GLASNIK MATEMATIČKI
Vol. 61(81)(2026), 1 – 15

AUBERT DUALS OF STRONGLY POSITIVE REPRESENTATIONS FOR METAPLECTIC GROUPS

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Abstract. We determine the Aubert duals of strongly positive representations of the metaplectic group $\widetilde{Sp}(n)$ over a non-Archimedean local field F of characteristic different from two. Using the classification of Matić and an explicit analysis of Jacquet modules, we describe these duals in terms of precise inducing data. Our results extend known descriptions for classical groups to the metaplectic group case and clarify the role of Aubert duality for non-linear covering groups, providing a foundation for future applications to the study of unitary representations for these groups. Furthermore, we are able to show that the same method applies to odd general spin groups, yielding an explicit description of Aubert duals in that setting as well.

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The study of classifying unitary representations of connected reductive groups defined over a p-adic field F is one important subject in the Langlands program, which is still widely open. Even the construction of unitary non-tempered representations is not well understood. Among the tools, we expect the Aubert involution to help construct unitary representations under a conjecture that the Aubert involution preserves unitarity. Very briefly, the Aubert duals describe the duality structure within the Grothendieck group of admissible representations [3, 4].

For connected classical groups, the structure and behavior of Aubert duals have been extensively studied. In particular, significant progress has been made in describing Aubert duals explicitly in terms of Langlands data. In the half-integral reducibility case, Jantzen [11] obtained a detailed description of Aubert duals for classical p-adic groups, while more recently Atobe and Mínguez [2] provided a general framework for computing Aubert duals

2020 *Mathematics Subject Classification.* 22E35, 22E50, 11F70.
Key words and phrases. Aubert duals, metaplectic groups, Jacquet modules.

for p-adic symplectic and odd special orthogonal groups using the endoscopic classification of Arthur [1]. Explicit descriptions in the case of strongly positive discrete series (and certain discrete series arising in the inductive classification) have also been obtained by Matić [16, 17], where the Aubert involution plays a crucial role in the construction of certain families of unitarizable representations (see also [10] for related results on the stability of Arthur-type representations under the Aubert involution).

In contrast, much less is known for non-linear groups and in particular for covering groups. For example, the metaplectic group $\widetilde{Sp}(n)$, the unique non-trivial two-fold central extension of F-rational points of the symplectic group $Sp(n) := \mathrm{Sp}(n, F)$ is a fundamental example of a covering group. Note that the Aubert involution and its main properties have been extended to the setting of finite central extensions by Ban and Jantzen [6], while explicit descriptions of Aubert duals for concrete classes of representations of covering groups have remained largely unexplored. Therefore, based on the foundational work in [6], it is natural and timely to pursue explicit descriptions of Aubert duals for certain classes of representations of metaplectic groups.

The main goal of this paper is to explicitly determine the Aubert duals of strongly positive representations of $\widetilde{Sp}(n)$, where $\widetilde{Sp}(n)$ is the unique non-trivial two-fold central extension of F-rational points of a symplectic group $Sp(n) := \mathrm{Sp}(n, F)$ and F is a non-Archimedean local field of characteristic different from two. In [14], Matić obtained a complete classification of strongly positive representations of $\widetilde{Sp}(n)$, showing that they can be realized uniquely as irreducible subrepresentations of certain induced representations (see also the appendix of [12]). A natural next step is to determine the Aubert duals of those strongly positive representations. We show that their Aubert duals can be described in terms of explicit data of those induced representations.

Note that we expect possible further strong application on the construction of unitary representations. Very briefly, this project is motivated by analogous results for classical groups such as symplectic groups and special orthogonal groups, where in this case Aubert duality has been shown to preserve unitarity for strongly positive representations and, therefore, it plays an essential role in describing unitary representations for those cases [7, 16]. However, for the metaplectic case, new difficulties arise: the non-linearity of the group complicates the structure of Jacquet modules, and standard techniques must be carefully adapted to the covering setting; we leave this application for future work. Our work should be viewed as the metaplectic counterpart of Matić's explicit description of Aubert duals for classical groups [16]. Although we follow [16] closely, the corresponding result for metaplectic groups does not appear explicitly in the literature. Therefore, we record it here as we believe that it provides a key structural ingredient in the development of the unitary duals for metaplectic groups.

We note that the Aubert involution has also been used in the analysis of parabolically induced representations, as it allows one to transfer structural information on the composition factors of parabolically induced representations. Such an approach has been employed in various contexts, for instance in the study of composition factors and reducibility phenomena (see, e.g., [13]). The perspective adopted in the present paper is in a similar general spirit, in that the Aubert involution is used as a tool to organize and extract information from Jacquet modules, leading to explicit descriptions of representations.

We now describe the content of the paper. In Section 1, we introduce notation and preliminaries such as properties of Aubert duals. In Section 2, we describe several lemmas that are needed to describe the Aubert duals of certain representations. Finally, in Section 3, we describe explicitly the Aubert duals of strongly positive representations of $\widetilde{Sp}(n)$. We also consider the case of odd general spin groups in Section 4 and we are able to show that the structure is similar to the case of metaplectic groups.

1. Notation and preliminaries

Let $\mathrm{Sp}(n, F)$ be the symplectic group of rank n defined over a non-Archimedean local field F of characteristic different from two and let $Sp(n)$ be its F-rational points. Let $\widetilde{Sp}(n)$ be the metaplectic group of rank n, the unique non-trivial two-fold central extension of the symplectic group $Sp(n)$. In other words, the following holds:

$$1 \to \mu_2 \to \widetilde{Sp}(n) \to Sp(n) \to 1,$$

where $\mu_2 = \{1, -1\}$. The multiplication in $\widetilde{Sp}(n)$ (which is as a set given by $Sp(n) \times \mu_2$) is given by Rao's cocycle. Let $\widetilde{GL}(n)$ be a double cover of F-rational points of a general linear group $GL(n) := \mathrm{GL}(n, F)$, where the multiplication is given by

$$(g_1, \epsilon_1)(g_2, \epsilon_2) = (g_1 g_2, \epsilon_1 \epsilon_2 (\det\ g_1, \det\ g_2)_F)$$

with $\epsilon_i \in \mu_2$, where $(\cdot, \cdot)_F$ denotes the Hilbert symbol of the field F. Let α denote the character of $\widetilde{GL}(n)$ given by $\alpha(g) = (\det g, \det g)_F = (\det g, -1)_F$.

Let Σ denote the set of roots of $\widetilde{Sp}(n)$ with respect to a fixed minimal parabolic subgroup and let Δ stand for a basis of Σ. For $\Theta \subseteq \Delta$, we let $\widetilde{P}_\Theta$ be the standard parabolic subgroup of $\widetilde{Sp}(n)$ corresponding to Θ, which is defined as the preimage of the corresponding parabolic subgroup P of $Sp(n)$. If we write the Levi decomposition $P = MN$, then the unipotent radical N lifts to $\widetilde{Sp}(n)$. Therefore, we have a Levi decomposition $\widetilde{P}_\Theta = \widetilde{M}_\Theta N$. The Levi factor $\widetilde{M}_\Theta$ is not a product of the form $\widetilde{GL}(n_1) \times \cdots \times \widetilde{GL}(n_k) \times \widetilde{Sp}(n')$, but there is an epimorphism

$$\widetilde{GL}(n_1) \times \cdots \times \widetilde{GL}(n_k) \times \widetilde{Sp}(n') \twoheadrightarrow \widetilde{M}_\Theta.$$

For a parabolic subgroup $\widetilde{P}$ of $\widetilde{Sp}(n)$ with a Levi factor $\widetilde{M}$ and a representation σ of $\widetilde{M}$, we denote by $i_{\widetilde{M}}(\sigma)$ a normalized parabolically induced representation of $\widetilde{Sp}(n)$ induced from σ. For an admissible finite length representation π of $\widetilde{Sp}(n)$, the normalized Jacquet module of π with respect to the standard parabolic subgroup having a Levi factor $\widetilde{M}$ will be denoted by $r_{\widetilde{M}}(\pi)$.

Sometimes we use the following notation for normalized induced representations: every irreducible genuine representation of $\widetilde{M}$ is of the form $\pi_1 \otimes \cdots \otimes \pi_k \otimes \sigma$, where the representations $\pi_1, \ldots, \pi_k$, and σ are all genuine. Representations of $\widetilde{Sp}(n)$ that are parabolically induced from representation $\pi_1 \otimes \cdots \otimes \pi_k \otimes \sigma$ will be denoted by $\pi_1 \times \cdots \times \pi_k \rtimes \sigma$.

Let $\mathrm{Irr}(\widetilde{Sp}(n))$ (resp. $\mathrm{Irr}(\widetilde{GL}(n))$) be a set of irreducible genuine admissible representations of $\widetilde{Sp}(n)$ (resp. $\widetilde{GL}(n)$). For $\sigma \in \mathrm{Irr}(\widetilde{Sp}(n))$, let $r_k(\sigma)$ be the normalized Jacquet module of σ with respect to the standard maximal parabolic subgroup having a Levi factor $\widetilde{GL}(k) \times \widetilde{Sp}(n-k)$. We define $\mu^*(\sigma)$ by

$$\mu^*(\sigma) = \sum_{k=0}^{n} s.s.(r_k(\sigma)),$$

where $s.s.(r_k(\sigma))$ denotes the semisimplification of $r_k(\sigma)$. Tadić's structure formula for metaplectic groups is fully derived in [8, Proposition 4.5] and in this paper we use two special related cases from [15] (see Lemma 3.2 and [15, Theorem 6.1]).

Let $\rho \in \mathrm{Irr}(\widetilde{GL}(m))$ be unitary and cuspidal. We say that $[\nu^a\rho, \nu^{a+\ell}\rho] = \{\nu^a\rho, \nu^{a+1}\rho, \ldots, \nu^{a+\ell}\rho\}$ is a genuine segment, where $a \in \mathbb{R}$ and $\ell \in \mathbb{Z}_{\geqslant 0}$. We denote by $\delta([\nu^a\rho, \nu^{a+\ell}\rho])$ the unique irreducible subrepresentation of $\nu^{a+\ell}\rho \times \cdots \times \nu^a\rho$. Note that $\delta([\nu^a\rho, \nu^{a+\ell}\rho])$ is a genuine, essentially square-integrable representation attached to $[\nu^a\rho, \nu^{a+\ell}\rho]$. For an essentially square-integrable representation $\delta \in \mathrm{Irr}(\widetilde{GL}(n))$, there exists a unique $e(\delta) \in \mathbb{R}$ such that the representation $\nu^{-e(\delta)}\delta$ is square-integrable. Note that $e(\delta([\nu^a\rho, \nu^b\rho])) = \frac{a+b}{2}$.

For $\sigma \in \mathrm{Irr}(\widetilde{GL}(m))$, we let $\widetilde{\sigma}$ be a contragredient representation of σ.

We recall the subrepresentation version of the Langlands classification. For $1 \leq i \leq k$, suppose that $\delta_i \in \mathrm{Irr}(\widetilde{GL}(n_i))$ is essentially square-integrable such that $e(\delta_1) \leq e(\delta_2) \leq \cdots \leq e(\delta_k)$. Then the induced representation $\delta_1 \times \delta_2 \times \cdots \times \delta_k$ has a unique irreducible subrepresentation, which we denote by $L(\delta_1 \times \delta_2 \times \cdots \times \delta_k)$. This irreducible subrepresentation is called the Langlands subrepresentation.

Similarly, using [6, Theorem 3.1], we write a non-tempered $\pi \in \mathrm{Irr}(\widetilde{Sp}(n))$ as the unique irreducible subrepresentation of the induced representation of the form $\delta_1 \times \cdots \times \delta_k \rtimes \tau$, where $\tau \in \mathrm{Irr}(\widetilde{Sp}(n'))$ is tempered and $\delta_i \in \mathrm{Irr}(\widetilde{GL}(n_i))$ $(1 \leq i \leq k)$ are essentially square-integrable such that $e(\delta_1) \leq \cdots \leq e(\delta_k) < 0$. In this case, we write $\pi = L(\delta_1 \times \cdots \times \delta_k \rtimes \tau)$.

An irreducible representation σ of $\widetilde{Sp}(n)$ is called strongly positive or a strongly positive discrete series if for every embedding

$$\sigma \hookrightarrow \nu^{a_1}\rho_1 \times \cdots \times \nu^{a_k}\rho_k \rtimes \sigma_{cusp},$$

where ρ_i, $i = 1, \ldots, k$, are cuspidal unitary representations of $\widetilde{GL}(n_i)$ and σ_{cusp} is an irreducible cuspidal representation of $\widetilde{Sp}(n')$, we have $a_i > 0$ for each i.

The classification of strongly positive representations for $\widetilde{Sp}(n)$ is fully provided in [14] and every genuine strongly positive representation can be realized in a unique way (up to a certain permutation) as the unique irreducible subrepresentation of the induced representation of the following form:

$$(\prod_{i=1}^{m}\prod_{j=1}^{k_i}\delta([\nu^{\alpha_i-k_i+j}\rho_i, \nu^{\alpha_j^{(i)}}\rho_i])) \rtimes \sigma_{cusp} \tag{1.1}$$

where $\rho_i \in \mathrm{Irr}(\widetilde{GL}(n_i))$ $(1 \leq i \leq m)$ are mutually non-isomorphic, cuspidal, and α-self-contragredient (i.e. $\rho_i \simeq \alpha\widetilde{\rho_i}$), $\sigma_{cusp} \in \mathrm{Irr}(\widetilde{Sp}(n'))$ is cuspidal, $\alpha_i > 0$ such that $\nu^{\alpha_i}\rho_i \rtimes \sigma_{cusp}$ reduces, $k_i = \lceil \alpha_i \rceil$, where $\lceil \alpha_i \rceil$ denotes the smallest integer which is not smaller than α_i, and, for $i = 1, \ldots, m$, we have $-1 < \alpha_1^{(i)} < \alpha_2^{(i)} < \cdots < \alpha_{k_i}^{(i)}$ and $\alpha_j^{(i)} - \alpha_i \in \mathbb{Z}$ for $j = 1, \ldots, k_i$.

We finally recall the following definition and main properties of the Aubert involution from [6, Theorem 4.2 and 4.3].

Theorem 1.1. *Define the operator on the Grothendieck group of admissible representations of finite length of* $\widetilde{Sp}(n)$ *by*

$$D_{\widetilde{Sp}(n)} = \sum_{\Theta \subseteq \Delta} (-1)^{|\Theta|} i_{\widetilde{M}_\Theta} \circ r_{\widetilde{M}_\Theta}.$$

The operator $D_{\widetilde{Sp}(n)}$ *has the following properties:*

(1) $D_{\widetilde{Sp}(n)}$ *is an involution.*
(2) $D_{\widetilde{Sp}(n)}$ *takes irreducible representations to irreducible ones.*
(3) *If* σ *is an irreducible cuspidal representation, then* $D_{\widetilde{Sp}(n)}(\sigma) = (-1)^{|\Delta|}\sigma$.
(4) *For a standard Levi subgroup* $\widetilde{M} = \widetilde{M}_\Theta$, *we have*

$$r_{\widetilde{M}} \circ D_{\widetilde{Sp}(n)} = Ad(w) \circ D_{w^{-1}(\widetilde{M})} \circ r_{w^{-1}(\widetilde{M})},$$

where w *is the longest element of the set* $\{w \in W : w^{-1}(\Theta) > 0\}$.

If σ is an irreducible representation of $\widetilde{Sp}(n)$, we denote by $\hat{\sigma}$ the representation $\pm D_{\widetilde{Sp}(n)}(\sigma)$, taking the sign $+$ or $-$ such that $\hat{\sigma}$ is a positive element in the Grothendieck group of admissible representations of finite length of $\widetilde{Sp}(n)$. We call $\hat{\sigma}$ the Aubert dual of σ.

2. Several lemmas

In this subsection, we introduce three key lemmas used to prove our main theorem, i.e., Theorem 3.4.

Lemma 2.1. *Let $\sigma \in \mathrm{Irr}(\widetilde{Sp}(n))$ and suppose that $r_{\widetilde{M}}(\sigma) \geq \nu^{x_1}\rho_1 \otimes \cdots \otimes \nu^{x_m}\rho_m \otimes \sigma_{cusp}$, where $\rho_i \in \mathrm{Irr}(\widetilde{GL}(k))$ and $\sigma_{cusp} \in \mathrm{Irr}(\widetilde{Sp}(n'))$ are all cuspidal, and $\widetilde{M}$ is an appropriate standard Levi subgroup. Then*

$$r_{\widetilde{M}}(\hat{\sigma}) \geq \nu^{-x_1}\alpha\widetilde{\rho}_1 \otimes \cdots \otimes \nu^{-x_m}\alpha\widetilde{\rho}_m \otimes \sigma_{cusp}. \tag{2.1}$$

In particular, if $\sigma \in \mathrm{Irr}(\widetilde{Sp}(n))$ is strongly positive, then in (2.1) $-x_i < 0$ for $i = 1, \ldots, m$.

Proof. By assumption $r_{\widetilde{M}}(\sigma)$ contains

$$\nu^{x_1}\rho_1 \otimes \cdots \otimes \nu^{x_m}\rho_m \otimes \sigma_{cusp}$$

with respect to the appropriate standard Levi subgroup $\widetilde{M}$. Let w be as in Theorem 1.1 such that $w^{-1}(\widetilde{M}) = \widetilde{M}$. Theorem 1.1(4) implies

$$r_{\widetilde{M}} \circ D_{\widetilde{Sp}(n)}(\sigma) = Ad(w) \circ D_{\widetilde{M}} \circ r_{\widetilde{M}}(\sigma).$$

Since $\rho_1, \ldots, \rho_m, \sigma_{cusp}$ are irreducible genuine cuspidal representations,

$$D_{\widetilde{M}}(\nu^{x_1}\rho_1 \otimes \cdots \otimes \nu^{x_m}\rho_m \otimes \sigma_{cusp}) = \pm\nu^{x_1}\rho_1 \otimes \cdots \otimes \nu^{x_m}\rho_m \otimes \sigma_{cusp},$$

and

$$Ad(w)(\pm\nu^{x_1}\rho_1 \otimes \cdots \otimes \nu^{x_m}\rho_m \otimes \sigma_{cusp}) = \pm\nu^{-x_1}\alpha\widetilde{\rho}_1 \otimes \cdots \otimes \nu^{-x_m}\alpha\widetilde{\rho}_m \otimes \sigma_{cusp},$$

which completes the first assertion of the lemma. The second assertion follows directly due to definition of strongly positive representations. □

Lemma 2.2. *Let $\sigma \in \mathrm{Irr}(\widetilde{Sp}(n))$ be strongly positive and let σ_{cusp} be the partial cuspidal support of σ. Then $\hat{\sigma} = L(\delta_1 \times \cdots \times \delta_m \rtimes \sigma_{cusp})$, for irreducible genuine essentially square-integrable representations $\delta_1, \ldots, \delta_m$ of $\widetilde{GL}(n_i)$, such that $e(\delta_i) \leq e(\delta_{i+1}) < 0$ for $i = 1, \ldots, m-1$.*

Proof. By the Langlands classification [6, Theorem 3.1], $\hat{\sigma} = L(\delta_1 \times \cdots \times \delta_m \rtimes \tau)$, for irreducible essentially square-integrable representations $\delta_1, \ldots, \delta_m$ of $\widetilde{GL}(n_i)$, such that $e(\delta_i) \leq e(\delta_{i+1}) < 0$ for $i = 1, \ldots, m-1$, and a tempered representation $\tau \in \mathrm{Irr}(\widetilde{Sp}(n'))$ for some $n' \leq n$. If τ is not isomorphic to σ_{cusp}, by Casselman's criterion [5, Proposition 3.5], then there is an $x \geq 0$ and a cuspidal representation $\rho \in \mathrm{Irr}(\widetilde{GL}(t))$ such that τ is a subrepresentation of

$\nu^x\rho \rtimes \tau'$, for some $\tau' \in \mathrm{Irr}(\widetilde{Sp}(n''))$. Using Frobenius reciprocity, together with transitivity of Jacquet modules, we get a contradiction with Lemma 2.1. This completes the proof. ∎

We also need the following lemma, which generalizes [15, Lemma 3.4] to the case of metaplectic groups.

LEMMA 2.3. *Let $\sigma \in \mathrm{Irr}(\widetilde{Sp}(n))$ be strongly positive. Suppose that $\tau\otimes\sigma' \leq \mu^*(\sigma)$ for some $\tau \in \mathrm{Irr}(\widetilde{GL}(t))$ and $\sigma' \in \mathrm{Irr}(\widetilde{Sp}(n'))$. Then σ' is strongly positive.*

PROOF. The argument is analogous to that in [15, Lemma 3.4] and the main ideas are to use the strong positivity of σ and Frobenius reciprocity. We briefly explain its adaptation to the metaplectic group case without repeating the whole argument. Suppose that σ' is not strongly positive. Then, there exists a cuspidal representation $\nu^{c_1}\rho_1 \otimes \cdots \otimes \nu^{c_k}\rho_k \otimes \sigma_{cusp}$ ($\exists\ j$ such that $c_j \leq 0$) that appears in the Jacquet module of σ with respect to the appropriate parabolic subgroup. The Frobenius reciprocity implies that σ is a subrepresentation of $\nu^{c_1}\rho_1 \times \cdots \times \nu^{c_k}\rho_k \rtimes \sigma_{cusp}$, which contradicts the strong positivity of σ. ∎

3. MAIN THEOREMS ON THE AUBERT DUALS OF STRONGLY POSITIVE REPRESENTATIONS FOR $\widetilde{Sp}(n)$

In this subsection, we determine the Aubert duals of strongly positive representations of $\widetilde{Sp}(n)$. Note that our main idea of the proof follows similarly as in the classical group case and therefore we emphasize how we adapt ideas of the proof in [16] to metaplectic groups. In case the proof in [16] is omitted, we provide all the details.

We first consider the special case, the set of strongly positive representations whose cuspidal supports are the representation σ_{cusp} and twists of the representation ρ by unramified characters of the form $|\det(\cdot)|_F^s$ with $s \geq 0$, denoted $D(\rho, \sigma_{cusp})$, where $\rho \in \mathrm{Irr}(\widetilde{GL}(m))$ is α-self-contragredient (i.e. $\rho \simeq \alpha\widetilde{\rho}$) and cuspidal for some $m \in \mathbb{N}$ and $\sigma_{cusp} \in \mathrm{Irr}(\widetilde{Sp}(n'))$ is cuspidal for some $n' \in \mathbb{N}$. Via the method of the theta correspondence, Hanzer and Muić prove that there is a unique non-negative real number a such that $\nu^s\rho \rtimes \sigma_{cusp}$ ($s \in \mathbb{R}$) reduces if and only if $|s| = a$ [9, Theorem 4.1]. We call such a as the reducibility point of ρ and σ_{cusp}, which we denote by α_ρ.

LEMMA 3.1. *When the reducibility point α_ρ above is 0, σ_{cusp} is the only strongly positive representation in $D(\rho;\sigma_{cusp})$.*

PROOF. This is exactly as in [16, Section 3]. We briefly write how we applied the idea in [16, Section 3] to the case of metaplectic groups. Let $\sigma_{sp} \in D(\rho;\sigma_{cusp})$ be strongly positive and noncuspidal. Then we have

$$\sigma_{sp} \hookrightarrow \nu^{x_1}\rho\times\cdots\times\nu^{x_{m-1}}\rho\times\nu^{x_m}\rho\rtimes\sigma_{cusp} \cong \nu^{x_1}\rho\times\cdots\times\nu^{x_{m-1}}\rho\times\nu^{-x_m}\rho\rtimes\sigma_{cusp}$$

for some $x_i > 0$ for $i = 1, \ldots, m$ with $m \geq 1$ since $\nu^{x_m}\rho \rtimes \sigma_{cusp} \cong \nu^{-x_m}\rho \rtimes \sigma_{cusp}$ is irreducible. Here, we use our assumption $\rho \simeq \alpha\widetilde{\rho}$. This contradicts that σ_{sp} is strongly positive. Therefore, σ_{cusp} is the only strongly positive representation contained in $D(\rho; \sigma_{cusp})$. $\square$

Due to Lemma 3.1, we can assume that the reducibility point $\alpha_\rho > 0$ is positive. Let $k = \lceil \alpha_\rho \rceil$. The main results in [14] imply that there is a bijection between the set of strongly positive representations in $D(\rho; \sigma_{cusp})$ and the set of all ordered k-tuples $(a_1, \ldots, a_k)$ such that $a_i - \alpha_\rho \in \mathbb{Z}$, for $i = 1, \ldots, k$, and $-1 < a_1 < a_2 < \ldots < a_k$. The non-cuspidal strongly positive representation corresponding to such k-tuple $(a_1, \ldots, a_k)$ will be denoted by $\sigma_{(a_1,\ldots,a_k)}$, and it is the unique irreducible subrepresentation of

$$\delta([\nu^{\alpha_\rho-k+1}\rho, \nu^{a_1}\rho]) \times \delta([\nu^{\alpha_\rho-k+2}\rho, \nu^{a_2}\rho]) \times \cdots \times \delta([\nu^{\alpha_\rho}\rho, \nu^{a_k}\rho]) \rtimes \sigma_{cusp}. \tag{3.1}$$

In case $\alpha_\rho - k + i > a_i$, we allow the segment $[\nu^{\alpha_\rho-k+i}\rho, \nu^{a_i}\rho]$ to be an empty set and correspondingly delete $\delta([\nu^{\alpha_\rho-k+i}\rho, \nu^{a_i}\rho])$ from (3.1).

We now recall the following Tadić's structure formula, which is a special case, i.e., $D(\rho; \sigma_{cusp})$-case of [15, Theorem 6.1].

Lemma 3.2 ([15]). *Let $\sigma_{(a_1,\ldots,a_k)}$ be an irreducible strongly positive representation in $D(\rho; \sigma_{cusp})$. Then we have*

$$\mu^*(\sigma_{(a_1,\ldots,a_k)}) = \sum L(\delta([\nu^{b_1+1}\rho, \nu^{a_1}\rho]) \times \cdots \times \delta([\nu^{b_k+1}\rho, \nu^{a_k}\rho])) \otimes \sigma_{(b_1,\ldots,b_k)},$$

where the sum runs over all ordered k-tuples $(b_1, \ldots, b_k)$ such that $b_i - \alpha_\rho \in \mathbb{Z}$ and $\alpha_\rho - k + i - 1 \leq b_i \leq a_i$ for $i = 1, \ldots, k$. Again, in case $\alpha_\rho - k + i > a_i$, we allow the segment $[\nu^{\alpha_\rho-k+i}\rho, \nu^{a_i}\rho]$ to be an empty set and correspondingly delete $\delta([\nu^{b_i+1}\rho, \nu^{a_i}\rho])$.

For $\sigma_{(a_1,\ldots,a_k)} \in D(\rho; \sigma_{cusp})$, Lemma 2.2 implies that there exist irreducible essentially square-integrable representations $\delta_1, \ldots, \delta_s$ with $e(\delta_i) \leq e(\delta_{i+1}) < 0$ for $i = 1, \ldots, s-1$ such that $\widehat{\sigma_{(a_1,\ldots,a_k)}}$ is of the form $L(\delta_1 \times \cdots \times \delta_s \rtimes \sigma_{cusp})$. For $i = 1, \ldots, s$, we can write $\delta_i = \delta([\nu^{-x_i}\rho, \nu^{-y_i}\rho])$ with $x_i > 0$ and $y_i > 0$ such that $x_i - \alpha_\rho, y_i - \alpha_\rho \in \mathbb{Z}$, and we denote $x_i - y_i$ by z_i.

We now start to describe the explicit data, i.e., its exponents in the Jacquet modules. Let $k' := \min\{i : a_i \geq \alpha_\rho - k + i\}$, i.e., k' is the minimal index i such that the segment $[\nu^{\alpha_\rho-k+i}\rho, \nu^{a_i}\rho]$ is nonempty.

Lemma 3.3. 1. *Let $\sigma_{(a_1,\ldots,a_k)} \in D(\rho; \sigma_{cusp})$ be as above and let $\sigma_{(b_1,\ldots,b_k)} \in D(\rho; \sigma_{cusp})$ be as in Lemma 3.2 such that*

$$r_{\widetilde{M}}(\sigma_{(a_1,\ldots,a_k)}) \geq \nu^{y_1}\rho \otimes \nu^{y_1+1}\rho \otimes \cdots \otimes \nu^{x_1}\rho \otimes \nu^{y_2}\rho \otimes \cdots \otimes \nu^{x_{i-1}}\rho \otimes \sigma_{(b_1,\ldots,b_k)}, \tag{3.2}$$

where $\widetilde{M}$ is an appropriate standard Levi subgroup. Then, there exists $j \in \{k', \ldots, k - z_i\}$ such that $y_i + r = b_{j+r}$ for $r = 0, 1, \ldots, z_i$. Furthermore, if $j \geq 2$, then $b_j \geq b_{j-1} + 2$.

2. $x_{m+1} = x_m - 1$ *and* $y_{m+1} < y_m$ *for* $m = 1, \dots, s-1$. *Also,* $x_1 = a_k$.

PROOF. We follow the arguments in [16] but write in detail for completeness. We first prove the part 1. Lemma 3.2 implies $b_l \le a_l$ $(l = 1, \dots, k)$ and

$$(3.3)\quad r_{\widetilde{M'}}(\sigma_{(b_1,\dots,b_k)}) \ge \nu^{y_i}\rho \otimes \nu^{y_i+1}\rho \otimes \cdots \otimes \nu^{x_i}\rho \otimes \nu^{y_{i+1}}\rho \otimes \cdots \otimes \nu^{x_s}\rho \otimes \sigma_{cusp},$$

where $\widetilde{M'}$ is an appropriate standard Levi subgroup.

From Lemma 3.2, a comparison of the unitary exponents y_i and b_l $(l = 1, \dots, k)$ shows that there exists a $j \in \{k', \dots, k\}$ such that $y_i = b_j$ and $b_j \ge b_{j-1} + 2$ if $j \ge 2$. Again, comparing the exponents $y_i + 1$, $b_j - 1$, and b_{j+1}, we have $y_i + 1 = b_{j+1}$ and $b_{j+1} = b_j + 1$. Repeating the same argument, we obtain $y_i + r = b_{j+r}$ for $r = 0, 1, \dots, z_i$ and $j \le k - z_i$.

Now we prove the second claim of the lemma. For $1 \le m \le s-1$, let $\sigma_m \in D(\rho; \sigma_{cusp})$ be such that

$$(3.4)\quad r_{\widetilde{M}_m}(\sigma_{(a_1,\dots,a_k)}) \ge \nu^{y_1}\rho \otimes \nu^{y_1+1}\rho \otimes \cdots \otimes \nu^{x_1}\rho \otimes \nu^{y_2}\rho \otimes \cdots \otimes \nu^{x_{m-1}}\rho \otimes \sigma_m,$$

and

$$(3.5)\qquad r_{\widetilde{M}'_m}(\sigma_m) \ge \nu^{y_m}\rho \otimes \nu^{y_m+1}\rho \otimes \cdots \otimes \nu^{x_m}\rho \otimes \sigma_{m+1},$$

where $\widetilde{M}_m$ and $\widetilde{M}'_m$ are appropriate standard Levi subgroups, $\sigma_1 = \sigma_{(a_1,\dots,a_k)}$, and $\sigma_s = \sigma_{cusp}$. We use this notation in the remainder of the proof. Note that Lemma 2.3 implies that each σ_m $(1 \le m \le s)$ is strongly positive. Suppose that there is $t \in \{1, \dots, s-1\}$ such that $x_t \le x_{t+1}$. We write $\sigma_t = \sigma_{(b_1,\dots,b_k)}$ since σ_t is strongly positive. The first part of the lemma implies that there is $j_1 \in \{k', \dots, k - z_t\}$ such that $y_t + r = b_{j_1+r}$ for $r = 0, 1, \dots, z_t$ (here, k' is exactly defined as in $\sigma_{(a_1,\dots,a_k)}$).

Since $b_1 < b_2 < \dots < b_k$, Lemma 3.2 implies that we can also describe σ_{t+1} in terms of classification of strongly positive representations as follows:

$$\sigma_{t+1} = \sigma_{\left(b_1,\dots,b_{j_1-1},b_{j_1}-1,b_{j_1+1}-1,\dots,b_{j_1+z_t}-1,b_{j_1+z_t+1},\dots,b_k\right)}.$$

Since $b_{j_1+z_t+1} - 1 > b_{j_1+z_t} - 1$ and $x_t \le x_{t+1}$, Lemma 3.2 implies that y_{t+1} cannot be any of $\{b_1, b_2, \dots, b_{j_1-1}, b_{j_1} - 1, b_{j_1+z_t} - 1\}$. Therefore, we obtain $y_{t+1} \ge b_{j_1+z_t+1} > b_{j_1+z_t} = x_t$. This is a contradiction with $e(\delta_t) \le e(\delta_{t+1})$. We conclude that $x_t > x_{t+1}$ for $1 \le t \le s-1$.

If we write σ_m as $\sigma_{\left(b_1^{(m)},\dots,b_k^{(m)}\right)}$, a comparison of the largest unitary exponents implies $x_m = b_k^{(m)}$. In particular, we have $x_1 = a_k$ and $x_s = \alpha_\rho$. For $m = 1, \dots, s$, we define $j_m = 1$ if $b_{j-1}^{(m)} = b_j^{(m)} - 1$ for all $j = 2, \dots, k$ and $j_m = \max\left\{j : b_{j-1}^{(m)} < b_j^{(m)} - 1\right\}$ otherwise. Using Lemma 3.2 again, we have $y_m = b_{j_m}^{(m)}$ and $\left(b_1^{(m+1)}, \dots, b_k^{(m+1)}\right) = \left(b_1^{(m)}, \dots, b_{j_m-1}^{(m)}, b_{j_m}^{(m)} - 1, \dots, b_k^{(m)} - 1\right)$ for $m = 1, \dots, s-1$. Therefore, we have $x_{m+1} = x_m - 1$ and $j_m \ge j_{m+1}$ for

$m = 1, \ldots, s-1$. This also implies $b^{(m+1)}_{j_{m+1}} \leq b^{(m)}_{j_m} - 1$, i.e., $y_{m+1} < y_m$ for $m = 1, \ldots, s-1$. □

With Lemma 3.3, we describe the Aubert dual of a strongly positive representation $\sigma_{(a_1,\ldots,a_k)} \in D(\rho; \sigma_{cusp})$. We write its proof in detail, since the proof is omitted in [16]. This needs to be written, since we need to verify that the argument holds for metaplectic groups.

THEOREM 3.4. *The Aubert dual of the strongly positive representation* $\sigma_{(a_1,\ldots,a_k)}$ *is the unique irreducible subrepresentation of the induced representation*

$$\left(\prod_{i=1}^{k} \prod_{j=-a_{k-i+1}}^{-a_{k-i}-2} \delta\left(\left[\nu^{j-i+1}\rho, \nu^{j}\rho\right]\right) \right) \rtimes \sigma_{cusp},$$

where $a_0 = \alpha_\rho - \lceil \alpha_\rho \rceil - 1$.

PROOF. We describe x_i and y_i for $i = 1, \ldots, s$. In the proof of Lemma 3.3, we show that $x_{m+1} = x_m - 1$, $x_m = b^{(m)}_k$, $y_m = b^{(m)}_{j_m}$ for $m = 1, \ldots, s-1$. Especially $x_1 = a_k$. From the definitions, note that $a_{i+1} = a_i + 1$ for $i = 1, \ldots, k'-2$.

If $a_{k-1} < a_k - 1$, then $j_1 = k, y_1 = a_k$, and $(b^{(2)}_1, b^{(2)}_2, \ldots, b^{(2)}_{k-1}, b^{(2)}_k) = (a_1, a_2, \ldots, a_{k-1}, a_k - 1)$. If $a_{k-1} < a_k - 2$, then $j_2 = k, y_2 = a_{k-1}$, and $(b^{(3)}_1, b^{(3)}_2, \ldots, b^{(3)}_{k-1}, b^{(3)}_k) = (a_1, a_2, \ldots, a_{k-1}, a_k - 2)$. We continue this process $a_k - a_{k-1} - 1$ times until we get $(a_1, a_2, \ldots, a_{k-2}, a_{k-1}, a_{k-1}+1)$ and $x_{a_k - a_{k-1} - 1} = y_{a_k - a_{k-1} - 1} = -a_{k-1} - 2$. Up to this step, the process gives the following product:

$$\prod_{j=-a_k}^{-a_{k-1}-2} \nu^j \rho.$$

Next, if $a_{k-2} < a_{k-1} - 1$, we continue the same process $a_{k-1} - a_{k-2} - 1$ times until we get $(a_1, a_2, \ldots, a_{k-3}, a_{k-2}, a_{k-2}+1, a_{k-2}+2)$ and $x_{a_k - a_{k-2} - 2} = -a_{k-2} - 3, y_{a_k - a_{k-2} - 2} = -a_{k-2} - 2$. Up to this step, the process gives the following product:

$$\prod_{j=-a_k}^{-a_{k-1}-2} \nu^j \rho \times \prod_{j=-a_{k-1}}^{-a_{k-2}-2} \delta([\nu^{j-1}\rho, \nu^j \rho]).$$

If we continue the above process until we get $(a_1, a_1+1, \ldots, a_1+k-2, a_1+k-1)$, this completely provides all information about x_i and y_i for $i = 1, \ldots, s$ and completes the proof. □

We now consider the general case. Let $D(\rho_1, \ldots, \rho_m; \sigma_{cusp})$ be a set of strongly positive representations whose cuspidal supports are the representation σ_{cusp} and twists of the representation ρ_i $(i = 1, \ldots, m)$ by unramified characters of the form $|\det(\cdot)|_F^s$ with $s \geq 0$, where ρ_i is an irreducible α-self-contragredient (i.e. $\rho_i \simeq \alpha\widetilde{\rho_i}$) representation of $\widetilde{GL}(m_i)$ for some $m_i \in \mathbb{N}$ and σ_{cusp} is an irreducible cuspidal representation of $\widetilde{Sp}(n')$ for some $n' \in \mathbb{N}$. Let σ be a strongly positive representation in $D(\rho_1, \ldots, \rho_m; \sigma_{cusp})$ and let α_i be the unique non-negative real number such that $\nu^{\alpha_i}\rho_i \rtimes \sigma_{cusp}$ reduces. Let $k_i = \lceil \alpha_i \rceil$ and $a_0^{(i)} = \alpha_i - \lceil \alpha_i \rceil - 1$, for $i = 1, \ldots, m$. Classification of strongly positive representations [12, 15] imply that for $i = 1, \ldots, m$ there exist $a_1^{(i)}, \ldots, a_{k_i}^{(i)}$ such that $-1 < a_1^{(i)} < \cdots < a_{k_i}^{(i)}$ and $a_j^{(i)} - \alpha_i \in \mathbb{Z}$ for $j = 1, \ldots, k_i$, such that σ is the unique irreducible subrepresentation of the induced representation

$$\left(\prod_{i=1}^{m}\prod_{j=1}^{k_i}\delta\left(\left[\nu^{\alpha_i-k_i+j}\rho_i, \nu^{a_j^{(i)}}\rho_i\right]\right)\right) \rtimes \sigma_{cusp}. \tag{3.6}$$

We now explain how we generalize the arguments for the proof of the special case, i.e., $D(\rho; \sigma_{cusp})$ to any strongly positive representation in $D(\rho_1, \ldots, \rho_m; \sigma_{cusp})$. Note that Lemmas 2.1, 2.2, and 2.3 apply to any strongly positive representation, and Lemma 3.2 has a general version in [15, Theorem 6.1].

Theorem 3.5. *The Aubert dual of the strongly positive representation $\sigma \in D(\rho_1, \ldots, \rho_m; \sigma_{cusp})$ is the unique irreducible subrepresentation of the induced representation*

$$\left(\prod_{i=1}^{m}\prod_{l=1}^{k_i}\prod_{j=-a_{k_i-l+1}^{(i)}}^{-a_{k_i-l}^{(i)}-2}\delta\left(\left[\nu^{j-l+1}\rho_i, \nu^{j}\rho_i\right]\right)\right) \rtimes \sigma_{cusp}.$$

Proof. The proof follows the same line as classical group case. We briefly explain the main ideas of the proof for metaplectic group case since the proof for classical group case is omitted in [16]. Let $\sigma \in D(\rho_1, \ldots, \rho_m; \sigma_{cusp})$ be a strongly positive representation which is realized as a unique irreducible subrepresentation of an induced representation of the form (3.6). Lemma 2.2 implies that there exist essentially square-integrable irreducible representations $\delta_1^{(i)}, \ldots, \delta_{k_i}^{(i)}$ for $i = 1, \ldots, m$ such that $\widehat{\sigma}$ is of the form

$$L\left(\left(\prod_{i=1}^{m}\delta_1^{(i)} \times \cdots \times \delta_{k_i}^{(i)}\right) \rtimes \sigma_{cusp}\right), \tag{3.7}$$

where for $i = 1, \ldots, m$ and $j = 1, \ldots k_i$ we can write

$$\delta_j^{(i)} = \delta\left(\left[\nu^{-x_j^{(i)}}\rho_i, \nu^{-y_j^{(i)}}\rho_i\right]\right)$$

with $x_j^{(i)} > 0$ and $y_j^{(i)} > 0$ such that $x_j^{(i)} - \alpha_i, y_j^{(i)} - \alpha_i \in \mathbb{Z}$.

First, we apply the whole argument of the proof of Lemma 3.3 to $\delta_1^{(1)} \times \cdots \times \delta_{k_1}^{(1)}$ in (3.7). In other words, we apply the arguments so that only twists of ρ_1 appear in the GL-part of cuspidal supports in (3.2) and (3.4) and then describe $x_j^{(1)}$ and $y_j^{(1)}$ for $j = 1, \ldots, k_1$. Furthermore, it is well known that for $1 \le s \ne t \le m$, $1 \le p \le k_s$, and $1 \le q \le k_t$, we have $\delta\left(\left[\nu^{-x_p^{(s)}}\rho_s, \nu^{-y_p^{(s)}}\rho_s\right]\right) \times \delta\left(\left[\nu^{-x_q^{(t)}}\rho_t, \nu^{-y_q^{(t)}}\rho_t\right]\right) \cong \delta\left(\left[\nu^{-x_q^{(t)}}\rho_t, \nu^{-y_q^{(t)}}\rho_t\right]\right) \times \delta\left(\left[\nu^{-x_p^{(s)}}\rho_s, \nu^{-y_p^{(s)}}\rho_s\right]\right)$ since $\rho_s \ncong \rho_t$. Then we apply the whole argument of the proof of Lemma 3.3 again to representation of index $i = 2$ to describe $x_j^{(2)}$ and $y_j^{(2)}$ for $j = 1, \ldots, k_2$. We continue this process for all $1 \le i \le m$. This completes the proof. □

4. The case of odd general spin groups

We briefly discuss the case of odd general spin groups. Let $\mathbf{G}(n)$ denote the split general spin group $\mathbf{GSpin}(2n+1)$ of semisimple rank n defined over F, that is, the connected reductive F-group of type B_n whose based root datum is dual to that of $\mathbf{GSp}_{2n}$. Equivalently, $\mathbf{G}(n)$ is the split reductive group of type B_n whose derived subgroup is the simply connected group $\mathbf{Spin}(2n+1)$. We write $G(n) = \mathbf{G}(n)(F)$.

Most of the arguments developed in the metaplectic case can be extended to the case of odd general spin groups with only minor modifications, and we therefore briefly provide the outline.

First, Theorem 1.1 remains valid for $G(n)$, since it is a connected reductive group and the Aubert involution is well-defined in this case. Lemma 2.1 also holds for $G(n)$ since the argument of the proof depends on analysis of Jacquet modules and properties of the Aubert involutions, although its formulation requires a slight adjustment due to the difference of the Weyl group action on the representations as follows.

Lemma 4.1. *Let $\sigma \in \mathrm{Irr}(G(n))$ and suppose that $r_M(\sigma) \ge \nu^{x_1}\rho_1 \otimes \cdots \otimes \nu^{x_m}\rho_m \otimes \sigma_c$, where $\rho_i \in \mathrm{Irr}(GL(k_i))$ and $\sigma_c \in \mathrm{Irr}(G(n'))$ are all cuspidal, and M is an appropriate standard Levi subgroup. Then*

$$r_{\widetilde{M}}(\hat{\sigma}) \ge \nu^{-x_1}\widetilde{\rho}_1\omega_{\sigma_c} \otimes \cdots \otimes \nu^{-x_m}\widetilde{\rho}_m\omega_{\sigma_c} \otimes \sigma_c, \tag{4.1}$$

where $\widetilde{\rho}_i$ is a contragredient representation of ρ_i and ω_{σ_c} is the central character of σ_c.

We now recall the results on the classification of strongly positive representations of odd general spin groups.

Theorem 4.2 ([12]). *Every strongly positive representation $\sigma \in \mathrm{Irr}(G(n))$ can be realized in a unique way (up to a certain permutation) as the unique* *irreducible subrepresentation of the induced representation of the following form:*

$$(\prod_{i=1}^{m}\prod_{j=1}^{k_i}\delta([\nu^{\alpha_i-k_i+j}\rho_i,\nu^{\alpha_j^{(i)}}\rho_i]))\rtimes\sigma_c, \tag{4.2}$$

where $\rho_i\in\mathrm{Irr}(GL(n_i))$ $(1\leq i\leq m)$ *are mutually non-isomorphic, cuspidal, and essentially self-contragredient (i.e.* $\rho_i\simeq\omega_{\sigma_c}\widetilde{\rho_i}$*),* $\sigma_c\in\mathrm{Irr}(G(n'))$ *is cuspidal,* $\alpha_i>0$ *such that* $\nu^{\alpha_i}\rho_i\rtimes\sigma_c$ *reduces,* $k_i=\lceil\alpha_i\rceil$*, and, for* $i=1,\dots,m$*, we have* $-1<\alpha_1^{(i)}<\alpha_2^{(i)}<\cdots<\alpha_{k_i}^{(i)}$ *and* $\alpha_j^{(i)}-\alpha_i\in\mathbb{Z}$ *for* $j=1,\dots,k_i$*.*

Lemma 2.2, Lemma 2.3, Lemma 3.1, and Lemma 3.3 also hold for $G(n)$ since their proofs depend on Casselman's criterion, Jacquet modules technique, Frobenius reciprocity, and classification of strongly positive representations (Theorem 4.2). Especially, Lemma 3.3 also depends on combinatorial arguments on the exponents of the representation of GL. The odd $GSpin$ analogue of Theorem 3.4 follows since its proof depends on Lemma 3.3. Finally, to generalize Theorem 3.4 to Theorem 3.5 in the case of odd $GSpin$ groups, we only need the irreducibility properties of the representations of GL. Therefore, odd $GSpin$ analogue of Theorem 3.5 follows. We now briefly prove the odd $GSpin$ version of Lemma 3.2.

LEMMA 4.3. *Let* $\rho\in\mathrm{Irr}(GL(k))$ *be an irreducible cuspidal and essentially self-contragredient representation and* $\sigma_c\in\mathrm{Irr}(G(n))$ *be a cuspidal representation. Let* $\sigma:=\sigma_{(a_1,\dots,a_k)}\in D(\rho;\sigma_c)$ *be a strongly positive representation similarly defined as in (3.1) using Theorem 4.2. Then we have*

$$\mu^*(\sigma_{(a_1,\dots,a_k)})=\sum L(\delta([\nu^{b_1+1}\rho,\nu^{a_1}\rho])\times\cdots\times\delta([\nu^{b_k+1}\rho,\nu^{a_k}\rho]))\otimes\sigma_{(b_1,\dots,b_k)},$$

where the sum runs over all ordered k-tuples $(b_1,\dots,b_k)$ *such that* $b_i-\alpha_\rho\in\mathbb{Z}$ *and* $\alpha_\rho-k+i-1\leq b_i\leq a_i$ *for* $i=1,\dots,k$*, and* α_ρ *is a reducibility point of* ρ *and* σ_c*.*

PROOF. We briefly explain how we generalize the main results in [15] to the case of odd general spin groups. First, odd $GSpin$ analogs of all results in [15, Section 2] are already constructed in [12, Section 3]. Moreover, [12, Lemmas 3.1–3.3] concern representations of general linear groups and therefore apply without modification in our setting. The symplectic analogue of Lemma 2.3 is given in [15, Lemma 3.4], whose proof relies mainly on strong positivity and Frobenius reciprocity. As these arguments do not depend on specific structural features of symplectic groups, they remain valid for odd general spin groups as well. Finally, the main arguments in the proofs of [12, Proposition 4.1, 4.2, and 4.5] are using Jacquet modules technique and compare its unitary exponents and therefore their odd $GSpin$ analogue exactly follows the ones for odd special orthogonal groups, and we do not repeat the same arguments. This completes the proof. □

Let ρ_i $(i = 1, \ldots, m)$ and σ_c be as in Theorem 4.2. Let $D(\rho_1, \ldots, \rho_m; \sigma_c)$ be a set of strongly positive representations whose cuspidal supports are σ_c and twists of the representation ρ_i $(i = 1, \ldots, m)$ by unramified characters of the form $|\det(\cdot)|_F^s$. Then we have the following statement.

THEOREM 4.4. *Let $\sigma \in D(\rho_1, \ldots, \rho_m; \sigma_c)$ be a strongly positive representation as in Theorem 4.2. Its Aubert dual is the unique irreducible subrepresentation of the following induced representation:*

$$\left(\prod_{i=1}^{m}\prod_{l=1}^{k_i}\prod_{j=-a^{(i)}_{k_i-l+1}}^{-a^{(i)}_{k_i-l}-2} \delta\left(\left[\nu^{j-l+1}\rho_i, \nu^j\rho_i\right]\right)\right) \rtimes \sigma_c.$$

ACKNOWLEDGEMENTS.

The first author would like to thank Ivan Matić for answering questions and for helpful discussions. The first author has been supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIP) (No. RS-2022-0016551 and No. RS-2024-00415601 (G-BRL)). The second author has been supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIP) (No. RS-2024-00415601 (G-BRL)). The authors would like to thank the referee for a number of corrections and very useful suggestions.

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Received: 6.1.2026.
Revised: 28.4.2026.

AUBERTOVI DUALI JAKO POZITIVNIH REPREZENTACIJA ZA METAPLEKTIČKE GRUPE

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Sažetak. Određujemo Aubertove duale jako pozitivnih reprezentacija metaplektičke grupe $\widetilde{Sp}(n)$ nad ne-Arhimedovim lokalnim poljem F karakteristike različite od dva. Koristeći Matićevu klasifikaciju i eksplicitnu analizu Jacquetovih modula, opisujemo ove duale pomoću preciznih inducirajućih podataka. Naši rezultati proširuju poznate opise za klasične grupe na slučaj metaplektičkih grupa te razjašnjavaju ulogu Aubertova dualiteta za nelinearne natkrivajuće grupe, pružajući temelj za buduće primjene u proučavanju unitarnih reprezentacija u tim slučajevima. Nadalje, pokazujemo da se ista metoda primjenjuje i na neparne opće spinske grupe, dajući eksplicitan opis Aubertovih duala i u tom okruženju.