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A new approach in time series analysis of psychophysiological data

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The research on chaos in biological systems has attracted a lot of interest recently. Some of such analyses indicated that time series of heart inter beat intervals may contain "hidden information" which is neither visually apparent, nor extractable by conventional methods of analysis. Previous correlational and spectral analysis of heart dynamics has identified some of internal and external effects on the normal cardiac activity, but phase interactions between the different frequency components of the processes stayed hidden. Recent studies showed that mathematical techniques based on chaos theory and the parameters they yield, may differentiate various mental load levels, as well as the stress effects, which may be reflected *via* specific chaotic dynamics of biological subsystems. In this paper, some of the mathematical tools, which are used to analyse chaotic behaviour in various biological systems, is presented (attractor phase portrait, correlation dimension and the largest Lypunov exponent).

The time series analysis is an appropriate technique for the analysis of continuous psychophysiological variables, which may appear in time as different types of signal such as steady states, linear oscillations, non-linear oscillation or noise.

In the case of linear oscillations, existing mathematical tools (cosinor analysis, autocorrelations, spectral analysis) are appropriate for description and explanation of such rhythms. In the case of more complex rhythms, however, with tendency of non-linear dynamics, these are not adequate. For example, measures of heart rate variability (HRV) as indicators of mental load are more likely to be coarse-grained measures, while spectral analysis of HRV shows broadband frequency characteristic, pointing towards non-stationarity or non-linearity (Sammer, 1998). One important advantage of non-linear methods is that the measures give information, not only about the structure of the empirical time series, but also about underlying system itself. Besides, the assumption of non-linearity is considered to be more realistic approach to the investigations of biological systems (Elbert et al., 1994).

The idea of existence of "chaotic" activities in biological systems, and the central nervous system in particular, is not a new one. This approach for understanding complexity in nature has its roots in the work of Newton, Rayleigh, and Poincare, but it is only recently that the procedures and concepts were developed to a point where they are beginning to have an important impact on a wide variety of fields including physiology and psychophysiology. This approach has significantly modified the manner in which physiological processes are viewed and described. For example, some processes formerly perceived as erratic, or random, are now viewed in terms of patterns and potential lawful relationships. Even the term chaos, itself, has changed in meaning. Previously the term suggested randomness, but it now connotates the idea of underlying structure and the potential for describing a complex system with the aid of relatively simple mathematical formulations. Chaos has been formally defined as "stochastic behaviour in a deterministic system", i.e. a system which displays apparent random behaviour, but has an underlying pattern of lawfulness.

In the past 10 years, there has been an increasing interest in the field of non-linear dynamics, or chaos, and it's application to psychophysiology. This approach my help in answering some questions about nature and dynamics of different biological systems.

Attractor Phase-Portrait

Research in psychophysiology often involves the interpretation of signals reflected in time series that are irregular. One source of problems is our ability to visually recognize patterns within these irregularities, which have been proven as impossible to systematically detect by use of ordinary statistical techniques. The result was static rather than dynamic view of behaviour. Dynamic view suggests that a time series may be seen as to reflect the effects of all

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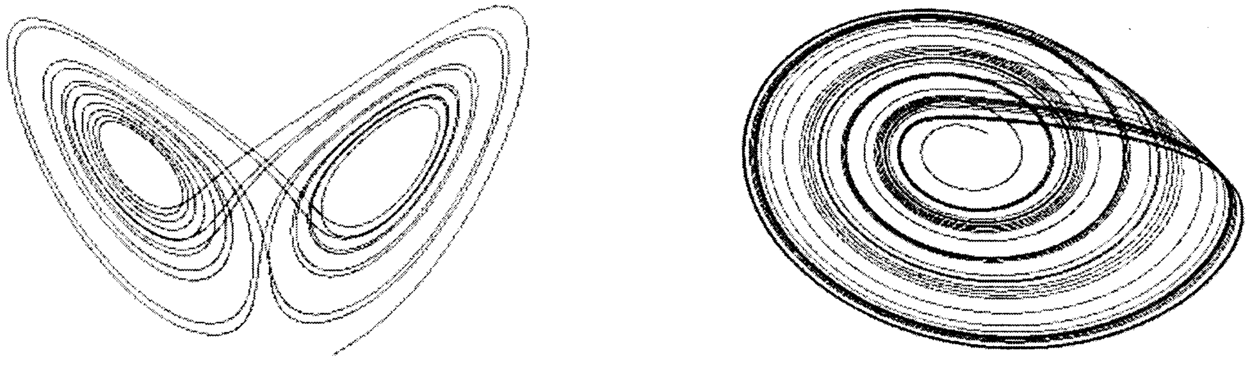

fig 1a Lorenz attractor

fig 1b Rossler attractor

Figure 1. Most often attractors generated by coupled process equations

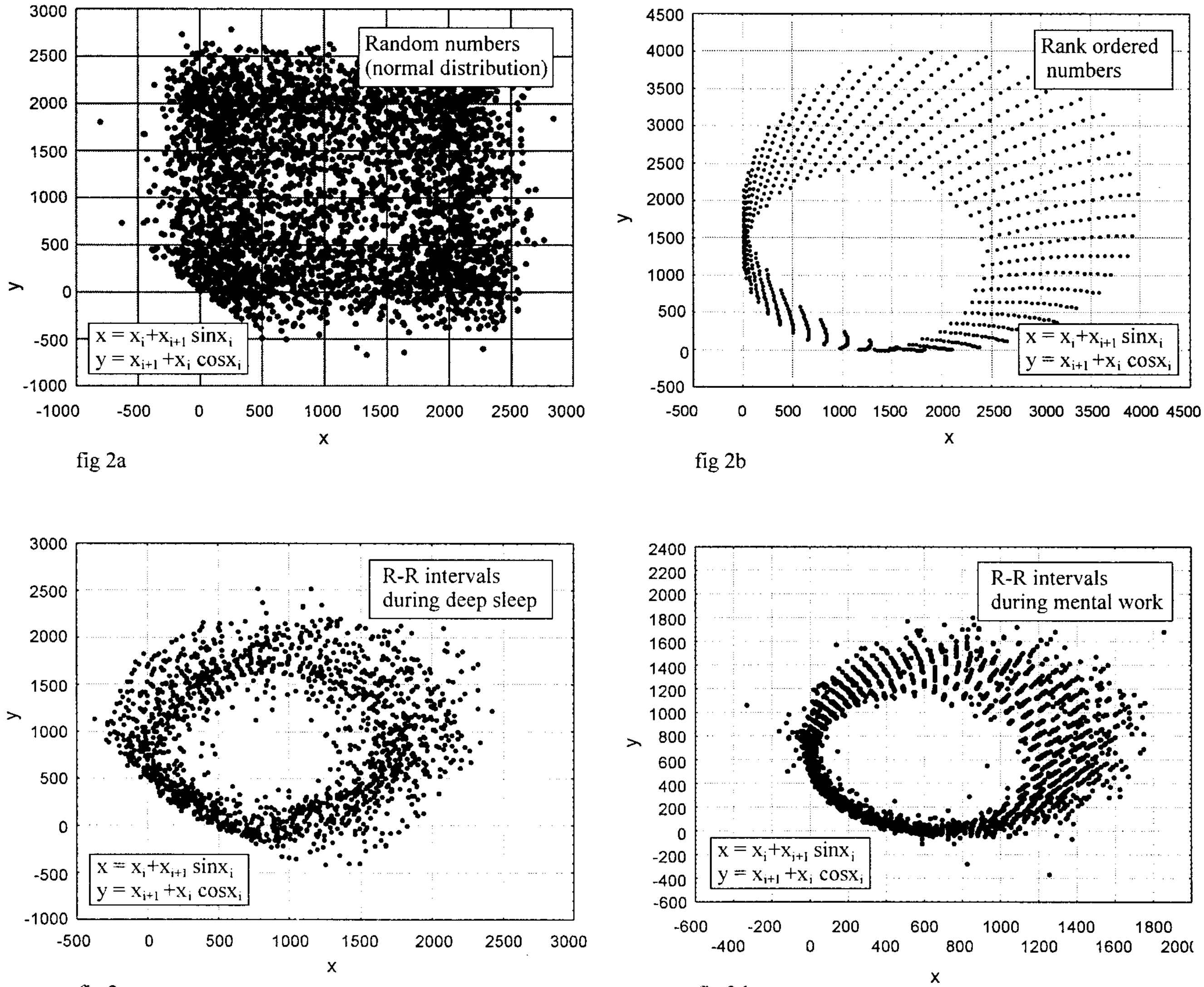

fig 2a

fig 2b

fig 2c

fig 2d

Figure 2. Attractors of computer generated data and empirical data (inter beat intervals)

other variables participating in the dynamics of the system. One breakthrough in this area was possibility to project the dynamics of a system onto a phase-space diagram. The phase-space of a dynamic system is a mathematical space with orthogonal coordinate directions representing each of the variables needed to specify the instantaneous state of the system (Baker & Gollub, 1994). This is particularly attractive to many investigators in relatively long time measurements, because it is very simple. Time series of a measured variable X_t is sampled at a time interval producing the discrete data set. One then forms n-duplets (triplets..) of points X_t, X_{t+lag}.. and plots of the trajectory, traced out by these n-dimensional phase–space diagram. This method of reconstructing a phase-space trajectory is called "the method of time delays" or "embedding". A reasonable choice of the delay gains importance through the fact that we always have to deal with a finite amount of "noisy" data. Depending on the type of structure we want to explore we have to choose a suitable time delay (by autocorrelation function, mutual information, false neighbours statistic etc.), but the proper choice of the delay and embedding dimension cannot be established except in the context of a specific application (Hegger et al, 1999).

Examination of the phase – portrait sometimes reveals a banded structure that is reminiscent of "strange attractors" seen in numerical simulations of mathematical systems that are commonly accepted as being chaotic. For example, EEG and ECG signals have underlying patterns, which may be generated by coupled process equations (Sabelli, 2001): $A_{t+1}= A_t+B_t$ inA_t and $B_{t+1}= B_t+A_t$ cosA_t and often formed Rossler, Lorenz or Henon attractors.

The attractors which are generated by process equations on random numbers, rank ordered numbers and empirical data (heart inter beat intervals) are presented on figures 2 a to 2 d.

In psychophysiology, measuring and quantifying complexity of dynamical system presented as a strange attractor, includes primarily measures of complexity and measures of predictability.

Measures of complexity

Fractal dimension is a term related to fractal objects, which are known as self-similar mathematical structures produced by simple repetitive mathematical operations. The main characteristic of a fractal is its dimension, which is different from the Euclidian and can be fractional or real numbers. Some of the well-known fractals are Cantor set, Koch snowflake, Mandelbrot set etc. The determination of some form of fractal dimension is probably the most commonly used basis on which claims of chaotic dynamics have been made in biological systems. If the irregular waveform produces attractor phase-portrait, we can calculate the dimension of this object using one of several different algorithms. Another context in which the term "fractal dimension" appears in literature is when one makes a measurement of the fractal dimension of the waveform itself, treating it as a fractal curve. Thus, by one measure, a straight line has a dimension of 1.0, while a very wiggly curve will have the dimension close to 1,5. This method is also known as "length of coastline" analysis (Guevara et al., 1988).

Some of the most important fractal dimensions for practical applications are: Huseldorff-Besicovitch dimension, Correlational dimension and Hurst dimension. In the field of psychophysiology, some recent studies report about differences in dimensional complexity (DC_x), where variables, such as cognitive task difficulty or cortical arousal, are manipulated to test quantitative hypotheses regarding brain-state changes (Watters, 1999). Various investigations reported that increased cognitive effort would be reflected in an increase in dimensional complexity of EEG signals (Murata & Iwase, 2001, Dhamala at al, 2002, Watters, 1999). The experiments show a direct relationship between complexity and the difficulty of the task. It seems, therefore, that higher levels of mental load recruit a larger number of independent neural processes, that contribute to the complex brain dynamics. Dhamala et al. (2002) suggest the possibility of relative change in correlation dimension as a useful global measure of brain dynamics, e.g., in determining the levels of mental activity, even if little is known about the underlying neurological processes. Similarly, non-linear dynamics has been used to characterize cardiac activity in terms of the DC_x of the signal. The results of many studies indicated that the complexity of heart dynamics is also related to the type of the task, and that the predictability of heart dynamics is also related to the amount of mental load (Sammer, 1998). Moreover, in extreme situations, a reduction in the complexity of cardiac dynamics may immediately precede coronary attack (Goldberger & Rigney, 1999). The most popular attempts to characterize attractors and dimensional complexity have been based on the correlation dimension as proposed by Grassberger and Procaccia (1983), where DC_x is typically computed as the slope of the correlation integral from a reconstructed state-space. One way to view this term is to consider a dynamical system, which would result in a circular or elliptic limit attractor with dimension of one. In addition, the power spectral density function could be computed. The spectrum will be broadband for chaotic data, as well as for

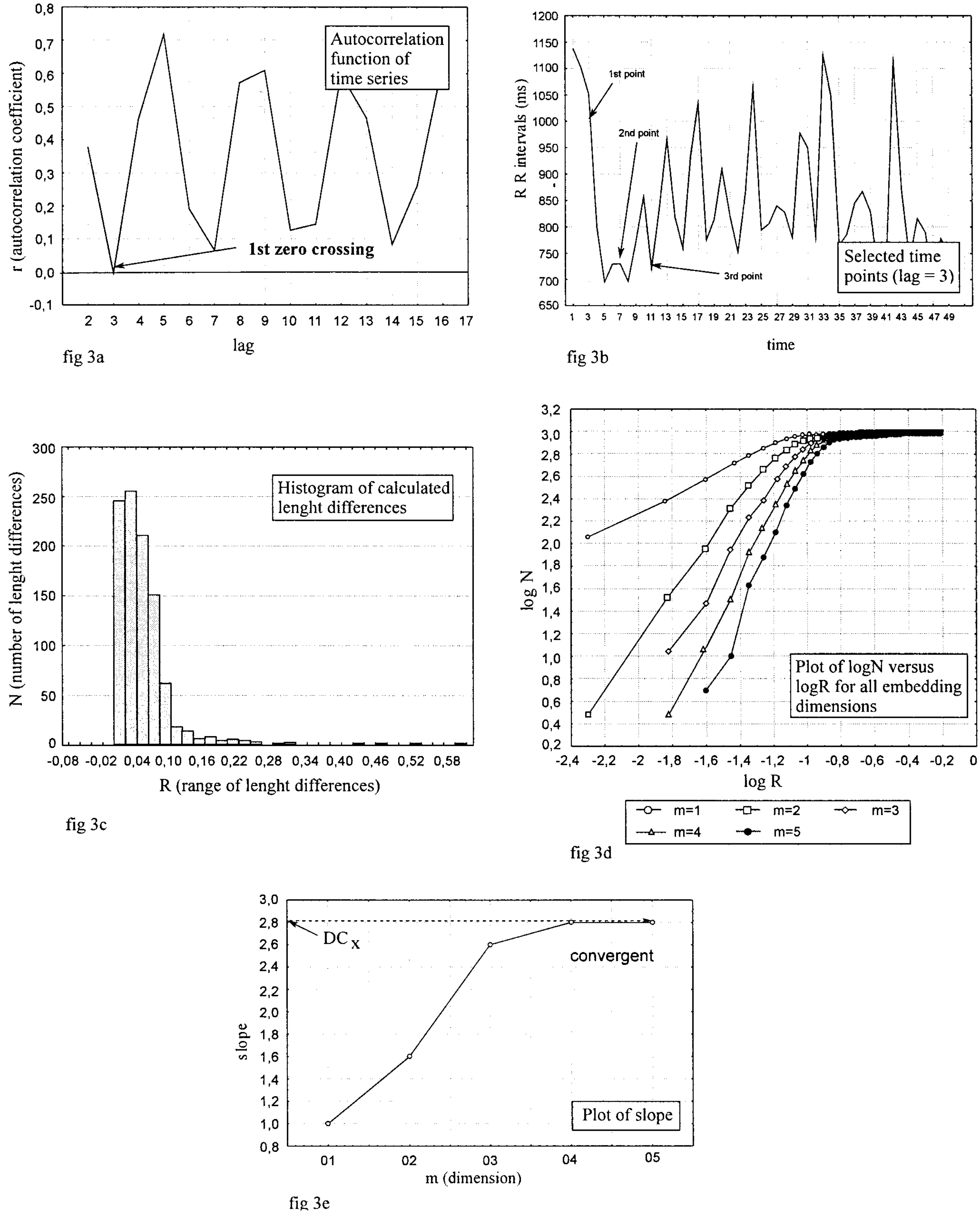

Figure 3. Computation of Grassberger – Procaccia correlation dimension

white noise. In contrast, if the signal is comprised of superimposed sinusoidal waves, it will have spikes at the frequency of each sinusoid.

Grassberger – Procaccia correlation dimension

The correlation dimension of time series is defined as cumulative number of rank-ordered vector-difference lengths within a specific rang. Vector differences are worked out as follows:

1) Calculate autocorrelation function (figure 3a) and find a point where correlation coefficient crosses zero on time-delay axis;

2) This delay (lag) is then used to select time points on x-axis to apply as coordinates for an embedding dimension (figure 3b);

3) Plots of selected points result in vector *i* and vector *j* and their difference makes one value in vector difference length histogram. Compute vector difference as Euclidian distance (figure 3c);

4) Make the plot of log N (number of difference lengths in specific rang) versus log r (rang) for all embedding dimensions (figure 3d);

5) Make the plot of slope found in linear scaling regions for each embedding dimension. DC_x is the slope value where convergence is observed (figure 3e).

Measures of predictability (instability)

The correlation dimension is not necessarily correlated with predictability. For example, noise is very complex but there is no predictability. On the other hand, a waveform, which is linearly built up by very many sine waves, may be understood as a complex signal. But it remains very predictable (Sammer, 1998). Non-linear systems theory provides so-called Lypunov exponent to estimate some aspects of predictability of system behaviour.

Lyapunov exponent

Chaotic attractor will always exhibit sensitivity to the initial conditions (SIC). That is, two nearby points in the phase-space diverge as the orbits (trajectories) progress. Indeed, the points are known to diverge exponentially, and this divergence is described by Lypunov exponent. The Lypunov exponent and chaos occurs when at least one exponent is positive. The first step is the transformation of the time series into phase-space representation. The second step involves measuring if the distances between two trajectories of the reconstructed system dynamics in the phase-space, grow, shrink or remain unchanged for a given time. This is repeated several times over the whole attractor and finally, the mean divergency/convergency is computed indicating how chaotic the system behaviour was originally. A value of zero indicates a periodic, totally predictable system; positive exponents indicate chaotic system behaviour, while negative exponents suggest that system does not show chaotic behaviour (noise). Therefore, to gain information about the irregularity of the system dynamics, it should be sufficient to compute the largest Lypunov exponent. There are several algorithms for estimating the Lypunov exponent directly from a time series, but the simplest mathematical method is the Wolf method (cited in Elbert at all, 1994).

Wolf method

1) Reconstruct the attractor.

2) Choose an arbitrary trajectory from the attractor's base (fiducial trajectory) and follow its evolution through the attractor to the end of the data set.

3) Continuously look for points in its locally nearest neighbourhood and measure the separation (Euclidian distance) of the pairs over time. New neighbouring points have to be substituted whenever the evolved distance exceeds some specified value. For simplicity in actual practice, instead of using a variable time step between two substitutions, one can simply take a fixed time step.

4) The average divergence rate (Lypunov exponent) is then finally computed as follows:

$$\lambda = \frac{1}{t_m - t_0} \sum_{k=1}^{m} \log \frac{r(t_k)}{r(t_{k-1})}$$

where:

λ is largest Lypunov exponent,
k is the chosen time delay (number of time points),
t_k is the corresponding time period for k time points,
$r(t_k)$ is the separation that evolved from the initial distance,
$r(t_{k-1})$ is initial distance,
m is the number of replacements made in time,
t_m is the average time of replacement,
t_0 is starting time period,

As can be seen from the Table 1, the biggest correlation dimension, i.e. dimensional complexity of the data was obtained, as expected, on random numbers, followed by R-R intervals during mental work, while it was absent in rank

Table 1

Correlation dimensions and largest Lypunov exponents for examples from Figure 2.

	correlation dimension (DC_x)	largest Lypunov exponent (l)
Random numbers	6.03	0
Rank ordered numbers	0	4.9
R-R intervals during deep sleep	3	0.002
R-R intervals during mental work	4.3	0.2

ordered numbers. On the other hand, the largest Lypunov exponent was the biggest in the rank ordered numbers, because the system is fully determined by non linear equations, while in random numbers the chaotic determinism does not exist. It should also be emphasised that the degree of deterministic chaos in R-R intervals is significantly higher in mental work situation than during relaxation, or deep sleep. Taking into account both calculated parameters, the degree of chaotic determinism can be well approximated in the data of some time series.

CONCLUSIONS

In general, there has been increasing evidence to support the case that chaos plays a positive role in the physiology of the organism. Goldberger and West (1987) suggest that the dynamics of a healthy physiological system would produce apparently highly irregular and highly complex types of variability, whereas disease and even aging are associated with less complexity and more regularity. For example, it has been shown for epileptic seizures that the degree of chaos is significantly different, both before and during the seizure (Graf & Elbert, 1989). Moreover, dimensional complexity and predictability, as empirical measures, have their contribution in many studies, dealing with dynamics of cortical arousal and its role in cognition. As a discriminating statistics, these parameters have much to offer in interpreting quantitative changes in cortical activity patterns, which previous attempts have not adequately explained (Watters, 1999). This property has caused that these parameters are more and more adopted as dependent variables in experimental designs, so the variability due to a particular factor, such as type of cognitive activity, can be isolated and tested for its significance against the control condition. For example, Sammer (1998) suggested that largest Lypunov exponent is sensitive to the amount of workload, while correlation dimension is sensitive for the type of task (mental, physical).

Finally, it must be pointed out that these methods, presented here, are a small number of mathematical procedures, which are based on the chaos theory. They are applied nowadays in various scientific disciplines, including psychophysiology. It also has to be emphasised, that their utility is not questionable. Their implementation in psychophysiology for practical and scientific purposes, will be increasing and attracting the interest of more and more researchers.

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