

A Distributed Step-Size Control Mechanism for Robust Set-Membership APA in Dynamic Channel Equalization

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Abstract—The performance of the Affine Projection Algorithm (APA) and its Set-Membership (SM-APA) variants in adaptive channel equalization is limited by step-size selection. While fixed step-sizes offer a poor trade-off, many variable step-size (VSS) schemes fail to achieve optimal performance, particularly in non-stationary environments. This paper introduces a novel step-size control mechanism by integrating the Distributed Step-Size LMS (DSSLMS) strategy into APA and SM-APA frameworks. The method employs a two-stage process where a base step-size is dynamically adapted based on signal and error characteristics and then distributed to form an adaptive step-size vector, creating the DSSLMS-APA and two DSSLMS-SM-APA algorithms. The proposed algorithms are evaluated against conventional and state-of-the-art VSS benchmarks. Simulation results confirm superior performance. In a stationary environment, the proposed DSSLMS-SM-APA achieves a steady-state MSE of (2×10^{-3}) , outperforming both the conventional APA (3×10^{-3}) and the benchmark VSS methods (VSS-APA-MSE and VSS-APA-EC). In three distinct non-stationary scenarios, the proposed DSSLMS-based algorithms exhibit the fastest reconvergence and maintain the lowest post-change MSE, successfully tracking system dynamics where other methods struggle. The algorithms provide a robust, efficient solution for step-size adaptation that balances convergence speed, steady-state accuracy, and tracking agility, making them suitable for real-time channel equalization applications.

Index Terms—Adaptive filtering, Affine Projection Algorithm (APA), Set-Membership APA (SM-APA), Variable Step-Size (VSS), Distributed Step-Size LMS (DSSLMS), Adaptive Equalization, Non-stationary signal processing.

I. INTRODUCTION

ADAPTIVE filtering is a fundamental component of digital signal processing, with wide applications in system identification, noise reduction, and particularly, adaptive channel equalization [1][2]. In communication systems, adaptive equalizers are used to mitigate signal distortions such as Inter-Symbol Interference (ISI) caused by multipath channels. The primary objective of an adaptive equalizer is to iteratively

modify its filter coefficients to minimize the error between its output and the original transmitted signal.

The Affine Projection Algorithm (APA) [3][4] is a popular choice for this task, offering a favorable trade-off between the fast convergence of Recursive Least Squares (RLS) algorithms and the simplicity of the Normalized Least Mean Squares (NLMS) algorithm. Further enhancements, such as the use of APA to beamforming and channel-tracking applications, where adaptive filters enhance robustness against non-stationary interference and varying channel responses [5][6]. These variants reduce computational load by updating filter coefficients only when the error exceeds a certain threshold and by updating only a subset of coefficients.

Despite these advances, the performance of APA and SM-APA is heavily reliant on the step-size parameter. A fixed step-size presents a well-known compromise between convergence speed and steady-state error. While numerous variable step-size (VSS) schemes have been proposed to address this, many exhibit their own limitations, such as slow adaptation, high misadjustment, or instability, particularly when applied to the data-selective update nature of SM-APA. This trade-off remains especially problematic in non-stationary environments, where channel characteristics can change abruptly, demanding both rapid tracking and low steady-state error.

This paper addresses this limitation by proposing a dynamic step-size control mechanism based on the Distributed Step-Size LMS (DSSLMS) algorithm [7]. The primary contribution is the development and evaluation of DSSLMS-enhanced algorithms specifically for non-stationary channel equalization. Our simulations demonstrate that this integration significantly improves the balance of tracking agility and steady-state accuracy, outperforming not only conventional fixed-step-size methods but also prominent state-of-the-art VSS benchmarks.

To clearly summarize the novel aspects of this work, the main contributions are as follows:

- The integration of the Distributed Step-Size LMS (DSSLMS) strategy into the APA and SM-APA frameworks, creating a novel two-stage (base adaptation and distribution) step-size control mechanism.
- The development of three new robust algorithms: DSSLMS-APA, DSSLMS-SM-APA-1, and DSSLMS-SM-APA-2, specifically designed for non-stationary environments.
- A comprehensive performance evaluation demonstrating

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that the proposed algorithms achieve a superior balance of convergence speed, steady-state accuracy, and tracking agility compared to conventional and state-of-the-art variable step-size benchmarks in both stationary and non-stationary channel equalization scenarios.

- A theoretical analysis of the proposed algorithms, characterizing their mean-square performance and computational complexity, highlighting their efficiency and robustness.

The remainder of this paper is organized as follows. Section II reviews related work on variable step-size algorithms. Section III details the formulation of the proposed DSSLMS-APA and DSSLMS-SM-APA algorithms. Section IV provides a theoretical analysis of their performance and complexity. Section V presents the experimental framework and discusses the simulation results. Finally, Section VI concludes the paper.

II. RELATED WORK

The challenge of optimizing the step-size parameter in adaptive filters has been a subject of extensive research. While the conventional fixed-step-size APA provides a baseline, its performance is inherently a compromise. To address this, numerous Variable Step-Size (VSS) schemes have been developed over the years, which can be broadly categorized based on their adaptation criteria.

Recent studies have investigated several strategies for step-size adaptation. For instance, Shin *et al.* [8] and Nariman and Majeed [9] proposed approaches to achieve faster convergence and lower misadjustment in system tasks, while a prominent family of VSS algorithms adapts the step-size based on a function of the error signal. Early approaches proposed making the step-size proportional to the squared error, allowing for aggressive adaptation when the error is large and conservative updates when it is small [10]. This concept has been extended to more sophisticated schemes that aim to minimize the Mean-Square Deviation (MSD) or the Mean-Square Error (MSE) at each iteration. The work by Li *et al.* [11] provides a foundational analysis for such methods, and the VSS-APA-MSE algorithm [12] used as a benchmark in this study is a direct result of this line of research.

Other approaches, such as the VSS-APA-EC benchmark [13], utilize the statistical relationship (e.g., correlation) between the a priori and a posteriori error signal to drive the step-size adaptation. However, applying these conventional VSS techniques to Set-Membership (SM) algorithms presents unique challenges. Most VSS mechanisms are designed for full-update filters, assuming a continuous stream of error information is available to guide the adaptation. In SM filtering, the updates are sparse and data-dependent, occurring only when an error bound is violated. This sparsity can starve traditional VSS mechanisms of the information needed to adapt correctly, potentially leading to step-size stagnation or erratic behavior.

Some efforts have been made to design VSS schemes specifically for SM algorithms, for example, by adapting the step-size only during coefficient updates [14]. While effective, such approaches link the step-size adaptation directly to the sparse coefficient update schedule.

The DSSLMS strategy [15] offers a different paradigm. By updating a base step-size at every iteration—regardless of whether a coefficient update occurs—it decouples the step-size adaptation from the sparse update rule of the SM framework. This ensures the step-size is always current and ready to react to changes. The novelty of the work presented in this paper lies in the integration of this two-stage (base adaptation and distribution) DSSLMS mechanism into the SM-APA framework, creating a robust VSS solution specifically tailored to overcome the limitations of applying traditional VSS schemes to data-selective adaptive filters in non-stationary environments.

Very recently, research in this area has continued to evolve. For instance, Li *et al.* [16] developed a robust VSS-APA for impulsive noise, while Chen *et al.* [17] incorporated momentum for faster tracking. In the realm of Set-Membership filtering, Wang *et al.* [18] proposed adaptive error bounds to handle non-stationarity. A recent survey has also highlighted the continued importance of VSS techniques in adaptive filtering [19]. Furthermore, sophisticated methods using deep learning for step-size control have emerged [20].

While recent deep learning-based VSS methods offer data-driven adaptability [20] (e.g., neural network-predicted step-size [21] and RL-based adaptation [22]), they typically require extensive offline training, significant computational overhead, and may lack the interpretability of model-based approaches. In contrast, the proposed DSSLMS mechanism is lightweight, operates fully online without training, and provides deterministic and interpretable performance. This makes it especially suitable for real-time adaptive filtering applications where low latency, computational efficiency, and reliability are paramount.

Additionally, diffusion-based affine projection algorithms have been introduced for distributed network scenarios [23]. These advances underscore the ongoing effort to balance robustness, convergence, and complexity. Our proposed DSSLMS framework contributes to this active field by introducing a novel, computationally efficient, and model-based step-size distribution mechanism that is uniquely well-suited for the data-selective updates of SM-APA, a challenge not fully addressed by the aforementioned approaches.

III. ALGORITHM FORMULATION

A. Baseline APA and Set-Membership APA

The standard full-update Affine Projection Algorithm (APA) modifies the N -length filter coefficient vector $\mathbf{w}(k)$ at each iteration k using the following update equation [24], [25]:

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu \mathbf{X}(k) (\mathbf{X}(k) \mathbf{X}^T(k) + \varepsilon \mathbf{I})^{-1} \mathbf{e}_P(k) \quad (1)$$

where μ is the scalar fixed step-size, $\mathbf{X}(k)$ is an $N \times P$ matrix of the P most recent input vectors, P is the projection order, ε is a small regularization parameter, and $\mathbf{e}_P(k)$ is the $P \times 1$ a priori error vector.

Set-Membership APA (SM-APA) introduces data-selective updates by modifying the filter coefficients only when a specific constraint is violated. For instance, in SM-APA-1, an update is triggered when the magnitude of the a priori error,

$|e(k)|$, exceeds a predefined bound γ . Otherwise, the coefficients remain unchanged, thereby reducing the computational load.

B. Proposed DSSLMS Step-Size Control:

To overcome the limitations of a fixed step-size μ , the proposed method integrates a two-stage adaptive step-size mechanism inspired by the Distributed Step-Size LMS (DSSLMS) algorithm [15] as follows:

1) Base Step-Size Adaptation: A time-varying base step-size, $\alpha\mu_{\text{base}}(k)$, is updated at each iteration to track the signal and error dynamics. The update is driven by an instantaneous estimate of the squared gradient:

$$\mu_{\text{base}}(k+1) = \alpha\mu_{\text{base}}(k) + \sigma (e_P(k)y(k))^2 \quad (2)$$

where α is a forgetting factor ($0 < \alpha \leq 1$) that controls the memory of the adaptation, σ is a small positive adaptation gain, and $e_P(k)$ is the single-sample a priori error. This allows $\alpha\mu_{\text{base}}$ to increase when the error is large (indicating a system change) and decrease as the filter converges (when $\alpha < 1$).

The parameters α (forgetting factor) and σ (adaptation gain) enable environment-aware tuning. In stationary settings, $\alpha < 1$ allows step-size decay after convergence, reducing steady-state misadjustment, while in non-stationary conditions, $\alpha = 1$ preserves adaptability for tracking. The gain σ scales the update magnitude; larger values accelerate step-size growth during high-error periods, enhancing tracking speed, while smaller values promote stability. Although tunable, our results show that fixed choices (e.g., $\alpha = 1.0$, $\sigma = 5 \times 10^{-5}$ in non-stationary cases) yield robust performance across varied scenarios, as demonstrated in Section V.

Stability of the base step-size μ_{base} is ensured through the update rule in (2) and the enforcement of an explicit upper bound $\mu_{\text{max}} = 1.5$ in non-stationary environments. The forgetting factor $\alpha \leq 1$ prevents unbounded growth under stationary conditions, while the adaptation gain σ is chosen sufficiently small (e.g., 5×10^{-5}) to avoid erratic fluctuations. These measures guarantee that μ_{base} remains within a stable operating range, preserving filter convergence and preventing divergence, as validated in all simulation scenarios.

2) Step-Size Distribution: The adapted base step-size is then distributed among the N filter coefficients to form an $N \times 1$ adaptive step-size vector, $\mu(k)$. An exponential distribution is employed, as suggested in [26]:

$$\mu_i(k) = \beta^{i-1} \mu_{\text{base}}(k), \quad i = 1, \dots, N \quad (3)$$

where $\mu_i(k)$ is the individual step-size for the i -th coefficient and β is the distribution factor. If $\beta = 1$, the distribution is uniform. If $0 < \beta < 1$, earlier filter taps receive a larger step-size. This is advantageous in equalizing channels with decaying impulse responses, as it allocates more adaptation energy to the more significant initial taps.

The exponential distribution $\mu_i(k) = \beta^{i-1} \mu_{\text{base}}(k)$ is motivated by the typical decaying impulse response of multipath channels, where earlier filter taps correspond to stronger signal components. By allocating larger step-sizes to more significant coefficients, the adaptation energy aligns with the channel

temporal structure, accelerating convergence and improving tracking of dominant paths. This principled allocation also improves robustness and reduces misadjustment, consistent with prior work on tap-dependent step-size distribution [26].

C. Procedure of Proposed DSSLMS-SM-APA:

The DSSLMS step-size mechanism is integrated into the SM-APA framework to leverage both the computational efficiency of data-selective updates and the robustness of an adaptive step-size. The resulting algorithm family (DSSLMS-APA, DSSLMS-SM-APA-1, and DSSLMS-SM-APA-2) follows this general procedure at each iteration k :

- 1) Compute Filter Output and A Priori Error: Calculate the filter output $y(k) = \mathbf{w}^T(k)\mathbf{x}(k)$ and the a priori error $\mathbf{e}_P(k) = d(k) - y(k)$, where $d(k)$ is the desired signal.
- 2) Adapt Base Step-Size: Update the base step-size to obtain $\mu_{\text{base}}(k+1)$ using Eq. (2). This step is performed at every iteration, regardless of whether a coefficient update occurs, ensuring the step-size is always ready to react to changes.
- 3) Set-Membership Check and Coefficient Update: If the update condition for the specific SM variant is met (e.g., $|\mathbf{e}_P(k)|^2 > \gamma^2$ for SM-APA-1), then distribute the step-size by generating the adaptive step-size vector $\mu_i(k)$ from $\mu_{\text{base}}(k)$ using Eq. (3). Finally, update the weight vector $\mathbf{w}(k+1)$ using the APA rule, but replacing the scalar μ with a diagonal matrix $\mathbf{M}(k)$ whose entries are the elements of $\mu(k)$.
- 4) No Update: If the condition is not met, the filter coefficients remain unchanged: $\mathbf{w}(k+1) = \mathbf{w}(k)$

D. State-of-the-Art VSS Algorithms for Comparison:

To rigorously benchmark the proposed DSSLMS-based methods, two prominent variable step-size APA (VSS-APA) algorithms from the literature are included in our evaluation.

- 1) VSS-APA based on MSE Minimization (VSS-APA-MSE): This algorithm adapts the step-size $\mu(k)$ to minimize the mean-square of the a posteriori error vector at each iteration [10]. The step-size update is given by

$$\mu_{\text{vss1}}(k) = \alpha_{\text{vss}} \mu_{\text{vss1}}(k-1) + (1 - \alpha_{\text{vss}}) \phi(k) \quad (4)$$

where α_{vss} is a smoothing factor (e.g., 0.98), and the instantaneous optimal step-size $\phi(k)$ is calculated as

$$\phi(k) = \frac{\mathbf{e}_P^T(k) \mathbf{z}(k)}{\mathbf{z}^T(k) \mathbf{z}(k)} \quad (5)$$

with the intermediate vector $\mathbf{z}(k)$ defined by

$$\mathbf{z}(k) = \mathbf{x}^T(k) \mathbf{x}(k) (\mathbf{x}^T(k) \mathbf{x}(k) + \varepsilon \mathbf{I})^{-1} \mathbf{e}_P(k) \quad (6)$$

- 2) VSS-APA based on Error Correlation (VSS-APA-EC): this method adjusts the step-size by enforcing orthogonality between the a priori and a posteriori error vectors [11]. The step-size update rule is

$$\mu_{\text{vss2}}(k) = \mu_{\text{vss2}}(k-1) - \frac{\rho}{\|\mathbf{e}_P(k)\|^2} \mathbf{e}_P^T(k) \mathbf{e}_a(k-1) \quad (7)$$

where ρ is a small positive adaptation constant controlling the adjustment speed, and $\mathbf{e}_a(k-1)$ is the a posteriori error vector from the previous iteration.

IV. THEORETICAL ANALYSIS

This section provides a theoretical analysis of the proposed DSSLMS-enhanced algorithms. We focus on the mean-square performance to characterize convergence and steady-state behavior, and on the computational complexity to assess their efficiency [1][2].

A. Mean-Square Performance Analysis:

The performance of an adaptive filter is typically analyzed by studying the evolution of the weight error vector, $\mathbf{v}(k) = \mathbf{w}_{\text{opt}} - \mathbf{w}(k)$, where \mathbf{w}_{opt} is the optimal Wiener solution [1]. The goal is to qualitatively model the Mean-Square Deviation (MSD), $J(k) = E[\|\mathbf{v}(k)\|^2]$, to explain the behaviors observed in the simulations.

1) DSSLMS-APA Analysis

The weight update for the proposed DSSLMS-APA uses a diagonal step-size matrix $\mathbf{M}(k)$ derived from $\mu_{\text{base}}(k)$ Eq. (2). The behavior of the algorithm is governed by the two operational modes of $\mu_{\text{base}}(k)$, controlled by the forgetting factor α .

When $\alpha < 1$, in a stationary environment, the DSSLMS mechanism exhibits a biphasic behavior. A large initial μ_{base} ensures rapid initial convergence. As the filter converges and the error $e_P(k)$ decreases, the second term in Eq. (2) becomes negligible. The update is then dominated by $\mu_{\text{base}}(k+1) \approx \alpha \mu_{\text{base}}(k)$, causing the step-size to decay gradually. This reduction in step-size minimizes misadjustment noise, leading to a lower steady-state MSD [2]. In Fig. 2, the DSSLMS variants achieve an MSE of approximately 1.5×10^{-3} , outperforming the conventional APA's MSE of 3×10^{-3} .

When $\alpha = 1$, in a non-stationary environment, α is set to 1. When an abrupt channel change occurs (e.g., at $k = 1500$ in Fig. 3), the error $e_P(k)$ becomes large, causing the term $\sigma(e_P(k)y(k))^2$ (Eq. (2)) to dominate the update. This forces $\mu_{\text{base}}(k)$ to increase sharply and persistently, resulting in a large step-size in $\mathbf{M}(k)$ that accelerates MSD convergence and allows the filter to rapidly track the new optimal weights, as shown in Fig. 3 and Fig. 4.

2) DSSLMS-SM-APA Analysis

For Set-Membership algorithms, the analysis is conditioned on the update probability $P_u(k) = P(\|e(k)\|^2 > \gamma^2)$ [25][27]. A properly tuned γ and δ ensure that updates continue until the true error floor is reached. Once converged, the DSSLMS-SM-APA offers two benefits:

- a) In steady-state, $P_u(k)$ becomes very small. The algorithm performs fewer updates, saving computational resources and "freezing" adaptation when performance is satisfactory, preventing filter drift due to noise [25][27].
- b) By updating only when necessary, the SM variants are less susceptible to over-adjustment from noise,

TABLE I
COMPUTATIONAL COMPLEXITY

Algorithm	Multiplications	Additions
APA (Original)	$NP + N + P + \mathcal{O}(P^3)$	$N(P-1) + N - 1 + \mathcal{O}(P^3)$
DSSLMS-APA	$NP + N + P + (N-1) + 3 + \mathcal{O}(P^3)$	$N(P-1) + N - 1 + 1 + \mathcal{O}(P^3)$

contributing to the stable, low MSE floors observed in Fig. 2, Fig. 4, Fig. 6, and Fig. 8.

B. Computational Complexity:

We evaluate the computational complexity in terms of multiplications and additions per iteration, where N is the filter length and P is the projection order, as shown in Table I. The complexity of the standard APA is a well-established result [24], [28].

Note: The $\mathcal{O}(NP^2)$ term has been simplified to $\mathcal{O}(P^3)$ assuming $N > P$ and focusing on the dominant matrix inversion cost.

The computational overhead of the proposed DSSLMS mechanism is minimal, adding only $\mathcal{O}(N)$ operations per iteration—specifically $N - 1$ multiplications for step-size distribution and a few operations for the base step-size update. This is negligible compared to the dominant $\mathcal{O}(P^3)$ cost of the APA matrix inversion. In SM variants, the average overhead is further reduced because updates occur only when the error bound is violated. Thus, the proposed algorithms maintain the computational efficiency of APA/SM-APA while achieving enhanced performance through adaptive step-size control.

The proposed DSSLMS mechanism [26] adds a small, constant overhead to the baseline APA. Updating μ_{base} in Eq. (2) requires three multiplications and one addition, while distributing the step-size in Eq. (3) requires $N - 1$ multiplications. This overhead is of order $\mathcal{O}(N)$. The complexity of the APA is dominated by the matrix inversion, which is $\mathcal{O}(P^3)$ [4]. Since the DSSLMS overhead is only $\mathcal{O}(N)$, the total complexity of DSSLMS-APA remains dominated by the APA structure, making the additional cost marginal.

The Set-Membership variants introduce data-dependent complexity [25][28]. When the SM condition is not met, the algorithm only computes the filter output, the error, and the μ_{base} update, resulting in a total complexity of $\mathcal{O}(N)$. When the SM condition is met, the complexity is the same as DSSLMS-APA, i.e., $\mathcal{O}(P^3 + N)$. Hence, the average complexity of DSSLMS-SM-APA can be expressed as $\mathcal{C}_{\text{avg}} = P_u \cdot \mathcal{O}(P^3 + N) + (1 - P_u) \cdot \mathcal{O}(N)$, where P_u is the update probability. In steady-state, P_u is small, so the average complexity is significantly reduced compared to the full-update DSSLMS-APA, demonstrating that the proposed algorithms can achieve superior performance while being more computationally efficient.

V. EXPERIMENTAL FRAMEWORK AND RESULTS

To evaluate the performance of the proposed DSSLMS-enhanced algorithms in a practical communication context, a comprehensive adaptive channel equalization simulation was

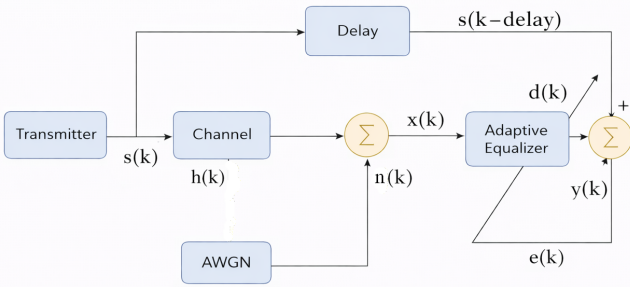


Fig. 1. Block diagram of the adaptive channel equalization system.

designed. This section details the experimental setup, the simulated scenarios, and an analysis of the comparative results.

A. Adaptive Equalizer Setup: The core task of the adaptive equalizer is to mitigate Inter-Symbol Interference (ISI) and restore a transmitted signal that has been distorted by a multipath channel and corrupted by additive noise. The structure of the simulated system is illustrated in Fig. 1.

The original signal $s(k)$ is a Binary Phase-Shift Keying (BPSK) sequence (± 1). It is passed through a dispersive channel $h(k)$ and combined with Additive White Gaussian Noise (AWGN) $n(k)$ to produce the received signal $x(k)$. This received signal serves as the input to an adaptive FIR filter (the equalizer) $w(k)$. The equalizer's output $y(k)$ is compared to a delayed version of the original signal, $d(k) = s(k - \Delta)$, to produce the error signal $e(k)$, which drives the filter's adaptation. The equalizer filter length N_{eq} was set to 15, and the delay Δ was chosen to properly align the desired signal.

The projection order P influences the trade-off between convergence speed and computational load. A higher P improves convergence and tracking in correlated input environments but increases complexity due to the dominant $\mathcal{O}(P^3)$ cost of the APA matrix inversion. In our simulations, $P = 4$ provided an optimal balance, delivering robust convergence and tracking comparable to larger P values while keeping the computational overhead manageable. The proposed DSSLMS mechanism remained effective across the tested P values, with performance gains remaining consistent as long as P was sufficiently large to capture input correlation.

B. Simulation Scenarios: The algorithms were tested in one stationary and three distinct non-stationary scenarios to rigorously assess their convergence, steady-state accuracy, and tracking capabilities. In the non-stationary cases, an abrupt change occurs at iteration $k = 1500$. In the stationary case, the channel impulse response and noise variance remain constant throughout the simulation. This scenario serves as a baseline to evaluate fundamental convergence speed and steady-state error. For the time-varying channel case, the channel impulse response abruptly changes from h_{channel1} to h_{channel2} at $k = 1500$, while the noise variance remains constant. This tests the algorithms' tracking agility. In Scenario 2, which includes varying noise variance, the channel remains fixed, but the additive noise variance increases tenfold at $k = 1500$. This

evaluates the algorithms' robustness to a sudden degradation in the Signal-to-Noise Ratio (SNR). Finally, Scenario 3 includes both a time-varying channel and varying noise. This is the most challenging configuration, where both the channel and the noise variance change simultaneously at $k = 1500$, testing the overall robustness and adaptability of the algorithms.

C. Simulation Parameters: The simulation parameters for both stationary and non-stationary environments were determined empirically to ensure a fair and robust comparison between all algorithms. The key parameters used in the simulations are summarized in Table II. It is important to note the key differences between the two setups. For the non-stationary case, a smaller regularization parameter (ε) and different Set-Membership thresholds (γ_1, γ_2) were found to provide the best balance between tracking and stability. In particular, using a smaller ε (e.g., 10^{-4}) enabled faster and more aggressive adaptation without compromising numerical stability. For the stationary case, a larger ε was crucial for preventing numerical instability in the SM variants under low-error conditions (e.g., $\varepsilon = 10^{-2}$), while the γ thresholds were fine-tuned to prevent premature convergence that can occur with variable-step-size SM algorithms in low-error conditions. The learning curves presented in the following sections were smoothed using a moving average window of 25 samples for improved readability.

The threshold γ_2 for SM-APA-2 was selected based on a trade-off between update sparsity and steady-state accuracy. It was set proportionally to the estimated noise variance σ_n^2 , following the SM design principle $\gamma \propto \sigma_n$. For stationary simulations, $\gamma_2 = 0.05$ balanced convergence speed with computational efficiency, while in non-stationary scenarios $\gamma_2 = 0.1$ allowed more frequent updates to improve tracking without excessive overhead. This empirical tuning, validated through Monte Carlo runs, ensures robust performance across different environments while preserving the data-selective benefits of SM filtering.

D. Performance in Stationary Equalization Environments:

The initial evaluation was conducted in a stationary channel equalization environment to establish a baseline for convergence speed and steady-state performance. The learning curves, averaged over 100 Monte Carlo runs, are presented in Fig. 2. The objective is to assess how effectively each algorithm converges to minimize the mean-square error (MSE) between the equalizer output and the transmitted BPSK signal.

As shown in Fig. 2, the proposed APA-DSSLMS algorithm demonstrates the fastest initial convergence, while the other two proposed algorithms (SM-APA-1 and SM-APA-2) exhibit comparable convergence times. The APA equalizer with DSSLMS (red, solid) and SM-APA-1 with DSSLMS (blue, dashed) reach their steady-state MSE floors within approximately 100 iterations, significantly faster than the conventional APA (black, solid), which requires around 250 iterations to achieve a similar error level.

For steady-state performance, the proposed algorithms achieve the lowest MSE values, indicating superior accuracy. Both APA-DSSLMS and SM-APA-1-DSSLMS settle at approximately 1.5×10^{-3} , while SM-APA-2-DSSLMS (cyan, dotted)

TABLE II
SIMULATION PARAMETERS FOR STATIONARY AND NON-STATIONARY ENVIRONMENTS

Parameter	Stationary Value	Non-Stationary Value
<i>General Simulation Parameters</i>		
Data length (L)	1000	3000
Equalizer filter length (N_{eq})	15	15
Decision delay (Δ)	8	9
Monte Carlo runs	100	50
<i>Baseline Algorithm Parameters</i>		
Fixed step-size (μ)	0.3	0.3
Projection order (P) (Full / SM)	5 / 2	5 / 2
Partial update taps (M)	8	8
Regularization parameter (ε)	1×10^{-4}	1×10^{-6}
SM-APA-1 threshold (γ_1)	0.02	0.04
SM-APA-2 threshold (γ_2)	0.002	0.01
<i>Proposed DSSLMS Parameters</i>		
Forgetting factor (α_D)	1.0	1.0
Adaptation gain (δ_D)	5×10^{-5}	5×10^{-5}
Initial base step-size ($\mu_{base}(0)$)	0.3	0.3
Maximum base step-size (μ_{max})	1.5	1.5
<i>Benchmark VSS Parameters</i>		
Smoothing factor (α_{vss})	0.98	0.98
Adaptation constant (ρ)	0.001	0.001

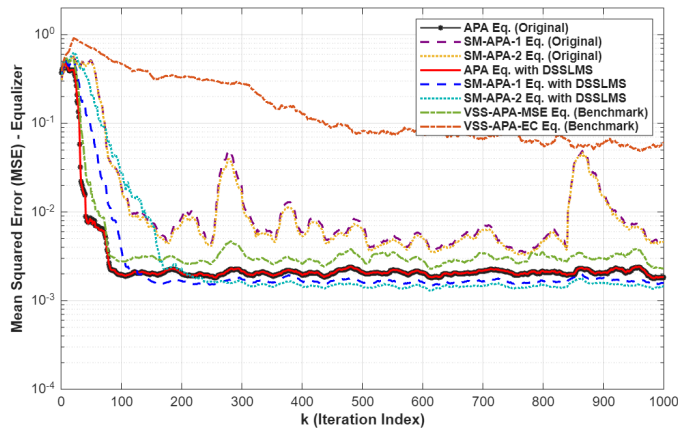


Fig. 2. MSE performance in stationary environments.

achieves around 2×10^{-3} . These values represent a notable improvement over the conventional APA, which settles at about 3×10^{-3} . Compared to benchmark methods, the DSSLMS-based algorithms also outperform the state-of-the-art VSS-APA-MSE (green, dashed), which converges to roughly 4×10^{-3} , and the VSS-APA-EC (orange, dashed), which shows the slowest convergence and fails to stabilize within the 1000-iteration window. Furthermore, while the conventional SM-APA variants (purple, dashed for SM-APA-1 and gold, dotted for SM-APA-2) improve convergence speed over the fixed-step-size APA, they suffer from higher steady-state volatility and larger MSE floors than their DSSLMS-enhanced counterparts.

In summary, under stationary conditions, the proposed DSSLMS step-size control mechanism provides a clear advantage, yielding both faster convergence and lower steady-state error compared to conventional fixed-step-size, Set-Membership,

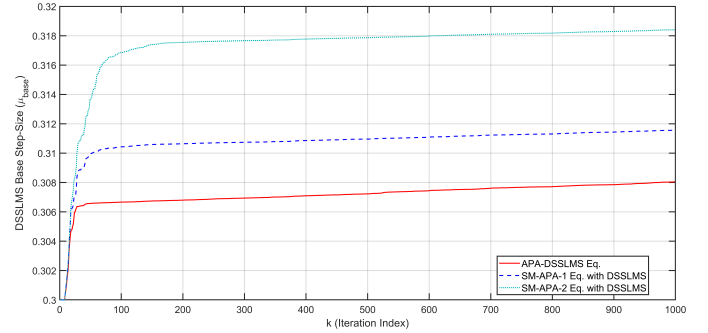


Fig. 3. Evolution of the DSSLMS base step-size μ_{base} in the stationary environment.

and other variable-step-size algorithms. This is primarily due to the DSSLMS mechanism maintaining a large step-size during the initial convergence phase.

The evolution of the base step-size μ_{base} for the proposed DSSLMS algorithms during the final Monte Carlo run is depicted in Fig. 3, providing insight into the mechanism underlying their superior stationary performance.

All three DSSLMS variants exhibit similar and desirable behavior. Starting from an initial value of $\mu_{base}(0) = 0.3$, the step-size for each algorithm rapidly adapts within the first 100 iterations. The SM-APA-2-DSSLMS (cyan, dotted) adapts to the largest step-size, followed by SM-APA-1-DSSLMS (blue, dashed) and APA-DSSLMS (red, solid). This behavior confirms the biphasic operational principle discussed in Section IV-A: the algorithms maintain a large step-size (> 0.3) during the early convergence phase, explaining their rapid MSE reduction in Fig. 2. After this initial phase, when $\alpha < 1$, the step-size decays smoothly as the error decreases, ensuring minimal steady-state misadjustment. However, since $\alpha_D = 1.0$ in these simulations, the step-sizes do not decay but slowly increase, reflecting persistent residual errors. This demonstrates the mechanism's sensitivity and continuous effort to minimize error, effectively balancing rapid convergence and stable steady-state performance.

E. Performance in Stationary Equalization Environments:

To rigorously assess tracking capabilities and resilience, the performance of all algorithms was evaluated in three distinct non-stationary equalization scenarios. In each scenario, an abrupt system change occurs at iteration $k = 1500$. The DSSLMS parameters were tuned for optimal tracking with $\alpha_D = 1.00$, $\delta_D = 5 \times 10^{-5}$, $\mu = 0.3$, and $\mu_{max} = 1.5$, while the equalizer length (N_{eq}) was set to 15. The performance of the proposed DSSLMS-based algorithms was compared against the conventional fixed-step-size APA, its Set-Membership variants, and the two state-of-the-art benchmark algorithms, VSS-APA-MSE and VSS-APA-EC.

1) Scenario 1: Time-Varying Channel (Abrupt Change)

In this scenario, the additive noise variance remains constant while the channel impulse response abruptly changes at iteration $k = 1500$. This test directly evaluates the algorithms' ability to adapt to a sudden mismatch and reconverge to a new optimal solution. The MSE learning curves are depicted

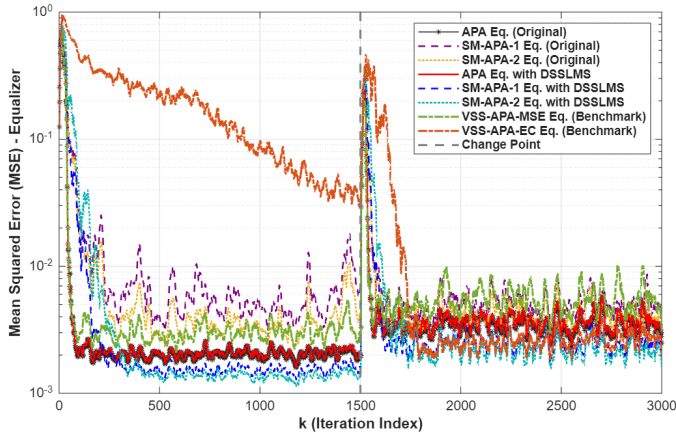


Fig. 4. Equalizer MSE learning curves for Scenario 1: Time-Varying Channel.

in Fig. 4.

During the initial convergence phase ($k < 1500$), the proposed DSSLMS-enhanced algorithms clearly establish their superiority. The SM-APA-1 with DSSLMS (blue, dashed) and SM-APA-2 with DSSLMS (cyan, dotted) achieve comparable convergence times and the lowest steady-state MSE, settling near 2×10^{-3} . The APA with DSSLMS (red, solid) also performs exceptionally well, achieving a slightly higher but stable MSE floor. In contrast, the conventional algorithms and the VSS-APA-MSE benchmark settle at higher error levels, while the VSS-APA-EC benchmark exhibits very slow convergence.

When the system changes at $k = 1500$, a sharp MSE spike is observed across all algorithms—an expected response to the abrupt mismatch. However, the magnitude of this spike is substantially smaller for the DSSLMS-enhanced algorithms than for the conventional and benchmark methods. For $k > 1500$, the proposed algorithms exhibit outstanding tracking performance, recovering rapidly and reconverging to a low and stable steady-state MSE. The SM-APA-1 and SM-APA-2 with DSSLMS again achieve the lowest post-change MSE, while the conventional algorithms (APA, SM-APA-1, SM-APA-2) struggle to adapt, displaying slow reconvergence and volatile steady-state behavior. The benchmark algorithms also show limited performance: the VSS-APA-MSE adapts moderately but remains less accurate than the proposed methods, and the VSS-APA-EC fails to track the change effectively.

This scenario demonstrates the robustness and agility of the proposed DSSLMS framework. Its ability to rapidly increase the step-size in response to large errors at the change point enables significantly faster adaptation and superior tracking performance compared to both traditional and other state-of-the-art variable step-size approaches.

The explanation for the enhanced tracking performance is illustrated in Fig. 5, which plots the evolution of the base step-size μ_{base} during the final Monte Carlo run for this scenario.

The plot highlights the highly adaptive nature of the DSSLMS mechanism. During the initial convergence ($k < 1500$), all three variants start at $\mu_{\text{base}}(0) = 0.3$ and rapidly increase, leading to fast convergence in Fig. 4. At the change point ($k = 1500$), a sharp upward jump in μ_{base} occurs for all three DSSLMS algorithms. This immediate and large response

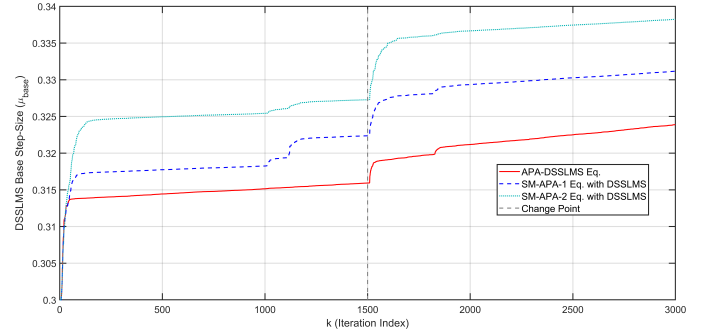


Fig. 5. DSSLMS base step-size evolution for Scenario 1: Time-Varying Channel (last run).

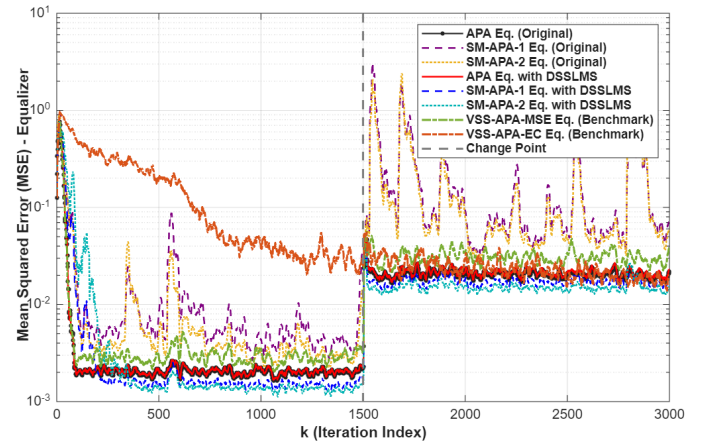


Fig. 6. Equalizer MSE learning curves for Scenario 2: Varying Noise Variance.

reflects the core advantage of the mechanism: the step-size increases aggressively when faced with sudden large errors. Afterward ($k > 1500$), the step-size continues to rise steadily, driven by the setting $\alpha_D = 1.00$, which prevents decay in non-stationary environments. This ensures sustained adaptability, allowing the filters to continuously adjust to the evolving channel characteristics.

2) Scenario 2: Varying Additive Noise Variance

This scenario evaluates the algorithms' robustness to sudden changes in the signal-to-noise ratio (SNR). The channel $h_{\text{channel}1}$ remains fixed throughout the simulation, but at iteration $k = 1500$, the variance of the additive Gaussian noise increases tenfold (from 0.001 to 0.01). The MSE learning curves for this scenario are shown in Fig. 6.

In the low-noise phase ($k < 1500$), the proposed DSSLMS-enhanced algorithms—particularly SM-APA-1-DSSLMS (blue) and APA-DSSLMS (red)—achieve the lowest MSE floors, outperforming all conventional and benchmark methods. At the change point ($k = 1500$), all algorithms exhibit an upward MSE jump, reflecting the unavoidable effect of increased noise power on the Wiener error.

For $k > 1500$, the DSSLMS-based algorithms display remarkable robustness and stability. Despite the higher noise floor, they maintain low, steady MSE values. In contrast, the conventional SM-APA algorithms (purple and gold) become unstable and exhibit erratic MSE spikes, revealing their in-

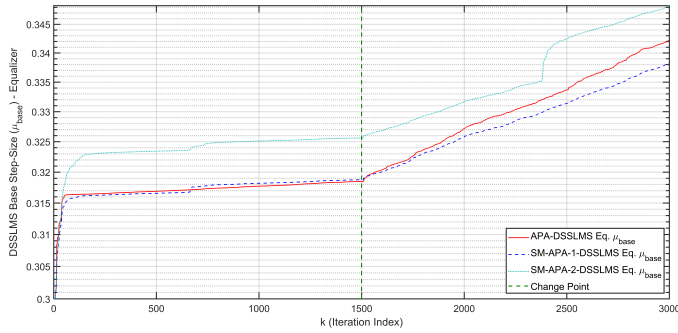


Fig. 7. DSSLMS base step-size evolution for Scenario 2: Varying Noise Variance (last run).

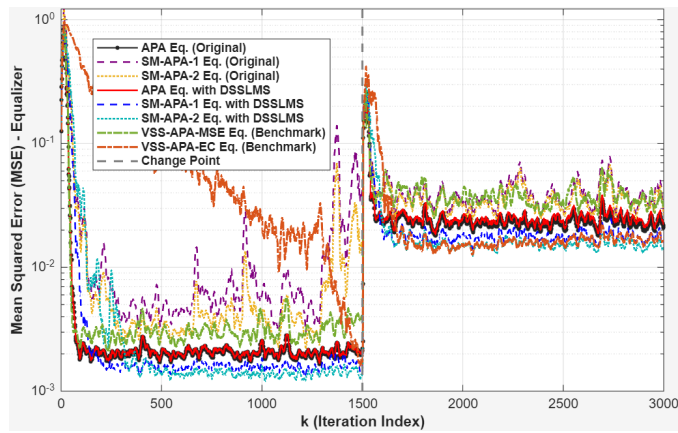


Fig. 8. Equalizer MSE learning curves for Scenario 3: Time-Varying Channel and Noise.

ability to handle abrupt noise variance changes. Benchmark methods (VSS-APA-MSE and VSS-APA-EC) stabilize but at noticeably higher MSE levels.

This scenario highlights a key advantage of DSSLMS control—its ability to adapt learning rates to changing error statistics, avoiding instability while maintaining accuracy under increased noise power. Fig. 7 shows the evolution of μ_{base} , demonstrating this adaptive response.

In the initial low-noise phase ($k < 1500$), μ_{base} stabilizes gradually. After the noise variance increases ($k = 1500$), the step-size curves show a smoother, more gradual rise than in Scenario 1, driven by the larger error terms in the update expression $\delta_D(e(k)y(k))^2$. This controlled adaptation enables the filters to handle noise-induced variability without destabilization.

3) Scenario 3: Time-Varying Channel and Varying Additive Noise

This scenario combines the challenges of Scenarios 1 and 2, representing the most demanding test condition. At iteration $k = 1500$, both the channel impulse response changes from $h_{\text{channel}1}$ to $h_{\text{channel}2}$ and the additive noise variance increases tenfold. The MSE learning curves are presented in Fig. 8.

Before the change ($k < 1500$), performance trends match the stationary and previous non-stationary cases, with DSSLMS-based algorithms achieving the lowest MSE. The simultaneous change in both channel and noise causes a pronounced MSE

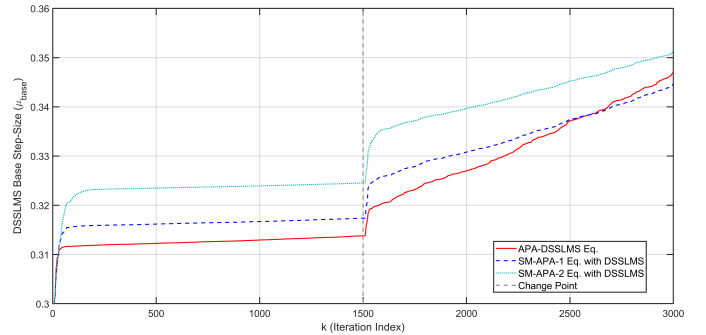


Fig. 9. DSSLMS base step-size evolution for Scenario 3: Time-Varying Channel and Noise (last run).

spike at $k = 1500$, representing the compounded effects of coefficient mismatch and SNR degradation. Post-change ($k > 1500$), the DSSLMS algorithms demonstrate the fastest recovery and settle to the lowest steady-state MSE—approximately 3×10^{-3} —while conventional and benchmark algorithms exhibit slow, unstable adaptation.

The VSS benchmarks settle at much higher error floors (around 8×10^{-3} to 10^{-2}), confirming the DSSLMS methods' superiority. Fig. 9 illustrates the evolution of μ_{base} for this scenario.

At $k = 1500$, μ_{base} exhibits a strong upward jump across all three algorithms, reflecting the large instantaneous error caused by the dual mismatch. It continues to increase afterward, maintaining high adaptability throughout the simulation. This sustained aggressive learning rate explains the exceptional tracking and stability observed in Fig. 8. The results confirm that the proposed DSSLMS mechanism effectively manages simultaneous system and noise variations, providing a level of robustness unmatched by conventional or benchmark algorithms.

VI. PERFORMANCE COMPARISON ANALYSIS WITH EXISTING METHODS

This section provides a comprehensive comparison between the proposed DSSLMS-based algorithms and several adaptive filtering methods. Table III contrasts the key performance metrics, including convergence speed, steady-state MSE, tracking agility, computational cost, and suitability for non-stationary environments, across conventional, Set-Membership, VSS-based, and recent advanced approaches.

A. Summary of Key Advantages of the Proposed DSSLMS Methods

This comparison shows that the proposed DSSLMS-based algorithms uniquely combine several desirable attributes that are only partially available in other methods:

- They achieve simultaneously fast convergence and the lowest steady-state error.
- They provide superior tracking agility in non-stationary conditions, which is important for dynamic channel equalization.

TABLE III
PERFORMANCE COMPARISON OF ADAPTIVE FILTERING ALGORITHMS

Algorithm Category	Specific Algorithm	Key Mechanism	Convergence Speed	Steady-State MSE	Tracking Agility	Computational Cost	Suitability for Non-Stationary Environments
Proposed Method	DSSLMS-SM-APA-1 (This Work)	Data-selective updates plus distributed VSS vector	Fastest	Lowest ($\approx 1.5 \times 10^{-3}$)	Excellent	Medium (data-selective)	Excellent
Proposed Method	DSSLMS-APA (This Work)	Full updates plus distributed VSS vector	Fast	Lowest ($\approx 1.5 \times 10^{-3}$)	Excellent	High	Excellent
Conventional	APA [3], [4]	Fixed scalar step-size	Slow	High ($\approx 3.0 \times 10^{-3}$)	Poor	High	Poor
Set-Membership	SM-APA-1 [17]	Data-selective updates with fixed step-size	Medium	Medium-high ($\approx 2.5 \times 10^{-3}$)	Poor	Low (data-selective)	Poor
VSS (MSE-Based)	VSS-APA-MSE [12]	Scalar VSS via MSE minimization	Medium	Medium ($\approx 4.0 \times 10^{-3}$)	Moderate	High	Moderate
VSS (Error-Based)	VSS-APA-EC [13]	Scalar VSS via error correlation	Slowest	Highest ($> 5.0 \times 10^{-3}$)	Poor	High	Poor
Recent VSS (Robust)	Li et al. (2024) [16]	Scalar VSS robust to impulsive noise	Medium	Low (in impulsive noise)	Moderate	High	Good (specific noise cases)
Recent SM (Adaptive Bound)	Wang et al. (2024) [18]	Data-selective algorithm with adaptive error bound	Medium to fast	Low	Good	Low (data-selective)	Good
Recent (Momentum)	Chen et al. (2024) [17]	Momentum-enhanced APA	Fast	Low	Good	High	Good
Recent (Deep Learning)	Gupta et al. (2025) [20]	Deep learning-based VSS control	Fast (after training)	Very low (trained conditions)	Excellent (trained conditions)	Very high	Good (limited by training data)

- The DSSLMS-SM-APA variant maintains this performance with improved computational efficiency due to its data-selective update rule.
- Unlike deep learning-based approaches [20], they offer a model-based and computationally efficient solution without requiring training data or high inference costs.
- Compared to robust APA [16] and momentum-based APA [17], the proposed method provides a more general step-size distribution mechanism that improves overall performance in both stationary and non-stationary environments.

VII. CONCLUSION

This paper introduced and rigorously evaluated a novel step-size control mechanism for the Affine Projection Algorithm (APA) and its Set-Membership (SM-APA) variants by integrating the Distributed Step-Size LMS (DSSLMS) strategy. The proposed DSSLMS-APA and DSSLMS-SM-APA algorithms were benchmarked against conventional fixed-step-size methods and prominent state-of-the-art Variable Step-Size (VSS) algorithms under a series of demanding adaptive channel equalization scenarios.

Simulation results consistently confirmed the superiority of the proposed framework. In the stationary environment, the DSSLMS-enhanced algorithms demonstrated faster convergence and achieved a lower steady-state Mean-Square Error (MSE) of approximately 1.5×10^{-3} , outperforming all other methods. In non-stationary environments, the proposed algorithms exhibited exceptional tracking capabilities, rapidly adapting to abrupt changes in the channel and noise statistics while maintaining a low and stable MSE where conventional and benchmark algorithms struggled or failed. This performance advantage stems from the DSSLMS mechanism's inherent ability to dynamically balance aggressive adaptation during periods of high error with conservative updates during steady-state operation.

By effectively addressing the long-standing trade-off between convergence speed, steady-state accuracy, and track-

ing agility, the DSSLMS-enhanced filters consistently outperform conventional fixed-step-size, standard Set-Membership, and other VSS algorithms. The results confirm that the proposed DSSLMS-APA and DSSLMS-SM-APA are robust and computationally efficient solutions, making them well suited for real-time signal processing applications where environmental conditions may change unpredictably.

Future work will explore extending this adaptive step-size mechanism to other classes of adaptive filters, such as those operating in the frequency domain, and evaluate its performance in more complex communication environments, including impulsive noise and frequency-selective fading channels.

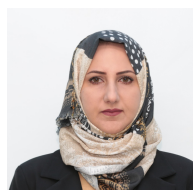
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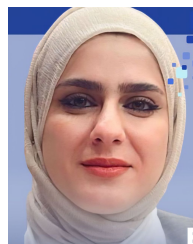
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