Optimization of humanoid robot locomotion behavior using hybrid technique combines between preview control algorithm and discrete algebraic Riccati equation (DARE)

Original Scientific Paper

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Abstract *– Achieving stable bipedal walking remains a significant challenge, as conventional Zero Moment Point (ZMP) methods often struggle to balance disturbance rejection with real-time performance. Other approaches improve computational efficiency or adaptive stability but still face limitations that hinder practical deployment on embedded platforms. In this study, we propose a hybrid control architecture that combines the Discrete Algebraic Riccati Equation (DARE) with preview control for optimal Center-of-Mass (CoM) trajectory generation. The framework integrates cubic-spline swing-leg trajectory planning and adaptive foot placement using damped numerical inverse kinematics, enabling smooth joint motion and precise foot placement. Simulation experiments were conducted on a 12-DoF humanoid robot in the PyBullet physics engine during forward walking on flat terrain. The proposed controller maintained the ZMP within the support polygon with a mean absolute error of 0.03 m, achieved real-time performance at 240 Hz with only 33% CPU usage, and demonstrated rapid stability recovery from a 10 N disturbance within 0.8 s. Additional performance metrics showed a torque efficiency of 9.96 Nm (simulation) , indicating that the method is suitable for low-power embedded platforms. These results highlight a computationally efficient, low-energy, for real-time bipedal locomotion. The proposed architecture improves the feasibility of cost-effective humanoid robots by enabling stable, adaptive walking without sacrificing performance.*

Keywords: *ZMP, Preview control, stability, Inverse kinematics, Biped robot*

Received: June 26, 2025; Received in revised form: October 31, 2025; Accepted: November 3, 2025

1. INTRODUCTION

Humanoid robots represent some of the most sophisticated achievements in robotics, combining advanced mechanical design with complex control architectures to mimic human dynamics. Their appeal lies in their ability to imitate human locomotion, communication, and even decision-making. This humanlike functionality makes humanoid robots suitable for diverse domains such as military operations, industrial automation, education, healthcare assistance, and entertainment [1-3]. However, despite their broad applicability, humanoid robots remain highly prone to instability due to their high number of degrees of freedom, nonlinear lower-body dynamics, and the complexity of joint coordination [4, 5]. Preventing falls and ensuring reliable balance are therefore central challenges in humanoid locomotion research.

The fundamental research problem is how to enable humanoid robots to walk dynamically and robustly in both structured and unstructured environments while maintaining balance and energy efficiency. Current methods are either too reliant on simplified assumptions, too computationally expensive for real-time deployment, or

insufficiently robust to external disturbances. As a result, the gap between simulation-based locomotion research and practical real-world application remains significant.

Balancing humanoid robots has long been studied using approaches inspired by biomechanics and physics-based dynamics. Traditional control frameworks rely on mathematical models such as the Zero Moment Point (ZMP), the Linear Inverted Pendulum Model (LIPM), and Capture Point (CP) dynamics [6-8]. For example, the LIPM simplifies analysis by assuming massless legs, making it possible to design controllers for flat surfaces. However, this simplification limits applicability to dynamic terrains and real-world disturbances [9, 10].

To address stability in unpredictable environments, researchers frequently adopt Model Predictive Control (MPC). MPC enables dynamic walking by predicting and optimizing future states of the robot [11-13]. However, MPC remains constrained by its high computational cost and sensitivity to modeling inaccuracies [14]. Similarly, the swinging limb of humanoid robots introduces vibration challenges, prompting innovative approaches such as harvesting vibrational energy with piezoelectric devices to reduce oscillations while generating usable power [15].

Trajectory generation plays a crucial role in humanoid locomotion, as stable walking patterns depend on dynamically consistent center of mass (CoM) trajectories. Quadratic Programming (QP) methods have been widely employed to enhance ZMP-based control and generate feasible trajectories in real time [16]. Yet, ensuring stability under multiple constraints while maintaining computational efficiency continues to pose difficulties. Offline ZMP tracking methods have also been explored [17], but their reliance on precomputed trajectories reduces their adaptability to dynamic conditions, and hardware validation remains limited.

Biomechanics-inspired methods that map human movement patterns to humanoid robots show promise in streamlining control [18]. Nevertheless, these approaches often lack robustness to disturbances and fail to optimize energy consumption. Among the most widely applied methods, ZMP remains central: it represents the point on the foot's support area where the resultant moments and forces balance out. To guarantee stability, this ZMP must remain within the support polygon of the foot [19-21] (Fig.1).

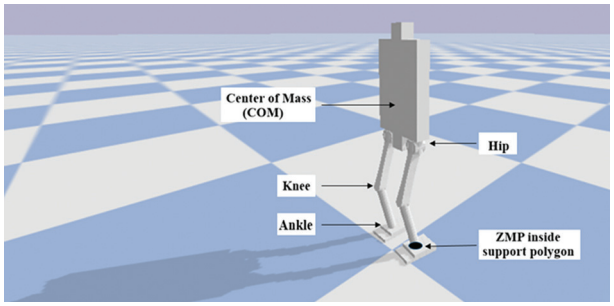

Fig. 1. Zero Moment Point (ZMP) inside support polygon, Center of Mass (COM) and robot joints

Shimmyo *et* al. [22] refined preview control by introducing a three-mass model to improve gait realism, though this approach remained largely open-loop and vulnerable to external disturbances. Adiwahono *et* al. [23] advanced locomotion robustness by implementing push recovery controllers that adapt footstep placement under external forces. However, challenges remain in responding to unpredictable, large, or constant pushes. Shimmyo *et* al. [24] proposed a 2D virtual plane combined with preview control, but the exclusion of angular momentum dynamics limited applicability under torque variations. Kajita *et* al. [25] introduced preview control as a method to track ZMP trajectories and generate stable biped walking patterns. This approach successfully anticipates future CoM motions but struggles with terrain irregularities and disturbances.

These studies reveal a persistent gap: while preview control and model-based methods excel in trajectory planning, they often lack the robustness and adaptability required for real-world environments. Conversely, robust control strategies that address disturbances frequently compromise on computational efficiency or trajectory optimality.

The objective of this study is to develop a hybrid control framework that combines the strengths of model-based preview control with optimization-based feedback methods. Specifically, we propose a hybrid architecture integrating the Discrete Algebraic Riccati Equation (DARE) with preview control to achieve optimal CoM trajectory planning. By doing so, we aim to ensure that the ZMP consistently remains within the support polygon while improving the adaptability of the robot to disturbances. The target platform for this research is a humanoid robot with 12 degrees of freedom (DOF), designed to replicate human locomotion mechanics. The guiding hypothesis is that a hybrid preview control and optimization-based feedback system can significantly improve the dynamic stability and disturbance rejection of humanoid locomotion compared to traditional open-loop preview control. The main contributions of this paper are as follows:

- A hybrid ZMP-based walking control framework that integrates the Discrete Algebraic Riccati Equation (DARE) with Preview Control for optimal Center-of-Mass (CoM) trajectory generation.
- A cubic-spline-based swing-leg trajectory generator that ensures smooth joint motion and continuous foot trajectories suitable for high-frequency, real-time control.
- An adaptive foot placement strategy using damped numerical inverse kinematics, enabling accurate foot positioning and improving stability during forward walking.
- A lightweight and computationally efficient control pipeline, achieving 240 Hz real-time operation with only 33% CPU usage, making the approach suitable for low-power embedded systems.

- Comprehensive simulation validation on a 12-DoF humanoid robot, demonstrating:
  I. ZMP tracking accuracy with a mean absolute error of 0.03 m
  II. Stable recovery from 10 N disturbances within 0.8 s
  III. Torque efficiency of 9.96 Nm, confirming suitability for practical deployment.
- A complete and robust locomotion solution that advances the feasibility of low-cost, embedded humanoid robots capable of stable and adaptive real-time walking.

This paper is structured as follows: Section 1 introduces the research problem, background, and objectives. Section 2 presents the simulation framework and the mathematical models used, including the integration of preview control with DARE optimization. Section 3 discusses the results of our simulations and highlights the advantages and limitations of the proposed approach. Finally, Section 4 concludes the study and outlines directions for future work, particularly toward experimental validation and application in real-world environments.

2. METHODS AND MATERIALS

2.1. RESEARCH DESIGN

This study employed a computational and simulation-based research design to investigate the control and trajectory generation of a humanoid bipedal robot. The research focused on the integration of Zero Moment Point (ZMP) theory, the Linear Inverted Pendulum Model (LIPM), preview control, cubic spline interpolation, and inverse kinematics (IK) to achieve dynamically stable walking patterns (Fig. 3). The design involved the development and implementation of a hybrid control architecture in a simulated environment, followed by analysis of robot performance in terms of stability, smoothness of motion, and trajectory tracking accuracy.

The target population for this study was humanoid bipedal robots designed to replicate human locomotion mechanics. For simulation purposes, the study utilized a single humanoid robot model with 12 degrees of freedom (DOF). This model represented a simplified yet sufficiently complex platform to test the proposed control algorithms. The robot's lower body consisted of six DOF, including hip, knee, and ankle joints for both legs, while the upper body was primarily used to maintain overall balance.

As this study was conducted in a simulated environment, the sampling technique involved selecting representative walking scenarios for analysis. The simulations included straight-line walking on flat surfaces and controlled disturbances to test robustness. Key sampling criteria included the number of steps, step length, and initial CoM positions to ensure a range of dynamic behaviors was captured. Each simulation trial represented a "sample" in evaluating the performance of the proposed methods.

2.2. DYNAMICAL MODEL

2.2.1. Zero Moment Point (ZMP) and Linear Inverted Pendulum Model (LIPM)

The ZMP and LIPM were used to model the bipedal robot's dynamics and balance control. The LIPM assumed massless legs and constrained the CoM to move at a fixed height within a 2D plane (sagittal or lateral) (Fig. 2). The simplified dynamics allowed real-time trajectory generation and were expressed as [25]:

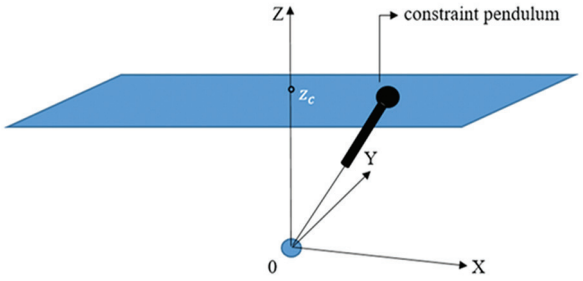

Fig. 2. Linear Inverted Pendulum constraint in (X, Y, Z) plane

$$Z = K_x \cdot x + K_y \cdot y + Z_c \quad (1)$$

Where:

K_x = intersection of x-axis

K_y = intersection of y-axis

Z_c = height of the mass on the Z-plane.

($K_x = K_y = 0$) when the plane is horizontal. Therefore, the constrained dynamics are given as equations (2) and (3):

$$\ddot{y} = \frac{g}{z_c} \cdot y - \frac{1}{m_{Zc}} \cdot \tau_x \quad (2)$$

$$\ddot{x} = -\frac{g}{z_c} \cdot x + \frac{1}{m_{Zc}} \cdot \tau_y \quad (3)$$

ZMP equations can be calculated from the two equations (4) and (5) below:

$$p_x = -\frac{\tau_y}{mg} \quad (4)$$

$$p_y = \frac{\tau_x}{mg} \quad (5)$$

Where p_x and p_y represent the zero moment point on the foot. After substituting Eq. (5) and (4) in Eq. (2) and (3), we get:

$$\ddot{y} = \frac{g}{z_c} \cdot (y - p_y) \quad (6)$$

$$\ddot{x} = -\frac{g}{z_c} \cdot (x + p_x) \quad (7)$$

After extracting p_x and p_y from these two equations, we can calculate ZMP as below:

$$p_y = y - \frac{z_c}{g}\ddot{y} \quad (8)$$

$$p_x = x - \frac{z_c}{g}\ddot{x} \quad (9)$$

Where p_x and p_y represented the ZMP on the robot's foot, Z_c was the CoM height, and g was gravitational acceleration. These equations were used to calculate desired footstep positions using inverse kinematics.

2.2.2. State-Space Model

The state space representation model is implemented in our approach due to its efficiency with predicting future states such as velocity, acceleration, and position of the robot joints. In this study we need to calculate the desired states of joints; therefore, the state space representation implemented in our work predicts these states of the center of mass easily. The state equations were expressed as [26]:

$$X(k+1) = A\,x(k) + B\,u(k) \quad (10)$$

$$P(k) = C\,x(k) \quad (11)$$

Where:

X is the state vector of the ZMP.

U is the control input.

P is the ZMP output, as below:

$$X(k) \equiv [x(kt)\ \dot{x}(kt)\ \ddot{x}(kt)]^T$$
$$U(k) = u\,x(kt)$$
$$P(k) = P\,x(kt)$$
$$A = \begin{bmatrix} 1 & t & t^2/2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix}$$
$$B = \begin{bmatrix} t^3/6 \\ t^2/2 \\ t \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 0 & z_c/g \end{bmatrix}$$

Where:

x (t): position,

$\dot{x}$ (t): velocity,

$\ddot{x}$ (t): acceleration,

t : time step,

A and B: matrices to find future states,

C: represents ZMP reference tracking.

2.2.3. Preview Controller

In this study we employed a preview control, which is a sophisticated control method that predicts future reference trajectories (such as Zero Moment Point (ZMP)) and maximizes the current control input to reduce tracking errors. The mathematical foundation of Preview Control is the cost function, which measures trade-offs between tracking accuracy, stability, and control effort. This cost function in the Discrete Algebraic Riccati Equation (DARE) formulation is as below [25]:

$$J = \sum_{k=0}^{\infty} (x_K^T\ Q x_k\ + u_k^T R u_k) \quad (12)$$

Where Q is the weight matrix to give a penalty in case that deviation in the state happened and the R matrix to penalize control effort. The control optimization law is given as

$$u_k = F x_k + \sum_{j=1}^{N} F_r\,[j](pref, k+j-1) \quad (13)$$

Where:

F = feedback gain,

$F[j]$ = preview gains,

N = number of steps.

This approach ensured local stability while allowing the robot to anticipate and react to CoM deviations.

2.2.4. Cubic Spline Interpolation

In the previous sections we calculate the desired footsteps of the robot and locate it on the actual joint angles. To enhance the behavior of the walking and make it imitate human walking, we employed an optimization method, so the cubic spline interpolation employed in this study made the joint translation smoother and provided continuous acceleration and velocity, as below [27]:

$$x(t) = C0t^3 + C1t^2 + C2t + C3 \quad (14)$$

Where:

Position: $x(t)$ provides the desired Cartesian position or joint angle at that moment.

Velocity is the first derivative $\dot{x}$(t).

Acceleration is the second derivative $\ddot{x}$(t).

This technique enhances the naturalness of humanoid walking and reduces abrupt changes in joint velocities.

2.2.5. Inverse Kinematics (IK)

Inverse kinematics was applied in this study to convert desired foot positions, derived from ZMP and preview control, into corresponding joint angles. The Damped Least Squares (DLS) method was implemented to handle singularities and reduce computational errors [27]:

$$q_{k+1} = q_k + \Delta q \quad (15)$$

Where Δq is the update rule can be calculated from:

$$\Delta q = J^T (J J^T + \lambda^2 I)^{-1} \Delta x \quad (16)$$

Where:

J is the Jacobian,

Λ is the damping factor.

The trajectory generation process, combining ZMP calculation, preview control, cubic spline interpolation, and inverse kinematics, is illustrated in (Fig. 3).

This architecture shows how desired CoM trajectories are converted into joint motions while maintaining stability and real-time performance.

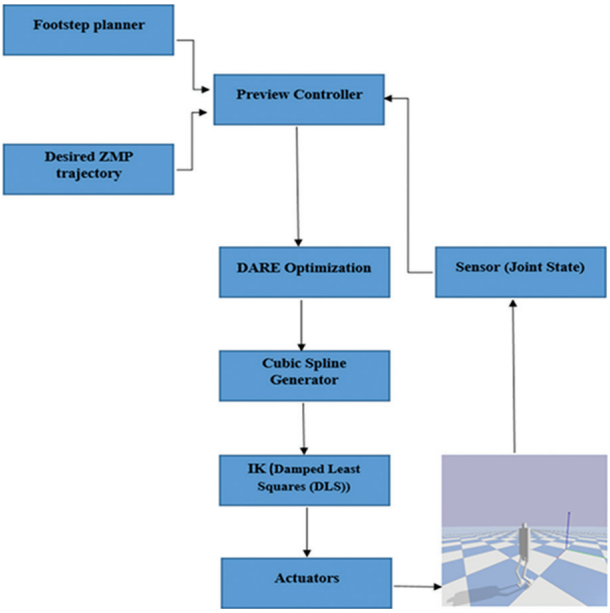

Fig. 3. Trajectory generation system architecture

Data were collected through simulation outputs from the robot's state-space model, including joint positions, velocities, accelerations, and the calculated ZMP locations. The simulation environment recorded time-series data for the center of mass (CoM), foot positions, and support polygon dynamics. Additionally, the system tracked deviations between desired and actual ZMP trajectories to evaluate tracking performance.

2.4. DATA ANALYSIS TECHNIQUES

Simulation data were analyzed quantitatively to evaluate ZMP tracking accuracy, CoM trajectory deviations, and stability margins within the support polygon. Root-mean-square error (RMSE) metrics were calculated for joint trajectories and foot placement accuracy. Additionally, visualization of CoM and ZMP trajectories over time provided qualitative insight into the robot's stability and gait smoothness.

The validity of the methods was ensured by using physically grounded models (ZMP, LIPM) and well-established control strategies (preview control, cubic splines, DLS IK). Reliability was assessed through repeated simulation trials under varying initial conditions and disturbances, confirming consistent results for ZMP tracking and CoM trajectory generation.

As this research was fully simulation-based, no human participants or live animals were involved. Ethical considerations focused on the responsible use of computational resources and transparent reporting of methods and results. All software models and simulations adhered to best practices for reproducibility and data integrity.

3. RESULTS AND DISCUSSION

The bipedal robot was simulated using the PyBullet physics engine in Python, an open-source environment widely used for robotics research, including dynamics-based and reinforcement learning studies. To evaluate performance, several control schemes were compared, namely, (a) biped robot locomotion using a PID controller, (b) preview control without Riccati, (c) preview control with manual feedback, and (d) preview control with Riccati (DARE).

3.1. BIPED ROBOT LOCOMOTION USING PID

When the walking controller was implemented using only PID feedback on the ZMP error, the robot showed some ability to regulate the CoM trajectory, but its performance was limited. PID reacted to deviations only after they occurred, which introduced a lag in response. This caused oscillations around the desired trajectory, especially when step transitions occurred. Moreover, PID required careful tuning of the gains (Kp, Ki, Kd) and remained sensitive to parameter variations and disturbances. In practice, this made the system unstable during continuous walking (Fig. 4), since the inverted pendulum dynamics are inherently unstable and demand anticipative control rather than purely reactive correction (Fig. 5 and Fig. 13 (a)).

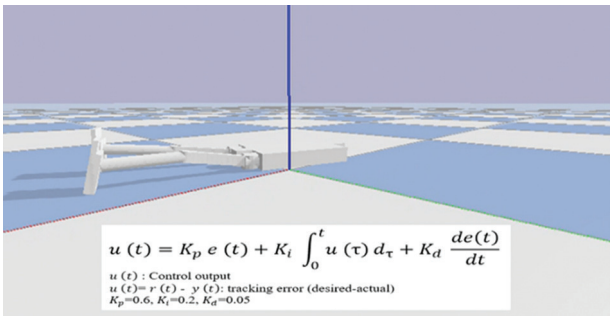

Fig. 4. Robot fall down due to instability

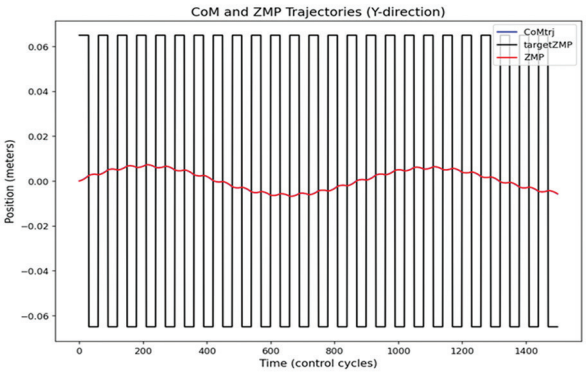

Fig. 5. CoM trajectories with PID controller

3.2. PREVIEW CONTROL WITHOUT RECCATI AND WITHOUT FEEDBACK

When the system was run with only feedforward preview gains (without Riccati and without any feedback),

the robot was unstable (Fig. 6). The preview gains allowed the system to anticipate the reference ZMP trajectory, but since there was no stabilizing feedback, any model mismatch or disturbance accumulated over time. Small errors in CoM tracking grew unbounded, leading to divergence. This shows that pure preview without stabilization is not sufficient for walking balance (Fig. 7 and Fig. 13 (b)).

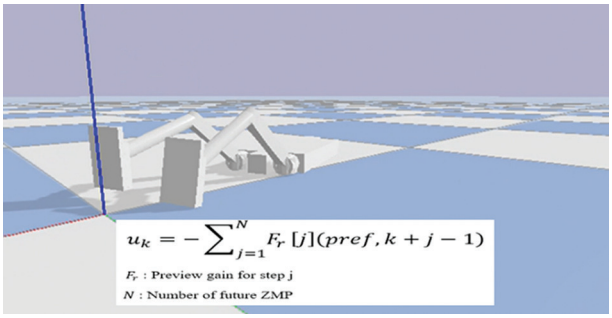

Fig. 6. Robot fall down without Reccati and Feedback

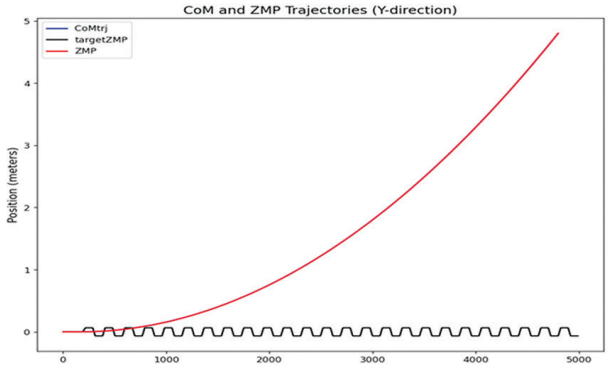

Fig. 7. CoM trajectories without Reccati and feedback

3.3. PREVIEW CONTROL WITH MANUAL FEEDBACK AND WITHOUT RECCATI

In this configuration, the Riccati-based feedback term was disabled and replaced by small hand-tuned manual gains. The performance improved compared to pure preview-only mode, since the manual feedback provided basic stabilization of CoM motion. However, the system still lacked the optimality of the Riccati solution. The feedback was not adaptive and could not perfectly balance error minimization against control effort. As a result, the robot could walk more stably than with preview-only, but oscillations and drift were still visible (Fig. 8), especially under disturbances or during longer walking sequences (Fig. 9 and Fig. 13 (c)).

3.4. PREVIEW CONTROL (WITH RECCATI FULL LQR DESIGN)

This case gave the best performance. The Riccati equation solved the optimal control problem by combining feedback and feedforward terms. The preview controller was able to anticipate future ZMP references, while the Riccati-based feedback corrected deviations optimally in real time. This led to smooth CoM trajectories, accurate ZMP tracking, and a balanced trade-off between tracking precision and control effort (Fig. 10). The robot achieved stable continuous walking with minimal oscillation, even under small disturbances, (Fig. 13 (d)). After implementing the control architecture, the ZMP could be accurately calculated. Using inverse kinematics, the robot's feet were placed at the desired positions, enabling forward walking, (Fig. 11). Table 1 represent control performance comparison. (Fig. 12) represents a control diagram of the system.

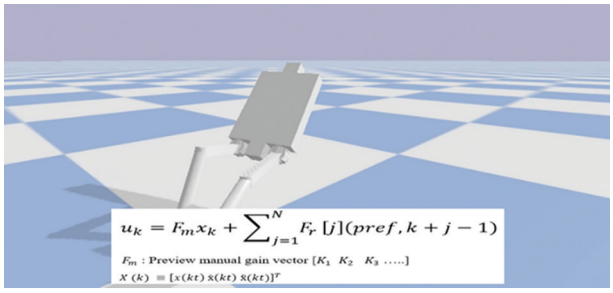

Fig. 8. Robot fall down with only Feedback

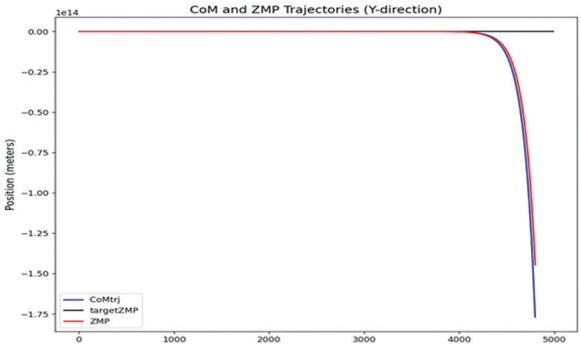

Fig. 9. CoM trajectories with feedback only

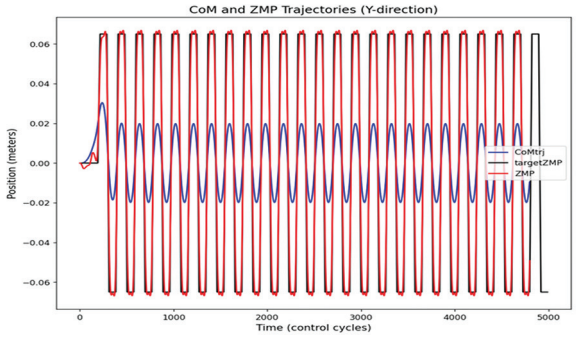

Fig. 10. COM trajectories with Preview Controller and Reccati

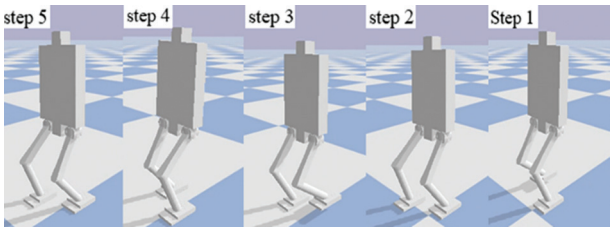

Fig. 11. Forward stable walking with Preview control combined with Reccati

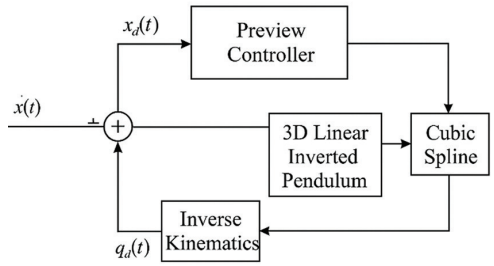

Fig. 12. System control diagram

Preview Control combinations with DARE performance metrics were evaluated for computational efficiency, frequency, ZMP tracking error, and recovery time, Table 2. The hybrid system combining numerical inverse kinematics (IK) with the Discrete Algebraic Riccati Equation (DARE) achieved high-frequency control at 240 Hz while maintaining low CPU usage (33%), (Fig. 14). The ZMP tracking error was measured at 0.03 m, indicating precise foot placement and CoM regulation. Recovery time from disturbances was approximately 0.8 s.

Table 1. Comparison between control performance

Method	Anticipaton of Future Steps	Feedback Quality	Stability	Control Effort	Performance
PID	None	Reactive, local error-based	Poor oscillations, unstable	Medium High (depends on tuning)	Limited, not suitable for walking
PC (without Reccati & feedback)	Future ZMP	None	Very poor divergence occurs	Low (no correction applied)	Fails through walking
PC (with only feedback)	Yes	Heuristic, hand-tuned	Moderate some stabilization	Medium	Better than PID, but non-optimal
PC + Reccati	Yes	Optimal (LQR-based)	Excellent stable & smooth	Optimally balanced (minimal for stability)	Best stable, robust walking

Table 2. Comparison between our system and existing methods

Metric	Our system	Wieber [28]	Erbatur&Kurt [29]	Yamamoto&Sugihara[30]
Control method	DARE + numerical IK	Linear MPC	Natural ZMP trajectory	QP – base control
Max_ZMP error(m)	0.03m	~0.02m	Not reported	0.017-0.02m
Disturbance Rejection	No test	33% robot mass	No test	20N
Compute Frequency	240H	1KH	Not reported	Not reported
CPU Usage	33%	RT kernel (core i7)	Not reported	Not reported
Torque Efficiency	9.96Nm (simulation)	Not reported	25-30% (energy reduction)	Not quantified
Recovery Time	0.8s	2 steps (for 18 kg impact)	No test (open loop)	~0.03-0.05s

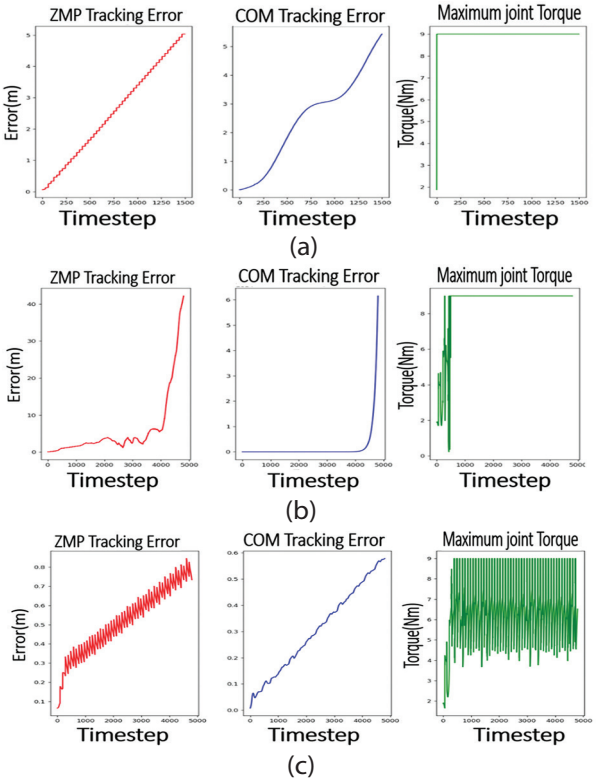

Fig. 13. Performance for each used controller

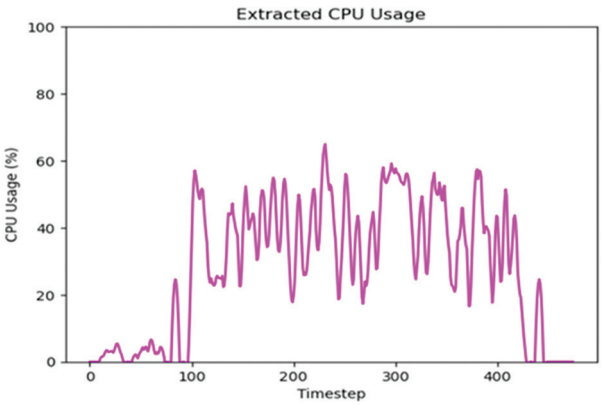

Fig. 14. CPU usage

A comparison with existing approaches highlighted the advantages of the proposed system: Wieber's Linear MPC tolerates perturbations up to 33% of the robot's mass at a control frequency of 1 kHz but requires a real-time kernel; Erbatur and Kurt's energy-optimized ZMP trajectories reduce energy use by 25–30% but lack real-time adaptability due to open-loop implementation; Yamamoto and Sugihara's QP-based controller achieves rapid recovery (~0.3–0.5 s) but is only effective for small perturbations (~20 N).

The results indicate that the proposed hybrid DARE + numerical IK control framework provides a balance between performance, efficiency, and deployability. The robot achieved forward walking while maintaining the ZMP within the support polygon, confirming that the combination of preview control and inverse kinematics effectively translates CoM trajectories into accurate foot placements.

Compared to prior literature, the system demonstrates several key advantages. Although Wieber's MPC shows higher disturbance tolerance, it relies on high computational resources and a real-time kernel, limiting deployment on embedded systems. In contrast, our system maintained low CPU usage (33%) while achieving competitive ZMP accuracy (0.03 m) and high-frequency control (240 Hz). Erbatur and Kurt's energy-optimized trajectories provide energy savings, but their open-loop design lacks adaptability to dynamic conditions; our approach addresses this by integrating feedback and preview components. Yamamoto and Sugihara's QP-based control recovers quickly from small disturbances but is restricted to minor perturbations, whereas our system achieves a recovery time of 0.8 s under simulated disturbances, suggesting reasonable robustness.

The use of cubic spline interpolation contributed to smooth joint motions, enabling continuous position, velocity, and acceleration profiles, which is critical for humanlike walking behavior. Furthermore, the damped least squares inverse kinematics approach avoided singularities and minimized computational losses, ensuring accurate foot positioning without excessive calculation overhead.

Implications of these findings include the feasibility of implementing real-time, dynamically stable bipedal walking on embedded systems with limited computational resources. The results demonstrate that hybrid control strategies can provide a practical compromise between optimization complexities and deployability, bridging the gap between high-performance simulations and real-world applications.

Limitations of the current study include the simulation-only environment and controlled flat terrain. While the system performed well in these conditions, real-world factors such as uneven terrain, external perturbations, sensor noise, and hardware limitations may affect performance. Additionally, disturbance rejection remains limited compared to high-end MPC implementations.

In summary, the proposed hybrid control framework demonstrates effective ZMP tracking, smooth CoM trajectory execution, low computational cost, and reasonable recovery time. These results highlight the potential of combining numerical IK with DARE-based preview control as a practical solution for real-time bipedal locomotion on embedded platforms, offering a balance between stability, efficiency, and deployability.

4. CONCLUSIONS

This study presents a hybrid control framework for bipedal humanoid locomotion that combines Zero Moment Point (ZMP) trajectory generation, Linear Inverted Pendulum Model (LIPM), inverse kinematics (IK), and preview control with the Discrete Algebraic Riccati Equation (DARE). Simulation results demonstrate that the robot maintains stability while walking, keeping the actual ZMP within the support polygon and achieving minimal tracking errors. These findings confirm that the approach effectively translates desired center-of-mass trajectories into accurate foot placements, enabling smooth, humanlike walking motions. A key contribution of this work is achieving high-frequency control (240 Hz) with low computational overhead (33% CPU usage), providing a practical alternative to traditional optimization-based methods such as Model Predictive Control (MPC). By integrating cubic spline interpolation, the system ensures continuous joint motion while reducing computational cost and avoiding singularities, demon strating real-time applicability.

The proposed framework has practical implications for embedded humanoid robots in applications such as service robotics, industrial automation, and assistive devices, where efficient and stable locomotion is essential. Future research will extend the method to higher degrees-of-freedom systems and dynamic environments, including uneven terrain. Hardware implementation is also planned to validate the simulation results under real-world disturbances, sensor noise, and energy constraints.

REFERENCES:

[1] F. Sygulla, D. Rixen, "A Force-Control Scheme for Biped Robots to Walk over Uneven Terrain Including Partial Footholds", International Journal of Advanced Robotic Systems, Vol. 17, No. 1, 2020, pp. 1-14. doi: 10.1177/1729881419897472.

[2] Y. Tong, H. Liu, Z. Zhang, "Advancements in Humanoid Robots: A Comprehensive Review and Future Prospects", IEEE/CAA Journal of Automatica Sinica, Vol. 11, No. 2, 2024, pp. 301-328. doi: 10.1109/JAS.2023.124140.

[3] Y. Tazaki, M. Murooka, "A Survey of Motion Planning Techniques for Humanoid Robots", Advanced Robotics, Vol. 34, No. 21-22, 2020, pp. 1370-1379. doi: 10.1080/01691864.2020.1803128.

[4] J. Cho, J. H. Park, "Model Predictive Control of Running Biped Robot", Applied Sciences, Vol. 12, No. 21, 2022, p. 11183. doi: 10.3390/app122111183.

[5] Ç. Kaymak, A. Uçar, C. Güzeliş, "Development of a New Robust Stable Walking Algorithm for a Hu-

manoid Robot Using Deep Reinforcement Learning with Multi-Sensor Data Fusion", Electronics, Vol. 12, No. 3, 2023, p. 568. doi: 10.3390/electronics12030568.

[6] S. Caron, A. Kheddar, "Dynamic Walking over Rough Terrains by Nonlinear Predictive Control of the Floating-Base Inverted Pendulum", Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems, Vancouver, Canada, 24-28 September 2017, pp. 5017-5024. doi: 10.1109/IROS.2017.8206385.

[7] A. Vedadi, K. Sinaei, P. Abdolahnezhad, S. S. Aboumasoudi, A. Yousefi-Koma, "Bipedal Locomotion Optimization by Exploitation of the Full Dynamics in DCM Trajectory Planning", Proceedings of the 9th RSI International Conference on Robotics and Mechatronics, Tehran, Iran, 17-18 November 2021, pp. 365-370. doi: 10.1109/ICRoM54204.2021.9663522.

[8] H. F. Al-Shuka, B. J. Corves, B. Vanderborght, W.-H. Zhu, "Zero-Moment Point-Based Biped Robot with Different Walking Patterns", International Journal of Intelligent Systems and Applications, Vol. 7, No. 1, 2014, pp. 31-41. doi: 10.5815/ijisa.2015.01.03.

[9] I. Al-Tameemi, O. Amanuel, "Bipedal Robots: A Systematic Review of Dynamical Models, Balance Control Strategies, and Locomotion Methods", Journal of Robotics and Control, Vol. 6, No. 3, 2025, pp. 1240-1254. doi: 10.18196/jrc.v6i3.25595.

[10] I. Altameemi, "A Comprehensive Survey of Push Recovery Techniques for Standing and Walking Bipedal Robots", Al-Khwarizmi Engineering Journal, Vol. 20, No. 3, 2024, pp. 20-35. doi: 10.22153/kej.2024.07.002.

[11] R. C. Luo, C.-W. Huang, W.-C. Hung, "Bipedal Robot Push Recovery Control Mimicking Human Reaction", Proceedings of the IEEE International Workshop on Advanced Motion Control, Auckland, New Zealand, 22-24 April 2016, pp. 334-339. doi: 10.1109/AMC.2016.7496372.

[12] E. Dantec *et* al. "Whole-Body Model Predictive Control for Biped Locomotion on a Torque-Controlled Humanoid Robot", Proceedings of the IEEE-RAS International Conference on Humanoid Robots, Ginowan, Japan, 28-30 November 2022, pp. 638-644. doi: 10.1109/Humanoids53995.2022.10000129.

[13] A. I. Haldar, N. D. Pagar, "Predictive Control of Zero Moment Point (ZMP) for Terrain Robot Kinematics", Materials Today: Proceedings, Vol. 80, 2023, pp. 122-127. doi: 10.1016/j.matpr.2022.10.286.

[14] H.-M. Joe, J.-H. Oh, "Balance Recovery through Model Predictive Control Based on Capture Point Dynamics for Biped Walking Robot", Robotics and Autonomous Systems, Vol. 105, 2018, pp. 1-10. doi: 10.1016/j.robot.2018.03.004.

[15] Y. O. N. Al-Azzawi, "Vibration Reduction of Walking Biped Humanoid Robot Using Energy Harvester", International Journal of Advanced Computer Technology, Vol. 5, No. 10, 2013, pp. 1-6. doi: 10.4156/ijact.vol5.issue10.3.

[16] S. Savin, "ZMP-Based Trajectory Generation for Bipedal Robots Using Quadratic Programming", Control and Signal Processing Applications for Mobile and Aerial Robotic Systems, IGI Global, 2020, pp. 266-285.

[17] B. Aghbali, A. Yousefi-Koma, A. G. Toudeshki, A. Shahrokhshahi, "ZMP Trajectory Control of a Humanoid Robot Using Different Controllers Based on an Offline Trajectory Generation", Proceedings of the RSI/ISM International Conference on Robotics and Mechatronics, Tehran, Iran, 13-15 February 2013, pp. 530-534. doi: 10.1109/ICRoM.2013.6510161.

[18] D. Astudillo, L. I. Minchala, F. Astudillo-Salinas, A. Vazquez-Rodas, L. Gonzalez, "A Simple Mapping Methodology of Gait Biomechanics for Walking Control of a Biped Robot", Proceedings of the IEEE International Conference on Electronics, Electrical Engineering and Computing, Lima, Peru, 15-17 August 2018, pp. 1-4. doi: 10.1109/INTERCON.2018.8526395.

[19] T. T. Huan, C. Van Kien, H. P. H. Anh, N. T. Nam, "Adaptive Gait Generation for Humanoid Robot Using Evolutionary Neural Model Optimized with Modified Differential Evolution Technique", Neurocomputing, Vol. 320, 2018, pp. 112-120. doi: 10.1016/j.neucom.2018.08.074.

[20] Q. Zhong, Y. Li, C. Zheng, T. Shen, "Humanoid Robot Cooperative Motion Control Based on Optimal Parameterization", Frontiers in Neurorobotics, Vol. 15, 2021, pp. 1-10. doi: 10.3389/fnbot.2021.699820.

[21] T. Brecelj, T. Petrič, "Zero Moment Line—Universal Stability Parameter for Multi-Contact Systems in Three Dimensions", Sensors, Vol. 22, No. 15, 2022, pp. 1-19. doi: 10.3390/s22155656.

[22] S. Shimmyo, T. Sato, K. Ohnishi, "Biped Walking Pattern Generation by Using Preview Control Based on Three-Mass Model", IEEE Transactions on Industrial Electronics, Vol. 60, No. 11, 2012, pp. 5137-5147. doi: 10.1109/TIE.2012.2221111.

[23] A. H. Adiwahono, C.-M. Chew, W. Huang, Y. Zheng, "Push Recovery Controller for Bipedal Robot Walking", Proceedings of the IEEE/ASME International Conference on Advanced Intelligent Mechatronics, Singapore, 14-17 July 2009, pp. 162-167. doi: 10.1109/AIM.2009.5230022.

[24] S. Shimmyo, T. Sato, K. Ohnishi, "Biped Walking Pattern Generation by Using Preview Control with Virtual Plane Method", Proceedings of the IEEE International Workshop on Advanced Motion Control, Nagaoka, Japan, 21-24 March 2010, pp. 414-419. doi: 10.1109/AMC.2010.5464096.

[25] S. Kajita *et* al. "Biped Walking Pattern Generation by Using Preview Control of Zero-Moment Point", Proceedings of the IEEE International Conference on Robotics and Automation, Taipei, Taiwan, 14-19 September 2003, pp. 1620-1626. doi: 10.1109/ROBOT.2003.1241826.

[26] B. Siciliano, L. Sciavicco, L. Villani, G. Oriolo, "Robotics: Modelling, Planning and Control", Springer, 2009.

[27] J. D. Faires, R. L. Burden, "Numerical Methods", Thomson/Brooks/Cole, 2015.

[28] P.-B. Wieber, "Trajectory Free Linear Model Predictive Control for Stable Walking in the Presence of Strong Perturbations", Proceedings of the IEEE-RAS International Conference on Humanoid Robots, Genoa, Italy, 4-6 December 2006, pp. 137-142. doi: 10.1109/ICHR.2006.321375.

[29] K. Erbatur, O. Kurt, "Natural ZMP Trajectories for Biped Robot Reference Generation", IEEE Transactions on Industrial Electronics, Vol. 56, No. 3, 2008, pp. 835-845. doi: 10.1109/TIE.2008.2005150.

[30] T. Yamamoto, T. Sugihara, "Foot-Guided Control of a Biped Robot through ZMP Manipulation", Advanced Robotics, Vol. 34, No. 21-22, 2020, pp. 1472-1489. doi: 10.1080/01691864.2020.1827031.