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Non-invasive Method for Parameter Identification in a Lumped Parameter Model of Pulmonary Circulation

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Introduction

Understanding the function of the right heart and pulmonary circulation becomes more and more important in the treatment of cardiac and pulmonary diseases [1]. Hemodynamic data for examination of pulmonary circulation are usually obtained invasively, what is unacceptable in healthy subjects. That is why we need a non-invasive clinical method for the estimation of pulmonary circulation function that would be suitable for all subjects (with normal function and with cardiorespiratory diseases).

The aim of this work is to develop a lumped parameter model of pulmonary circulation and to develop a method for parameter identification of this model, based on non-invasive (Echocardiography) measurements of velocity profiles through heart valves.

Mathematical model

Fig. 1 schematically shows the right heart, pulmonary artery (PA), lungs, pulmonary veins (PV) and left atrium (LA) as well as the electrical analogue scheme of the proposed lumped model of pulmonary circulation. Resistor R models the total pulmonary vascular resistance, capacitors C and C_1 model arterial compliances of proximal and distal parts, L represents the inertial effects within arteries, η is the wall resistance of the proximal part (the Voigt model) and Z models the impedance of the rest of the system. For the given model parameters and for input pulmonary valve flow ($Q_{pv} = v_{pv} A_{pv}$), it is possible to calculate pulmonary root pressure (p_{PA}). When p_{PA} is measured, it is possible to find optimal values of model parameters which minimize the RMS error between measured p_{PA} and p_{PA} calculated with the model.

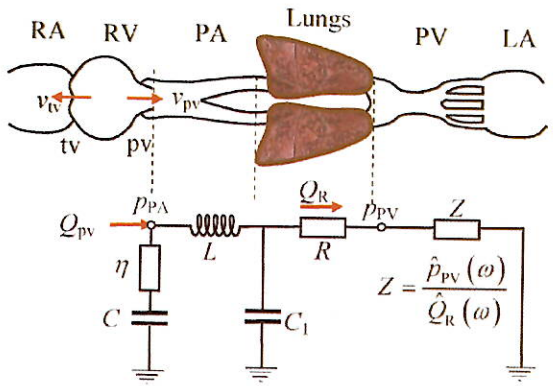

Fig. 1. Schematics of the pulmonary circulation and electrical analogue scheme of its lumped mathematical model. RA/RV = right atrium/ventricle, PA/PV = pulmonary arteries/veins, LA = left atrium, tv/pv = tricuspid/pulmonary valve, Z = impedance, ω = circular frequency

Measurements

By using Doppler Echocardiography, it is possible to measure pulmonary valve (v_{pv}) and tricuspid regurgitant blood velocity (v_{tv}). In each particular case, several measurements were recorded and the average data profiles were calculated. The "measured" p_{PA} is obtained from unsteady Bernoulli equation

$$p_{PA} = p_{RA} + \frac{1}{2}\rho v_{tv}^2 - \frac{1}{2}K\rho v_{pv}^2 - \rho l \frac{dv_{pv}}{dt}, \qquad (1)$$

where K and l are minor loss coefficient and inertial length through the pulmonary valve, respectively, p_{RA} is the average right atrium pressure, which is estimated from the width of vena cava inferior. Similarly, the average pressure in pulmonary veins (p_{PV}) is estimated from the mitral inflow pattern. The stroke volume calculated from the pulmonary and aortic valve velocity should be the same

$$V_{stroke} = \int_0^{T_{ej}} v_{pv} A_{pv} dt = \int_0^{T_{ej}} v_{av} A_{av} dt, \qquad (2)$$

where T_{ej} is ejection time. Since the aortic valve area can be measured more precisely, we use Eq. (2) to calculate A_{pv}.

Parameter identification

First, the pulmonary artery input impedance $Z_{in} = \hat{p}_{PA}/\hat{Q}_{pv}$ (the ratio of the pressure and flow phasors defined by the Fourier series) is calculated, and then p_{PA}^{WK5} is obtained based on measured Q_{pv}. This pressure is compared with the p_{PA} defined by Eq. (1), and the error defined as

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(p_{PA}^{WK5} - p_{PA}\right)^2} \qquad (3)$$

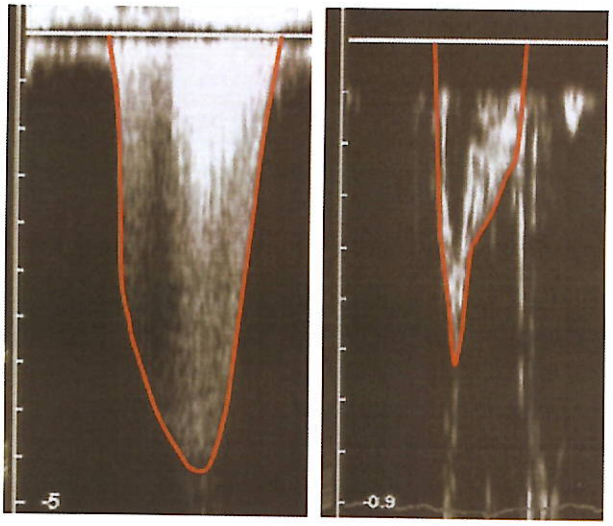

Fig. 2. Example of Doppler regurgitant tricuspid (left) and pulmonary velocity (right). Red lines are plotted for a digitization purpose

(where N is the number of points within T_{cj}) is minimized.

Results and Remarks

The method was applied to an elderly patient with pulmonary arterial hypertension using the following data: Cardiac Output: 5 l/min, Heart Rate 78 beat/min, pulmonary valve diameter D_{pv} = 2.73 cm, isovolumic contraction of RV time t_{eivc} = 34.4 ms, p_{RA} = 5 mmHg, p_{PV} = 10 mmHg, $l = 1.53 D_{pv}$, $K = 1$. Fig. 3 shows the measured pulmonary flow, and the comparison of the measured and calculated pressure during T_{cj}. p_{PA}^{WK5} from the model describes the "measured" pressure very well, and shows incisure immediately after pulmonary valve closing. Fig. 4 shows the pulmonary input impedance and the values of model parameters that minimize RMSE. The absolute value of Z_{in} shows its minimum value and zero crossing frequency of the phase angle is 5.4 Hz, what is in good agreement with the observations of elderly subjects.

The proposed method is capable to accurately identify PA model parameters and input impedance of pulmonary circulation by using the pressure data from the ejection time window only.

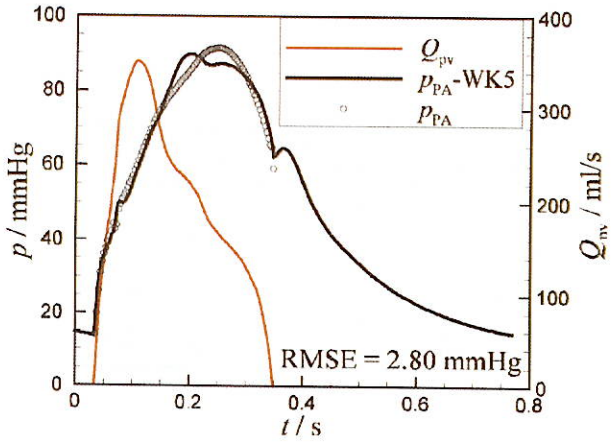

Fig. 3. Velocity through the pulmonary valve (thin red line), "measured" pulmonary root pressure from Eq. (1) (circles), and pulmonary root pressure from the five element lumped model (thick black line)

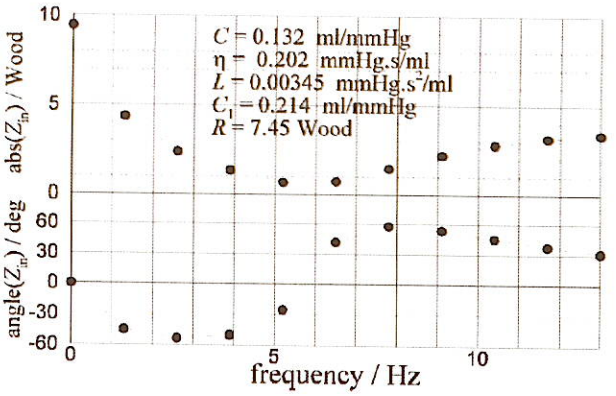

Fig. 4. Absolute value of pulmonary input impedance (upper part) and its phase angle (lower part)

The method is limited to the subject with nicely obtainable tricuspid regurgitant velocity and pulmonary valve flow.

References

[1] Peacock, A.J., Naeije, R., Rubin, L.J., Pulmonary circulation: Diseases and their Treatment. Hodder Arnold, 3rd ed., 2011.