Croatian Operational Research Review
CRORR x:x(202x), 1–15

Study of novel normalization technique on weighting and ranking methodology in multi criteria decision making: Several cases in financial performance

Zaky Pratiwi[1], **Zahedi**[1,*] **and Badai Charamsar Nusantara**[2]

[1] *Faculty of Mathematics and Natural Science, Universitas Sumatera Utara, Medan 20155, Indonesia*
E-mail: ⟨zakypratiwi12@gmail.com, zahedi@usu.ac.id⟩

[2] *Faculty of Engineering, Universitas Sumatera Utara, Medan 20155, Indonesia*
E-mail: ⟨badainusantara9@gmail.com⟩

Abstract. The evaluation of corporate financial performance typically entails the use of multiple financial ratios characterized by heterogeneous properties, including the potential presence of negative values. Within this context, Multi-Criteria Decision-Making (MCDM) approaches have been extensively employed due to their capacity to systematically integrate diverse evaluation criteria into a coherent and objective ranking framework. However, most conventional normalization techniques in MCDM are not specifically designed to accommodate negative data, thereby potentially resulting in distorted weights and inconsistent ranking outcomes. This study proposes a novel normalization approach, termed the Root Distance Ratio (RDR), which is derived from the Weitendorf principle and formulated through a non-linear root transformation. The effectiveness of the RDR method is evaluated by integrating it into four MCDM techniques PSI, ARAS, MABAC, and EDAS across three corporate financial data scenarios, namely datasets with entirely positive values, negative values within benefit criteria, and negative values within cost criteria. The results demonstrate that, across the evaluated cases, the RDR method produces normalized values within the $[0, 1]$ interval, generates more proportionally distributed weights, and yields more consistent ranking outcomes compared to conventional normalization techniques. These findings indicate that RDR enhances both the stability and interpretability of financial data-driven multi-criteria decision-making processes, particularly in the presence of negative values.

Keywords: criteria weight, financial performance, MCDM, normalization technique, root distance ratio

Received: May 5, 2025; accepted: March 30, 2026; available online: May 21, 2026

DOI: 10.17535/crorr.2026.0023

Original scientific paper.

1. Introduction

Financial performance data, typically represented by financial ratios including liquidity, solvency, activity, and profitability indicators [2, 16], often encompass both positive and negative values, reflecting varying degrees of financial health and risk exposure [6, 22]. These heterogeneous ratios constitute the empirical foundation of the present study's evaluation framework within the context of Multi-Criteria Decision Making (MCDM) context.

MCDM is widely employed to comprehensively evaluate corporate financial performance due to its capacity to integrate criteria characterized by differing scales, preference directions, and levels of importance [8]. MCDM process typically comprises three primary stages: normalization, weighting, and aggregation, which collectively produce structured and objective

*Corresponding author.

rankings that reflect the relative performance of alternatives [15]. Among these stages, normalization plays a pivotal role by transforming heterogeneous data into comparable scales [7]. However, most conventional normalization techniques are developed under the assumption of non-negative data [23], with outputs typically constrained within the $[0, 1]$ interval [10]. Empirical evidence further indicates that the choice of normalization method can substantially influence the resulting rankings. For example, [3], demonstrated that linear and vector normalization within the VIšekriterijumsko KOmpromisno Rangiranje (VIKOR) method produce divergent outcomes, thereby underscoring that normalization constitutes a critical determinant of overall MCDM validity rather than merely a preliminary processing step.

When datasets contain negative values, conventional normalization techniques often fail to preserve stability within the intended range, thereby generating distorted weights and irrational ranking outcomes. [24] reported that the application of normalization within the Preference Selection Index (PSI) method to datasets containing negative values produced outputs exceeding acceptable bounds, with a single criterion disproportionately dominating the weight distribution, thus biasing the resulting rankings. Such findings underscore the inherent vulnerability of MCDM frameworks when confronted with non-ideal or extreme data conditions.

Previous studies have proposed alternative normalization approaches to mitigate these effects. [23] identified Weitendorf normalization as the only technique capable maintaining stability under extreme data conditions, while [3], [9], and [21] confirmed that the choice of normalization method directly influences both weighting and ranking outcomes. Consistent with these findings, [19, 20] demonstrated that different normalization procedures can systematically alter preference structures and lead to inconsistent MCDM rankings, thereby reinforcing the critical importance of normalization stability. Furthermore, various hybrid or integrated MCDM approaches such as Best Worst Method (BWM)-Ranking Alternatives by Perimeter Similarity (RAPS) [1], entropy-fuzzy Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [11], Stepwise Weight Assessment Ratio Analysis (SWARA)-Weighted Aggregated Sum Product Assessment (WASPAS) [25], VIKOR [12], and PSI [24] have enhanced financial performance evaluation; however, they predominantly focus on datasets with positive values, thereby leaving a methodological gap in handling negative data.

Despite the development of numerous normalization techniques within the MCDM framework, substantial limitations persist, particularly when decision matrices contain negative values. Linear and vector-based methods are highly sensitive to scale disparities and sign variations, often resulting in biased weights and distorted ranking outcomes. Nonlinear and hybrid approaches, including those examined by [19, 20], may alter preference distributions or generate unstable value ranges when criteria differ significantly in magnitude. Furthermore, two-step procedures such as Alternative Ranking Order Method Accounting for Two-Step Normalization (AROMAN) [4], although designed to enhance scaling robustness, rely on underlying linear transformations that fail to ensure stability in the presence of negative or extreme financial ratios.

Although MCDM methods permit hybridization to mitigate bias, through the integration of PSI with Weitendorf normalization such combinations do not inherently ensure robustness. Stability in the normalization stage does not necessarily translate into consistent weighting and ranking outcomes. Accordingly, a clear research gap remains: many existing normalization techniques exhibit limited capability in handling negative values within decision matrices, particularly in preserving stability and proportionality throughout subsequent weighting and ranking processes. The primary motivation of this study is therefore to address this limitation by developing a normalization method that remains bounded, distributionally balanced, and robust under conditions involving negative data. To this end, this study proposes the Root Distance Ratio (RDR), a normalization technique designed to maintain values strictly within the $[0, 1]$ interval while improving distribution balance, thereby enhancing the rationality and consistency of MCDM results.

2. Failure of conventional MCDM methods on negative data

Four MCDM methods were employed to capture methodological diversity in multi-criteria evaluation: PSI [14], Additive Ratio Assessment (ARAS) [26], Multi-Attributive Border Approximation Area Comparison (MABAC) [18], and Evaluation based on Distance from Average Solution (EDAS) [5]. Each method is characterized by distinct computational mechanisms, rendering them suitable for assessing sensitivity to negative data in financial performance analysis. Their combined application facilitates a comprehensive comparison of how data irregularities influence normalization, weighting, and ranking outcomes.

2.1. Case scenario design based on real company data

This study employs 2023 financial data from publicly listed companies on the New York Stock Exchange (NYSE) and the National Association of Securities Dealers Automated Quotations (NASDAQ), representing three industry sectors. Based on these data, three analytical scenarios were constructed to capture varying financial conditions: Case X, comprising exclusively positive values; Case Y, incorporating a negative value in a benefit criterion (Return on Assets, ROA); and Case Z, involving a negative value in a cost criterion (Debt-to-Equity Ratio, DER). Table 1 presents the corresponding decision matrices, with negative values explicitly highlighted to denote anomalies.

Cases	**Alternative**	**Criteria**			
		C1 (CR)	**C2 (DER)**	**C3 (TATO)**	**C4 (ROA)**
X	X1	1.3545	0.9912	0.0010	0.2452
	X2	2.4942	0.6791	0.7618	0.1868
	X3	2.0630	0.6527	0.8297	0.1368
	X4	1.1094	0.4136	0.4637	0.3490
	X5	3.3149	0.4116	0.3980	0.0810
Y	Y1	1.0232	3.0506	0.8737	0.0475
	Y2	4.2760	0.1518	0.3107	0.0879
	Y3	1.2518	2.9604	0.3701	0.0172
	Y4	6.8233	0.1314	1.2536	0.2031
	Y5	0.8548	4.6928	0.0635	0.0074
	Y6	7.0275	0.1846	1.0032	0.1018
	Y7	1.8837	2.1803	0.3401	**-0.1035**
	Y8	2.4938	0.3683	0.0518	**-0.1883**
Z	Z1	2.8555	0.7762	0.7787	0.1665
	Z2	0.7467	**-2.0189**	0.0513	0.0051
	Z3	3.9872	0.2929	0.4342	0.1592
	Z4	3.0798	40.3118	0.0001	0.1053
	Z5	5.6900	0.2736	0.3965	0.1195
	Z6	1.2073	0.8386	0.4779	0.0360
	Z7	9.4312	0.1363	0.1660	0.0226

Table 1: *Decision matrix.*

As presented in Table 1, Case X represents a fully positive and stable evaluation condition, Case Y includes two companies exhibiting negative ROA values indicative of financial losses, and Case Z contains negative DER values reflecting potential financial distress or heightened bankruptcy risk. These three purposively selected cases illustrate varying financial conditions and enable assessment of whether conventional MCDM methods can yield rational and stable results under non-ideal data. Subsequent analyses focus on how negative values affect normalization, weighting, and ranking integrity.

2.2. Normalization using PSI, ARAS, MABAC, and EDAS

Normalization across all cases was conducted in accordance with the embedded procedures of each method PSI, ARAS, MABAC, and EDAS. Table 2 summarizes the resulting normalization ranges. In Case X, all methods produced values within the expected $[0, 1]$ interval [10], indicating stable performance under conditions involving exclusively positive data. Notably, MABAC, which incorporates the Weitendorf normalization principle, maintained consistent and logically coherent results. However, in Cases Y and Z, methods relying on linear normalization such as PSI, ARAS, and EDAS exhibited notable distortions. In PSI, negative ROA values in Case Y produced normalized outputs below zero ($Y_{74} = -0.5095$, $Y_{84} = -0.9273$), the cost criterion (DER) generated extreme values, whereby an alternative expected to perform best (Z72 = 0.1363) was instead ranked as the worst (-14.807). ARAS demonstrated similar instability, as negative denominators led to disproportionately scaled outcomes, and EDAS likewise failed to adequately accommodate extreme negative values, resulting in biased preference scores. Overall, the presence of even a single negative value proved sufficient to distort the entire normalization process across these methods.

Cases	**Methods**			
	PSI	**ARAS**	**MABAC**	**EDAS**
X	$0 \le \bar{X}_{ij} \le 1$	$0 \le \bar{X}_{ij} \le 1$	$0 \le \bar{X}_{ij} \le 1$	$0 \le \bar{X}_{ij} \le 1$
Y	$-1 \le \bar{X}_{ij} \le 1$	$-1 \le \bar{X}_{ij} \le 1$	$0 \le \bar{X}_{ij} \le 1$	$0 \le \bar{X}_{ij} \le 2$
Z	$-14 \le \bar{X}_{ij} \le 1$	$-1 \le \bar{X}_{ij} \le 1$	$0 \le \bar{X}_{ij} \le 1$	$-1 \le \bar{X}_{ij} \le 2$

Table 2: *Normalization value range*

2.3. Impact of negative values on weight calculation

Criterion weights were primarily determined using the PSI method due to its simplicity, objectivity, and ability to generate weights directly from normalized data. Table 3 summarizes the resulting weight distributions. Case X produced proportional and logical weights, whereas Cases Y and Z showed several negative and extreme values (for example, $w_{C4} = 0.9984$ in Case Y and $w_{C2} = 1.0040$ in Case Z). These distortions indicate that negative data cause instability in the weighting process, leading certain criteria to dominate or even contribute negatively to final rankings. Additional analyses employing Entropy and the Method based on the Removal Effects of Criteria (MEREC) weighting approaches further corroborated this issue, as both methods rely on logarithmic transformations that are undefined for zero or negative values. Consequently, conventional weighting techniques demonstrate limited robustness when applied to datasets containing negative financial ratios.

Cases	**Weights (w_j)**			
	C1	**C2**	**C3**	**C4**
Case X	0.2895	0.2978	0.1482	0.2645
Case Y	-0.0553	0.1192	-0.0623	0.9984
Case Z	-0.0023	1.0040	-0.0014	-0.0003

Table 3: *Weight values using conventional normalization.*

2.4. Ranking evaluation

Ranking results obtained from PSI, ARAS, MABAC, and EDAS are summarized in Table 4. In Case X, all methods produced identical rankings, indicating that the normalization, weighting, and aggregation procedures operated consistently and proportionally.

Cases	Alternative	Rank MCDM				Spearman Rank Correlation					
		I	II	III	IV	I vs II	I vs III	I vs IV	II vs III	II vs IV	III vs IV
X	X1	5	5	5	5	1	0.9	1	0.9	1	0.9
	X2	3	3	3	3						
	X3	4	4	4	4						
	X4	1	1	2	1						
	X5	2	2	1	2						
Y	Y1	4	4	4	4	0.976	1	0.976	0.976	1	0.976
	Y2	2	3	2	3						
	Y3	5	5	5	5						
	Y4	1	1	1	1						
	Y5	6	6	6	6						
	Y6	3	2	3	2						
	Y7	7	7	7	7						
	Y8	8	8	8	8						
Z	Z1	4	4	5	5	1	-0.25	-0.25	-0.25	-0.25	1
	Z2	1	1	1	1						
	Z3	5	5	4	4						
	Z4	2	2	7	7						
	Z5	6	6	3	3						
	Z6	3	3	6	6						
	Z7	7	7	2	2						

Table 4: *Alternative spearman rank correlation (I) PSI (II) ARAS (III) MABAC (IV) EDAS.*

However, Cases Y and Z revealed clear ranking irrationalities. In Case Y, inconsistencies emerged among mid-ranked alternatives (e.g., Y_2 and Y_6) due to the extreme dominance of criterion C_4 (ROA), whose weight reached 0.9983, causing all methods to rely almost entirely on ROA values. In Case Z, alternative Z_2 was ranked highest across all methods despite its negative DER (C_2), a value typically associated with bankruptcy risk. This anomaly resulted from an inflated weight (> 1) assigned to C_2, demonstrating that conventional normalization leads to distorted weights and rankings when negative data are present.

Extending this analysis to two-step normalization methods reveals that procedures such as AROMAN [4] likewise fail to rectify the ranking distortions induced by negative entries. Although AROMAN integrates a vector-based transformation with a min–max normalization, the vector component remains negative whenever the original criterion values are negative. These negative values propagate to the aggregation and weighting stages, causing several elements of the weighted decision matrix to remain negative. This becomes problematic in the ranking phase, because the computation of the benefit and cost utility terms requires exponentiation of aggregated values; if these values are negative, the resulting expressions become undefined or non-real. Conceptually, this limitation implies that AROMAN can only yield valid and comparable rankings when all normalized inputs entering its final stages are strictly positive, a condition satisfied under the proposed RDR normalization but not under AROMAN's internal steps when negative financial ratios are present. In essence, AROMAN becomes mathematically infeasible when negative normalized inputs appear, because its final utility stage requires raising aggregated values to fractional exponents. Any negative value entering this step produces a non-real output, making the ranking computation undefined and preventing the method from generating a valid ordering of alternatives.

2.5. Root cause of irrational weights and rankings

As discussed in Sections 2.2 and 2.3, the irrational weights and ranking outcomes observed in Cases Y and Z stem from the inability of conventional normalization techniques to adequately accommodate negative data. In the PSI method, excessive preference variance led to negative deviation values, as evidenced in Case Y, where the ROA variance reached 2.531 (deviation -1.531) and in Case Z where the DER variance increased to 177.898 (deviation -176.898). Under theoretically consistent conditions, preference variance should remain within the $[0, 1]$ interval to ensure the derivation of logical and interpretable weights.

These inflated variances arise from substantial discrepancies between average performance values and normalized results when values fall outside the ideal range, leading to uncontrolled dispersion and distorted weight distributions. Consequently, such distortions propagate from mean scores to variances, deviations, and ultimately to the final rankings. Therefore, there is a clear need for a normalization method that preserves valid value ranges and ensures stability, even in the presence of negative data.

3. Evaluation of existing normalization techniques

3.1. Comparison of all normalization methods

A comparative analysis was conducted on nine conventional normalization techniques summarized by [23], namely linear (N_1), Weitendorf (N_2), vector (N_3), linear sum (N_4), logarithmic (N_5), maximum linear (N_6), minimum linear (N_7), Jüttler-Körth (N_8), Peldschus (N_9) and AROMAN - Based Double Normalization (N_{10}).

Figure 1 illustrates that in Case X all methods (N_1–N_9) produced normalized values within the valid range $[0, 1]$, thereby demonstrating conformity with fundamental normalization principles. In these figures, the Lower Bound (LB) is defined as 0 and the Upper Bound (UB) as 1, collectively establishing the permissible normalization interval. Figure 2 shows that in Case Y a single negative ROA value (Y_{74} and Y_{84}) leads to instability across several normalization methods. Extreme results are observed in N_7 (26.285) and N_8 (-0.9272), while N_5 fails to compute outputs due to logarithmic limitations. Similar effects are also observed in N_1, N_3, N_4, and N_6.

In method N10, although two normalization stages are applied, alternatives containing negative values in the decision matrix still produce negative final normalized results (e.g., $Y_{74} = -0.024$ and $Y_{84} = -0.143$). This phenomenon arises from the aggregation mechanism embedded in the double-normalization procedure, which combines vector normalization $\frac{x_{ij}}{\sqrt{\sum x_{ij}^2}}$ and Weitendorf normalization $\frac{x_{ij}-x_{\min}}{x_{\max}-x_{\min}}$. The vector component preserves the sign of negative values, since dividing a negative value by $\sqrt{\sum x_{ij}^2}$ does not eliminate its sign, while the Weitendorf component maps values into the non-negative range. As a result, the negative influence from the vector normalization propagates through the aggregation process.

Because the aggregation parameter β is set to 0.5 to balance the contributions of both normalization techniques, the negative influence originating from the vector normalization component continues to affect the final result. Consequently, whenever negative values are present in the decision matrix, the double-normalization procedure generally yields negative normalized outcomes. Theoretically, a strictly non-negative result would only be attainable if the contribution of the vector component were eliminated or made negligible (i.e., $\beta \to 0$), effectively reducing the procedure to pure Weitendorf normalization. With $\beta = 0.5$, it is therefore highly unlikely for the double-normalization method to produce strictly positive normalized values when alternatives contain negative entries.

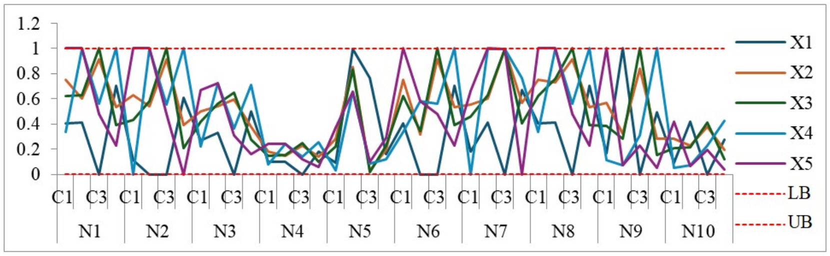


Figure 1: *The value of all normalization techniques for case X.*

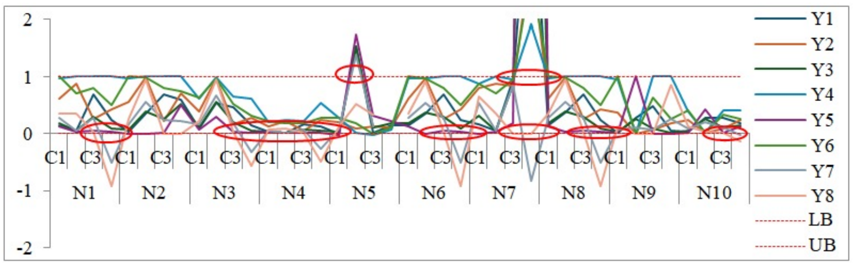


Figure 2: *The value of all normalization techniques for case Y.*

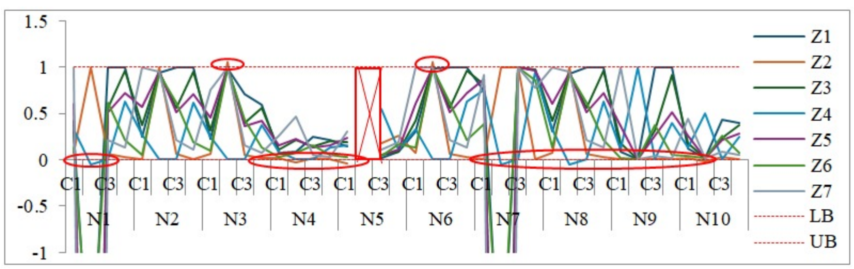


Figure 3: *The value of all normalization techniques for case Z.*

Figure 3 depicts similar instability in Case Z, where negative DER values affect multiple normalization methods. N_5 again fails to process the input values, while extreme values are observed in N_1 (-14.807 for Z_{72}) and N_6 (1.050 for Z_{22}). Other methods, such as N_3, N_4, and N_9, also exhibit distorted results. Using the same analytical reasoning as in Case Y, alternative Z_{22} also produced a negative value of -0.012 under the double-normalization method (N_{10}). This outcome occurs because the normalization technique does not explicitly distinguish between benefit and cost characteristics, thus reproducing the same distortion phenomenon observed previously. These distortions in normalized values subsequently give rise to irrational weighting outcomes and biased rankings. Among the evaluated techniques, only Weitendorf normalization (N_2) consistently maintained results within the $[0, 1]$ interval across Cases X, Y, and Z, thus confirming its stability and reliability for subsequent weighting and ranking analyses.

3.2. Application of Weitendorf as a partial solution

Weitendorf normalization was employed as a partial remedy to address the limitations of conventional linear normalization by integrating it into the PSI algorithm. The resulting criterion weights are presented in Table 5 and subsequently utilized for the ranking analysis.

Cases	**Weights (w_j)**			
	C1	**C2**	**C3**	**C4**
Case X	0.2425	0.2231	0.2518	0.2826
Case Y	-1.6402	-0.3046	0.1386	2.8061
Case Z	0.3933	0.2898	0.3330	-0.0161

Table 5: *Weight values using Weitendorf normalization.*

Despite all normalized values falling within the valid range, Table 5 indicates that Cases Y and Z still produced disproportionate weights, including negative and extreme values (e.g., -1.6402 and 2.8061). This anomaly arises from low average performance values that constrain deviation magnitudes, thereby impeding the generation of logical and proportionate weights. Thus, the issue lies not only in maintaining valid normalization ranges but also in ensuring a balanced value distribution. prevents mathematical inconsistencies, it remains insufficient in stabilizing weights or adequately improving value dispersion. Consequently, a more comprehensive approach is required one that simultaneously preserves valid normalization ranges and enhances distributional balance. To address this limitation, the present study introduces the RDR normalization technique as a more robust solution for generating consistent and interpretable weights, particularly under conditions involving negative data.

4. Proposed normalization technique: root distance ratio (RDR)

4.1. Motivation and theoretical basis

The irrationality of weight values in conventional normalization techniques, including Weitendorf, originates from low average performance values (N_j) produced during normalization. Alternatively, power transformations with positive integer exponents (x^p, $p > 1$) further reduce the magnitude of normalized values, resulting in $x^p < x$ for $0 < x < 1$, thereby exacerbating the problem of low average performance values. In contrast, root-based transformations ($x^{1/p}$) increase the magnitude of values within $(0, 1)$, making them more suitable for enhancing N_j. However, excessively high-order roots tend to compress the relative differences among alternatives, as all values tend to approach 1 (formally, $x^{1/p} \to 1$ as $p \to \infty$ for $0 < x \leq 1$), reducing the discriminative ability of the normalization.

Therefore, the square-root transformation ($p = 2$) is adopted as a balanced adjustment, as it increases normalized values while preserving sufficient differentiation among alternatives. Based on this reasoning, the Root Distance Ratio (RDR) normalization is defined as follows:

$$\bar{X}_{ij} = \begin{cases} \dfrac{\sqrt{x_{ij} - x_{\min}}}{\sqrt{x_{\max} - x_{\min}}}, & \text{for benefit criteria,} \\ \dfrac{\sqrt{x_{\max} - x_{ij}}}{\sqrt{x_{\max} - x_{\min}}}, & \text{for cost criteria.} \end{cases} \tag{1}$$

where x_{ij} represents the value of the i-th alternative on the j-th criterion, $x_{\max}$ denotes the maximum value for a given criterion, and $x_{\min}$ denotes the minimum value.

4.2. Distributional comparison: RDR vs Weitendorf

A comparative evaluation between the Weitendorf and RDR normalization techniques was conducted to examine RDR's adaptation of Weitendorf's principles and its capacity to enhance normalized values. Figures 4(a)-(c) show that Weitendorf results (red points) align closely with the linear reference line $y = x$, reflecting its linear transformation that maps negative values into the $[0, 1]$ range. In contrast, RDR results (green points) form a convex non-linear pattern above the line, illustrating the effect of the square root transformation on distance ratios. These visual and quantitative observations confirm that while both techniques maintain valid normalization ranges, RDR achieves a more balanced and expanded value distribution, promoting stability and rationality in the subsequent weighting process.

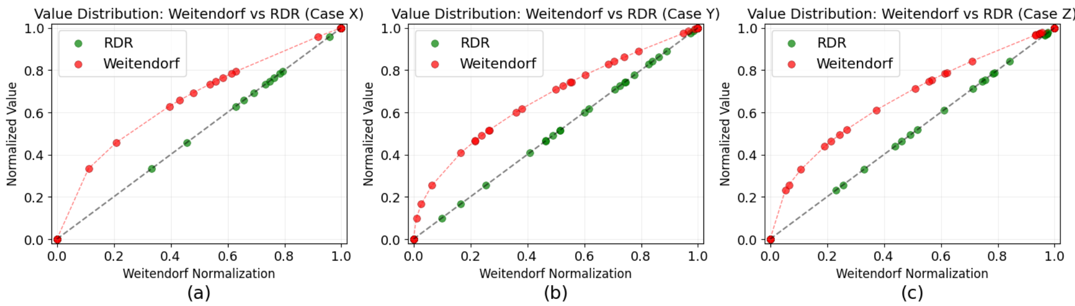


Figure 4: *Weitendorf vs RDR on (a) Case X, (b) Case Y, and (c) Case Z.*

4.3. PSI weighting integration using RDR

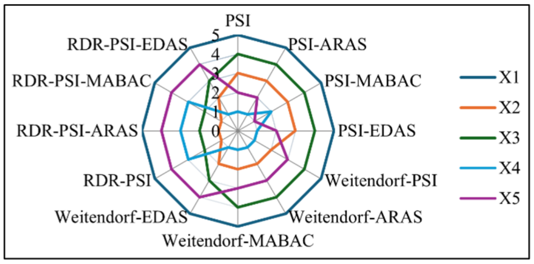
RDR normalization technique was implemented as a replacement for conventional normalization within the MCDM framework, applied to both weighting and ranking stages. In the PSI method, RDR substituted the original normalization component, producing the criterion weights shown in Table 6. As indicated in Table 6, all criteria across the three cases yielded proportional weights within the $[0, 1]$ interval. This improvement results from RDR's ability to elevate average performance values, thereby reducing deviation magnitudes and generating balanced, logically consistent weights.

Cases	**Weights (w_j)**			
	C1	**C2**	**C3**	**C4**
Case X	0.2544	0.2193	0.2377	0.2886
Case Y	0.0431	0.2747	0.1481	0.5341
Case Z	0.3634	0.1903	0.2924	0.1538

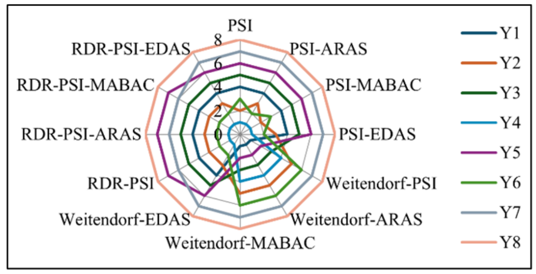
Table 6: *Weight values using RDR normalization.*

4.4. Substitution of RDR normalization in PSI, ARAS, MABAC, and EDAS rankings

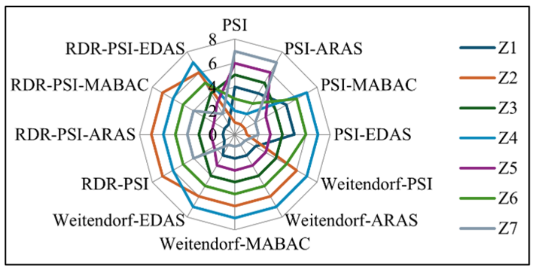
The evaluation of ranking consistency across the conventional, Weitendorf, and RDR approaches reveals clear differences in pattern. Without normalization substitution, radar charts for Cases X, Y, and Z (Figure 5) displayed irregular polygon shapes and large fluctuations among alternatives, indicating that rankings were highly dependent on the specific MCDM method. The use of Weitendorf normalization improved distribution balance, though deviations persisted in cases with negative values. In contrast, applying RDR normalization produced markedly stronger consistency, as reflected in the more symmetrical radar chart shapes in Figure 5. Rankings across PSI, ARAS, MABAC, and EDAS became closely aligned, showing that RDR enhances both value distribution and decision stability.



(a)



(b)



(c)

Figure 5: *Ranking results across methods for (a) Case X, (b) Case Y, and (c) Case Z.*

Cases	**Spearman Rank Correlation**					
	I vs II	**I vs III**	**I vs IV**	**II vs III**	**II vs IV**	**III vs IV**
X	1	1	0.7	1	0.7	0.7
Y	1	1	0.976	1	0.976	0.976
Z	1	1	0.857	1	0.857	0.857

Table 7: *Spearman rank correlation after the application of RDR (I) PSI (II) ARAS (III) MABAC (IV) EDAS.*

Spearman rank correlation results (Table 7) further confirm these findings. Correlations among methods increased substantially, indicating that RDR-based normalization yields more coherent and rational rankings than conventional approaches. The slight decrease observed in EDAS correlations does not suggest inconsistency but rather a more realistic alignment with the data's intrinsic structure, underscoring the robustness of the RDR normalization.

5. Evaluation using other datasets

5.1. Normalization simulation using equivalent interval ratios

In this section, a simulation is conducted using two datasets that possess identical interval ratios but differ in numerical ranges. Dataset A spans the interval 1–9 with a unit increment of 1 for nine alternatives (A_1–A_9), whereas Dataset B spans the interval 1–5 with an increment of 0.5 for the same nine alternatives (A_1–A_9), applied to both benefit ($*$) and cost ($**$) criteria.

This simulation evaluates the ability of normalization techniques to generate consistent outcomes when applied to datasets that have equivalent structural capacity yet different numerical scales. The analysis employs five normalization techniques: Root Distance Ratio (N_1), Linear (N_2), Vector (N_3), Sum-Linear (N_4), and AROMAN-based Double Normalization (N_5).

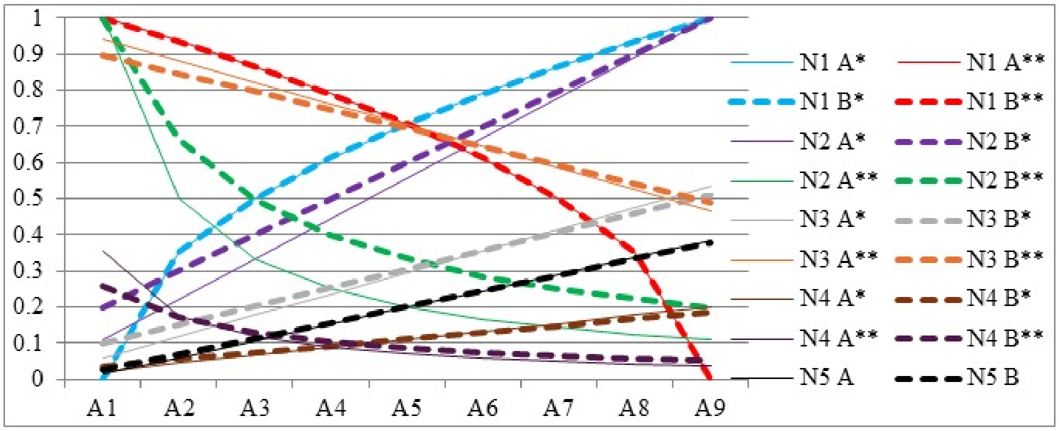


Figure 6: *Comparison of normalization results using equivalent interval ratios.*

Overall, all normalization methods employed in this simulation produced values within the $[0, 1]$ interval. This outcome is expected, as all raw data in both datasets are strictly positive. However, as illustrated in Figure 6, among the evaluated methods, N1 (RDR) demonstrates the closest overlap between Dataset A and Dataset B for both benefit and cost criteria.

This finding indicates that RDR is capable of producing highly consistent normalized values for datasets with equivalent interval structures, even when their numerical scales differ.

Although N_4 (Sum-Linear) and N_5 (AROMAN) yield curves that appear nearly overlapping, the resulting normalized values are not exactly identical, even though both datasets possess the same interval structure. Meanwhile, N_2 (Linear) and N_3 (Vector) exhibit substantially different normalization results, as reflected by the noticeable separation between the curves across the two datasets. Furthermore, the cost-type normalization under N_2 (Linear) fails to produce a truly linear pattern, revealing additional instability in the scale transformation process.

5.2. Weighting and ranking simulation based on case scenarios

Two simulated datasets, representing engineering and economic domains, were analyzed to compare the performance of linear and RDR normalization. Both datasets replicated real-world characteristics, including negative values in cost-type criteria (C_1 in Dataset 1 and C_4 in Dataset 2).

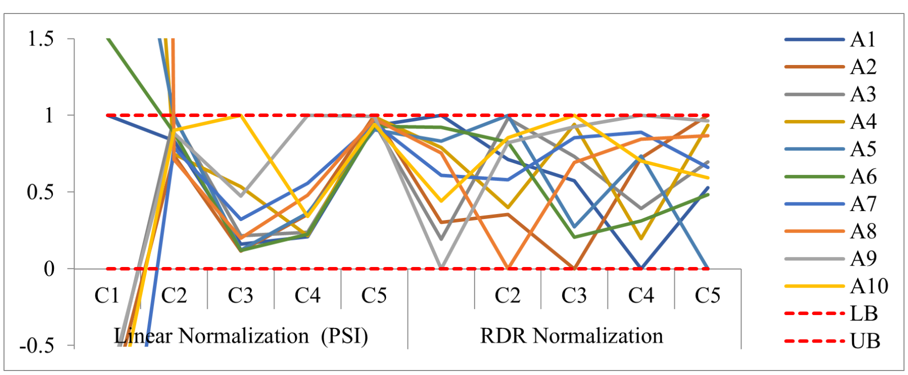


Figure 7: *Comparison of normalization value ranges in the first simulation.*

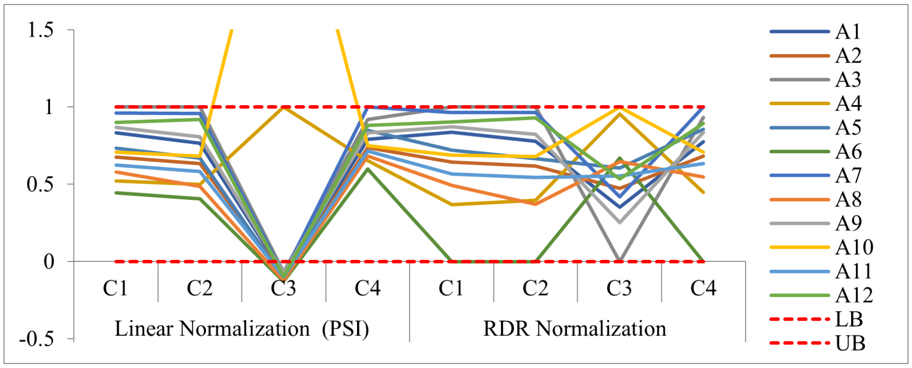
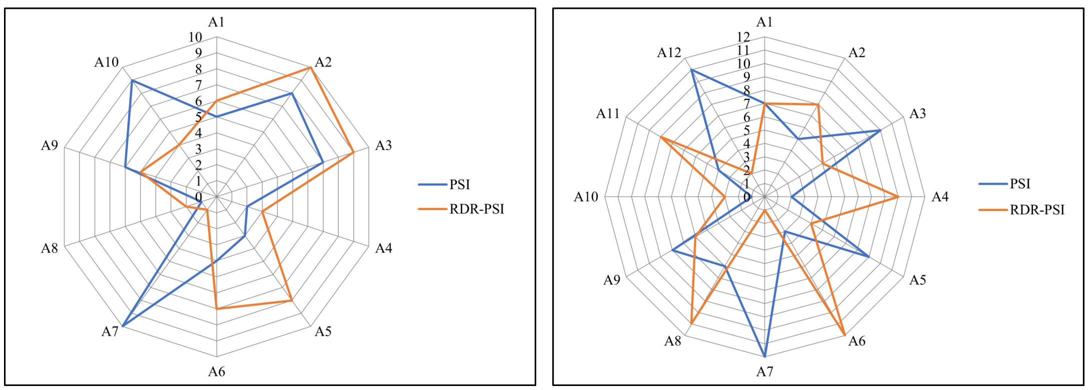


Figure 8: *Comparison of normalization value ranges in the second simulation.*



(a) (b)

Figure 9: *Ranking in the (a) first and (b) second simulation.*

Figures 7-9 illustrate the comparative outcomes across normalization, weighting, and ranking stages. Under linear normalization, several values exceeded the $[0, 1]$ range, producing distortions in both datasets. In Dataset 1, alternative A_7 under criterion C_1 yielded a negative normalized value despite representing the lowest-cost option, while in Dataset 2 the presence of negative values in criterion C_4 destabilized the entire normalization matrix. In contrast, the RDR approach maintained all values strictly within the $[0, 1]$ interval, thereby preserving proportional relationships among criteria and ensuring numerical stability.

The weighting results (Table 8) further underscore this improvement. Linear normalization produced inconsistent weights, including negative values and coefficients exceeding unity, thereby leading to unreliable aggregation. In contrast, RDR normalization generated balanced and theoretically consistent weights, all confined within the $[0, 1]$ interval, thus confirming greater accuracy and internal consistency within the PSI framework.

Data	Types of Normalization	C1	C2	C3	C4	C5
1	Linear Normalization (PSI)	1.00123	-0.00045	-0.00038	0.00041	-0.00049
	RDR Normalization	0.41079	0.18233	0.17075	0.23613	0.32588
2	Linear Normalization (PSI)	-0.12229	-0.11018	1.39137	-0.15891	
	RDR Normalization	0.18407	0.14974	0.30097	0.36523	

Table 8: *The criterion weight result.*

Ranking analyses (Figure 9) reinforce the same pattern. Linear PSI exhibited extreme preference disparities, such as the dominance of A_7 and the suppression of A_6 in Dataset 1, and the over-ranking of A_7 and A_{12} in Dataset 2. The RDR-PSI integration yielded smoother, more uniform radar profiles, where alternatives occupied logical positions with no unrealistic dominance.

Overall, distortions arising from normalization and weighting stages in the conventional PSI method were clearly propagated to the final ranking outcomes. The substituting of RDR normalization effectively mitigated these issues, resulting in coherent aggregate scores and enhanced decision reliability across both domains. Consequently, the RDR-PSI model demonstrates superior robustness and interpretability when applied datasets containing negative cost-type criteria.

6. Conclusion

This study investigated the limitations of conventional normalization techniques in Multi-Criteria Decision Making (MCDM) when applied to datasets containing negative values and proposed the Root Distance Ratio (RDR) as an alternative approach. The analysis focused on normalization stability, weight proportionality, and ranking consistency utilizing both real financial data (NYSE and NASDAQ) and simulated datasets.

The results indicate that linear normalization techniques are inadequate for handling negative values, leading to distorted weights and inconsistent ranking outcomes. In contrast, the RDR method maintained normalized values within the $[0, 1]$ interval, generated proportionate weight distributions, and produced generally consistent ranking patterns across the evaluated cases. These improvements were consistently observed across multiple MCDM methods (PSI, ARAS, MABAC, and EDAS) and further validated through rank correlation analysis and radar-based visualizations.

However, in specific scenarios involving negative cost-type criteria, such as the Debt-to-Equity Ratio (DER), minor variations in ranking positions may still occur, despite the stability of the normalization process. This suggests opportunities for further refinement, rather than indicating fundamental limitations of the proposed approach. Future research should therefore focus on developing an enhanced normalization framework capable of preserving both mathematical validity and semantic preference consistency, particularly under extreme or outlier conditions.

Acknowledgements

The authors would like to acknowledge support from Namira Rambe for her insightful discussions and analytical contributions, and from Nasya Nabila for her kind moral encouragement during the research and manuscript preparation.

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