i validacija. Ukupni rezultati koje su učenici postigli za svaku kompetenciju, procijenjeni primjenom Rubrike za procjenu kompetencija modeliranja (MCAR), prikazani su na Slici 1.

Slika 1

Maksimalni rezultat koji se mogao postići za svaku fazu kompetencija modeliranja bio je 600, što daje maksimalni ukupni rezultat od 3600 za sve kompetencije. Ukupni rezultat učenika u kompetencijama modeliranja iznosio je 829. Na temelju tih vrijednosti, kompetencija učenika u *zadatku s mostom* iznosila je 23 %. Prema tome, može se zaključiti da je ukupna razina kompetencija modeliranja učenika bila prilično niska.

Kao što je prikazano na Slici 1, učenici su ostvarili gotovo prosječne rezultate u kompetenciji razumijevanje problema (57 %), dok je njihova uspješnost u svim ostalim kompetencijama bila ispod prosjeka (38 %, 13 %, 17 %, 9 % i 2 %, redom). Posebno je zapažena niska razina uspjeha u matematizaciji i sljedećim kompetencijama. Validacija se pokazala kompetencijom s najnižim postignutim rezultatom. Ovi nalazi ukazuju da su učenici najviše poteškoća imali u prevođenju problema iz stvarnoga života u matematički jezik, u povezivanju rezultata sa stvarnim kontekstom te u validaciji ishoda. Sljedeći dijelovi pružaju detaljnu analizu svake kompetencije modeliranja.

Rezultati o razumijevanju problema

U procesu modeliranja od učenika koji pokazuje kompetenciju razumijevanja problema očekuje se da artikulira koherentno razumijevanje konteksta problema, da ispravno identificira zadane i nepoznate podatke te da uspostavi točne odnose među njima. Pregled razina razumijevanja problema učenika u *zadatku s mostom* prikazan je na Slici 2.

Slika 2

Kao što je prikazano na Slici 2, većina učenika nije bila na očekivanoj razini u pogledu razumijevanja problema. Ovaj nalaz ukazuje da su mnogi učenici imali poteškoća u potpunom shvaćanju zahtjeva zadatka. Konkretno, 19 učenika uspjelo je ispravno identificirati zadane i nepoznate podatke te uspostaviti odgovarajuće odnose među njima, većinom primjenom dijeljenja. Primjer odgovora učenika kategoriziranoga na razini 5 u kompetenciji razumijevanja problema prikazan je na Slici 2a:

Slika 2a

Dvanaest učenika identificiralo je zadane i nepoznate podatke, ali nisu uspjeli uspostaviti odnos među njima. Četrnaest učenika identificiralo je samo neke zadane i nepoznate podatke, dok pet učenika nije moglo navesti ni jedne. Slike 2b, 2c i 2d prikazuju primjere odgovora učenika kategoriziranih na razinama 3, 2 i 1, redom:

Slika 2b

Slika 2c

Slika 2d

Rezultati o pojednostavljivanju problema

Nakon kompetencije razumijevanja problema, ispitivana je uspješnost učenika u pojednostavljivanju problema. U ovoj fazi očekuje se da učenici identificiraju potrebne i nepotrebne varijable u problemu, reduciraju problem na njegove bitne komponente te postave realistične pretpostavke. U procesu rješavanja može se primijeniti više pojednostavljenih pristupa. Najjednostavnije pojednostavljenje bilo bi razmatranje duljine mosta, duljine vozila i udaljenosti između vozila kao potrebnih varijabli. Od učenika se očekivalo da uspostave odnose među ovim varijablama na temelju realističnih pretpostavki. Pregled uspješnosti učenika u ovoj kompetenciji prikazan je na Slici 3.

Slika 3

Kao što je prikazano na Slici 3, većina učenika nije bila potpuno kompetentna u pojednostavljivanju problema. Samo su dva učenika uspješno identificirala potrebne varijable i postavila realistične pretpostavke o njihovim međusobnim odnosima. Primjer odgovora učenika prikazan na Slici 3a ilustrira da su duljina mosta, duljina vozila i udaljenost između vozila prepoznati kao relevantne varijable, a učenik je kombinirao duljinu vozila s razmakom između vozila prije nego što je podijelio duljinu mosta:

Slika 3a

Suprotno tome, 21 učenik identificirao je potrebne varijable, ali je postavio samo djelomično zadovoljavajuće pretpostavke o njihovim odnosima. Na primjer, odgovor prikazan na Slici 3b pokazuje da je učenik zanemario razmak između vozila. Iako bi se to moglo prihvatiti kao pojednostavljujuća pretpostavka, učenik je nije izričito obrazložio u pisanom rješenju. Budući da je pretpostavka nultih razmaka između vozila u stvarnosti vrlo malo vjerojatna, ovaj i slični odgovori kategorizirani su na razinu 3:

Slika 3b

Devet učenika identificiralo je relevantne varijable, ali nije uspostavilo realistične pretpostavke o njihovoj međusobnoj povezanosti. Kao što je prikazano na Slici 3c, učenik je postavio pogrešnu pretpostavku o načinu kombiniranja varijabli i nastavio s ovom zabludom tijekom cijeloga postupka rješavanja:

Slika 3c

Na kraju, 18 učenika dali su irelevantne ili površne izjave koje nisu bile povezane s rješavanjem problema. Primjer takvog odgovora, kategoriziranoga na razinu 1, prikazan je na Slici 3d:

Slika 3d

Rezultati o matematizaciji

Nakon kompetencija razumijevanja problema i pojednostavljivanja problema, ispitivana je uspješnost učenika u matematizaciji. U ovoj fazi očekuje se da učenici konstruiraju valjane matematičke modele temeljem realističnih pretpostavki, objasne te modele i uspostave veze među njima. Raspodjela razina uspješnosti učenika u ovoj kompetenciji prikazana je na Slici 4.

Slika 4

Kao što je vidljivo na Slici 4, nijedan učenik nije uspio potpuno konstruirati, objasniti i međusobno povezati valjane matematičke modele temeljene na realističnim pretpostavkama. To ukazuje da su učenici u velikoj mjeri bili nesposobni prenijeti zadani problem iz stvarnoga svijeta u matematičku domenu.

Međutim, osam učenika konstruiralo je djelomično valjane modele temeljem pretpostavki koje su bile donekle prihvatljive. Na primjer, odgovor učenika prikazan na Slici 4a pokazuje da su duljina mosta i duljina vozila uzeti kao varijable, dok je udaljenost između vozila zanemarena. Stoga je učenikov model ocijenjen kao djelomično valjan:

Slika 4a

Suprotno tome, 11 učenika razvilo je nevaljane modele, iako su se djelomično oslanjali na razumne pretpostavke. Na primjer, odgovor prikazan na Slici 4b uzeo je u obzir duljinu mosta, duljinu vozila i udaljenost između vozila kao varijable, ali nije uspostavio matematičku jednadžbu koja bi vodila do rješenja. Osim toga, pretpostavka da vozilo ima 2 metra duljine nije bila u skladu sa stvarnim uvjetima:

Slika 4b

Na kraju, 31 učenik ili nije konstruirao nikakav model ili je proizveo nevaljane modele. Na primjer, odgovor prikazan na Slici 4c ilustrira pogrešno shvaćanje u kojem je učenik pokušao povezati duljinu mosta i duljinu vozila množenjem, što matematički nije ispravno:

Slika 4c

Rezultati o matematičkom radu

U procesu matematičkoga modeliranja od učenika se očekuje da koriste modele koje su konstruirali kako bi došli do ispravnih rješenja. Međutim, kao što je prikazano u prethodnom dijelu, većina učenika nije uspjela generirati valjane modele, što je izravno utjecalo na njihovu uspješnost u matematičkome radu. Raspodjela razina učenika u ovoj kompetenciji prikazana je na Slici 5.

Slika 5

Kao što je vidljivo na Slici 5, učenici nisu mogli doći do ispravnih rješenja pomoću valjanih matematičkih modela, prvenstveno zato što takvi modeli uopće nisu bili konstruirani. Većina učenika oslanjala se na pogrešne modele. Šesnaest učenika, međutim, ispravno je izvršilo operacije zahtijevane njihovim pogrešnim modelima. Na primjer, u Slici 5a učenik je izvršio izračune u skladu s (netočnim) pretpostavkama modela:

Slika 5a

S druge strane, tri učenika konstruirala su nevaljan model i riješila ga netočno. Ilustrativan primjer prikazan je na Slici 5b:

Slika 5b

Na kraju, 31 učenik nije došao ni do kakvog rješenja. Kao što je prikazano na Slici 5c, neki odgovori ostali su nejasni ili nepotpuni:

Slika 5c

Rezultati o interpretaciji

Još jedna potkompetencija u procesu modeliranja je interpretacija. U ovoj fazi očekuje se da učenici povežu svoja matematička rješenja sa stvarnim kontekstom na smislen način. Analiza je, međutim, pokazala da su modeli i rješenja većine učenika bila ograničena, osobito u vezi s varijablama duljine vozila i razmaka između vozila. Varijablu trake nije uzeo u obzir nijedan učenik. U ovoj potkompetenciji očekuje se da učenici barem pokušaju napraviti realistične interpretacije u vezi s primarnim varijablama (tj. duljinom vozila i razmakom između vozila). Raspodjela uspješnosti učenika u interpretaciji prikazana je na Slici 6.

Slika 6

Kao što je prikazano na Slici 6, nijedan učenik nije u potpunosti zadovoljio zahtjeve ove kompetencije i nitko nije dao cjelovitu interpretaciju ispravnoga matematičkog rješenja u stvarnom kontekstu. Devetnaest učenika djelomično je interpretiralo svoja (netočna ili nepotpuna) matematička rješenja. Na primjer, u Slici 6a učenik je konstruirao model koristeći duljinu jednoga vozila, ali je primijetio da se vozila u prometu razlikuju po veličini (npr. automobili, kamioni, teretna vozila). Ipak, učenik je zanemario varijabilnost u razmaku između vozila. Stoga interpretacija djelomično povezuje matematički rezultat sa stvarnim životom:

Slika 6a

Trideset i jedan učenik ili nije dao nikakvu interpretaciju svojih rješenja ili je dao irelevantne komentare u stvarnom kontekstu. Primjer je prikazan na Slici 6b:

Slika 6b

Rezultati o validaciji

U procesu modeliranja validacija zahtijeva da učenici kritički provjere svoja rješenja i, kada je potrebno, poduzmu korake za ispravljanje uočenih pogrešaka. Raspodjela uspješnosti učenika u validaciji sažeta je na Slici 7.

Slika 7

Kao što je prikazano na Slici 7, učenici su u velikoj mjeri bili neuspješni u validaciji. Nijedan učenik nije pokušao revidirati ili ispraviti svoj rad nakon uočavanja pogrešaka. Samo je osam učenika djelomično prepoznalo svoje pogreške, ali nisu poduzeli nikakvu korektivnu akciju. Na primjer, u Slici 7a učenik je priznao da je broj vozila koji je izračunao (šest) nerealističan, ali nije pokušao revidirati rješenje:

Slika 7a

Preostalih 42 učenika ili se uopće nisu bavili validacijom ili su dali nedovoljnu/netočnu validaciju. Kao što je prikazano na Slici 7b, jedan je učenik jednostavno potvrdio rješenje, a da nije prepoznao ili ispravio prethodne pogreške u odabiru varijabli (npr. ignoriranje razmaka između vozila) ili izračunskim pogreškama:

Slika 7b

Sveukupno, nalazi sugeriraju da većina učenika ili u potpunosti zanemaruje validaciju ili primjenjuje neučinkovite strategije validacije.

Diskusija i zaključak

Nalazi su pokazali da je ukupna razina kompetencija učenika u modeliranju 23 %. Ovaj rezultat ukazuje da su kompetencije učenika u modeliranju prilično niske te da su učenici nailazili na poteškoće u rješavanju problemskih zadataka vezanih uz modeliranje. Nalazi su također u skladu s prethodnim istraživanjima koja pokazuju da učenici osnovnih škola često postižu nisku razinu uspješnosti u rješavanju problemskih zadataka u modeliranju (Blum i Leiß, 2007; Ozulu i Sağırlı, 2021; Permata, Budiarto i Fuad, 2025).

Jedan od glavnih razloga zašto učenici nisu postigli očekivanu razinu kompetencije može biti njihovo ograničeno iskustvo s problemima tipa Fermi. Takvi problemi zahtijevaju vještine procjene, logičkoga razmišljnja i konstruiranja realističnih pretpostavki. Iako su ove vještine uključene u osnovne kurikule mnogih zemalja (Burkhardt, 2006), aktivnosti matematičkoga modeliranja rijetko su integrirane u nastavu (Maaß, 2005) ili u udžbenike (Tasarib, Rosli i Rambely, 2025). Pregled udžbenika kojima se koriste učenici u istraživanju (Altunkaynak, Tunç i Kavurmacı, 2024; Bektaş, Kahraman i Temel, 2019) pokazao je da također ne sadrže probleme tipa Fermi. Stoga je za poticanje kompetencija u modeliranju ključno da udžbenici sadrže veći broj aktivnosti modeliranja i da učitelji integriraju takve aktivnosti u svoju nastavu (Kaiser, 2020).

Tijekom procesa modeliranja uočeno je da razine uspješnosti učenika postupno opadaju kako kompetencije napreduju. Ovaj trend može se objasniti sekvencijalnom i međusobno ovisnom prirodom kompetencija. Međutim, ovaj pad djelomično je prekinut u fazi matematizacije. Naime, gotovo svi učenici uspjeli su dobiti rezultat pravilnim rješavanjem matematičkih modela koje su konstruirali. Niže ocjene u ovoj fazi proizlazile su iz toga što su učenici prethodno konstruirali netočne matematičke modele. Ovaj nalaz u skladu je s prethodnim istraživanjima koja pokazuju da učenici općenito ne nailaze na velike poteškoće u zadatcima koji zahtijevaju proceduralno znanje (Blum i Leiß, 2007; Ho, 2020).

Analiza po dimenzijama kompetencija pokazala je da su učenici iznadprosječno uspješni u razumijevanju problema, dok su u svim ostalim kompetencijama bili ispodprosječni. Posebno su većina učenika bila neuspješna u interpretaciji i validaciji. Ovaj nalaz u skladu je s literaturom koja ističe da interpretacija i validacija predstavljaju najzahtjevnije faze u zadatcima matematičkoga modeliranja (Blum i Leiß, 2007; Çiltaş, 2011; Çoksöyler, 2020; Kankanat, 2023; Ozulu i Sağırlı, 2021; Özer Keskin, 2008).

U literaturi se ovaj nedostatak uspjeha pripisuje različitim čimbenicima. Bukova Güzel i Uğurel (2010) naglašavaju da ove kompetencije nisu ograničene na matematičke procedure. One zahtijevaju složeni kognitivni napor kako bi se procijenila konzistentnost modela i prikladnost rješenja u stvarnom kontekstu, što učenicima stvara poteškoće. Doruk i Cihan (2024) ističu da nedovoljna povratna informacija i smjernice u vezi s ovim fazama tijekom nastave negativno utječu na razvoj viših kognitivnih vještina učenika. Korkmaz (2010) napominje da apstrakcija, kognitivna složenost i nedostatci u nastavnim metodama u naprednim fazama procesa modeliranja doprinose poteškoćama učenika u interpretaciji i validaciji. Slično tome, Blum i sur. (2007) tvrde da tradicionalni, umjetni oblici problema, koji se često koriste u obrazovanju, udaljavaju učenike od stvarnoga konteksta i time ograničavaju njihov uspjeh u validaciji.

Na temelju nalaza ovoga istraživanja, zaključeno je da bi bilo korisnije tretirati interpretaciju i validaciju kao odvojene kompetencije. Nedostatci učenika u fazi interpretacije mogu proizlaziti iz njihove ograničene sposobnosti uspostavljanja veza između matematike i svakodnevnoga života. Većina učenika nailazila je na poteškoće u horizontalnoj matematizaciji, tj. u prenošenju problema iz stvarnoga života u matematički svijet putem modela. Sličan problem uočava se i prilikom prenošenja matematičkoga rješenja natrag u stvarni kontekst. Postojeća istraživanja ističu da učenici osnovnih škola često pokazuju slabije vještine povezivanja matematike s kontekstom (English, 2017; Hagena, Leiss i Schwippert, 2017). Također, nedostatci u fazi validacije mogu se pripisati nedostatku argumentacijskih vještina, što je također potvrđeno u literaturi (Eraslan i Kant, 2015).

Tradicionalno, nastavne metode usmjerene na poboljšanje kompetencija u interpretaciji i validaciji temeljile su se na aktivnostima modeliranja. Međutim, brojna istraživanja pokazala su da takvi pristupi imaju ograničen učinak na razvoj ovih kompetencija (Biccards i Wessels, 2011; Çoksöyler, 2020; Ozulu i Sağırlı, 2021; Yurtsever, 2018). Stoga, umjesto primjene općih nastavnih metoda, učinkovitije je koristiti teorijske okvire koji izravno ciljaju specifične kompetencije. U tom smislu preporučuju se nastavna rješenja koja uključuju aktivnosti usmjerene na razvijanje vještina povezivanja i argumentacije kako bi se podržale kompetencije učenika u interpretaciji i validaciji. U skladu s tim, nastavnici matematike naglašavaju da interdisciplinarna i životno-orijentirana nastava poboljšava sposobnost učenika za rješavanje problema u svakodnevnim kontekstima (Burkhardt, 2014; English, 2016; Wang i sur., 2011). Okružja za učenje temeljena na aktivnom sudjelovanju i suradnji također doprinose razvoju metakognitivnih vještina učenika i pomažu im u prevladavanju poteškoća tijekom procesa modeliranja (Lesh i Doerr, 2003). Nadalje, učenje temeljeno na argumentaciji potiče razvoj kritičkoga mišljenja, podržava učinkovitu primjenu matematičkih konteksta i pruža mogućnosti za naprednije, kreativno i smisleno razmišljanje tijekom rješavanja problema (Reichersdorfer i sur., 2012; Zengin i Tapan-Broutin, 2023). Slično tome, Pirimoğlu i Gürel (2025) utvrdili su da aktivnosti matematičkoga modeliranja temeljene na argumentaciji pozitivno utječu na kompetencije u modeliranju i istraživačke vještine učenika.

Jedno od područja u kojem su učenici pokazali najslabije rezultate bila je matematizacija. Velik dio učenika nije uspio generirati valjane matematičke modele za rješavanje problema. Ovaj nalaz u skladu je s prethodnim istraživanjima koja ističu poteškoće učenika u konstruiranju matematičkih modela (Blum i Leiß, 2007; Krutikhina i sur., 2018). Glavni razlog ove poteškoće leži u nemogućnosti učenika da prepoznaju potrebne varijable i razviju odgovarajuće prikaze koji odražavaju odnose među tim varijablama. Ova su opažanja također u skladu s poteškoćama učenika u radu s višestrukim reprezentacijama (English, 2018; Ledezma i sur., 2022). Stoga nastavni pristupi koji potiču vještine reprezentacije mogu pomoći učenicima da učinkovitije izraze svoje ideje u matematičkom jeziku.

Studija je otkrila nekoliko poteškoća koje su spriječile učenike da postignu očekivanu razinu kompetencije. Te poteškoće mogu se sažeti kako slijedi:

- nemogućnost uspostavljanja veze između danih informacija i traženoga rezultata
- nemogućnost prepoznavanja varijabli potrebnih za rješavanje problema
- proizvodnja ograničenih i restriktivnih pretpostavki (npr. pretpostavka da je vozilo uvijek automobil, da su vozila postavljena bez razmaka i ignoriranje broja traka)
- nemogućnost izrade realističnih pretpostavki o vrijednostima varijabli
- nemogućnost izrade realističnih pretpostavki o odnosima među varijablama
- nemogućnost konstruiranja matematičkoga modela koji predstavlja odnose među varijablama
- nemogućnost formuliranja opće (prototip/model) reprezentacije
- nemogućnost povezivanja dobivenih rezultata sa stvarnim kontekstom
- nemogućnost prepoznavanja nedostataka u rješenju problema
- nemogućnost razvoja pristupa za rješavanje uočenih nedostataka.

Poteškoće s kojima su se učenici susretali tijekom procesa modeliranja također su zabilježene u prethodnim istraživanjima (Blum i Leiß, 2007; Eraslan i Kant, 2015; Tasarib, Rosli i Rambely, 2025). Međutim, u literaturi još nije predložena sustavna klasifikacija poteškoća koja obuhvaća cijeli proces modeliranja za učenike osnovnih škola. Većina istraživanja fokusirala se na učenike srednjih škola, na uske vrste problema ili na određene potkompetencije. Stoga se ovo istraživanje smatra originalnim doprinosom literaturi jer pruža detaljnu klasifikaciju poteškoća s kojima se učenici osnovnih škola susreću u procesu modeliranja. Buduća istraživanja mogla bi dodatno istražiti uzroke svake poteškoće.

U ovome istraživanju od učenika se očekivalo ne samo rješavanje problema iz stvarnoga života, već i razmatranje više mogućnosti te generiranje alternativnih pretpostavki. Međutim, većina učenika pokušala je riješiti problem oslanjajući se na jednu pretpostavku i određujući samo jednu vrijednost. Ova je tendencija ograničila njihovu kreativnost tijekom procesa modeliranja. Stoga se preporučuju aktivnosti u učionici koje potiču generiranje alternativnih pretpostavki i razvoj kreativnoga mišljenja.

Ovo istraživanje provedeno je s 50 učenika šestoga razreda u kontekstu problema mosta, utemeljeno na teorijskom okviru kompetencija u modeliranju i primjenjujući kvalitativni istraživački pristup. Nalazi se stoga odnose na uzorak, odabranu školu, situaciju problema i prihvaćeni teorijski okvir. Također, učenici su radili individualno tijekom rješavanja zadatka, što je moglo utjecati na rezultate; moguće je da bi grupni rad dao različite ishode. Slična istraživanja mogu se replicirati s različitim populacijama učenika, instrumentima prikupljanja podataka i istraživačkim dizajnima. Nadalje, istraživanje kognitivnih varijabli potencijalno povezanih s kompetencijama u modeliranju (npr. metakognitivna svijest, povezivanje matematičkih koncepata, strategije rješavanja problema) moglo bi pružiti vrijedne doprinose u ovome području.