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Mathematical Model of Blood Flow Through the Aortic Valve

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Introduction

In lumped models of circulatory system the aortic flow is governed by unsteady Bernoulli equation. The aortic valve is usually considered as the idealized check valve. This means that the valve opens instantaneously for the positive flow and closes instantaneously preventing the negative (reverse) flow from arteries to the left ventricle. The invasive measurements show that during the valve closing phase the negative flow always occurs. The negative flow cannot be obtained by the idealized valve model. Here we have proposed a model of the aortic valve that could also predict the negative aortic flow.

Mathematical model and numerical method

The characteristic opening and closing phases of the aortic valve are shown in Fig.1. Initially (at the beginning of systole) there is no blood flow through the aortic valve, and the valve leaflets are at rest (panel A in Fig. 1). When the left ventricle (LV) pressure exceeds the arterial pressure (due to LV contraction), the unsteady Bernoulli equation holds

$$M\frac{\mathrm{d}Q}{\mathrm{d}t}=p_{\mathrm{lv}}^{M}-p_{\mathrm{sa}}^{\mathrm{M}}-\frac{K\rho}{2A_{\mathrm{av}}^{2}}(Q-Q_{\mathrm{L}})^{2}, \qquad (1)$$

where Q is the absolute blood flow through the aortic root, M is inertance coefficient, $\rho=1050$ kg/m^3 is blood density, A_{av} is the aortic root area, $p_{\mathrm{lv}}^{\mathrm{M}}$ and $p_{\mathrm{sa}}^{\mathrm{M}}$ are measured left ventricle and arterial pressure, respectively, K is minor loss coefficient and Q_{L} is the flow rate that defines the volume V_{L} swept by valve leaflets

$$\frac{\mathrm{d}V_{\mathrm{L}}}{\mathrm{d}t}=Q_{\mathrm{L}}. \qquad (2)$$

During the first opening phase, leaflets move into the arterial space with $Q_{\mathrm{L}}=Q$, but there is no orifice. The orifice occurs after the leaflets have swept a certain volume (see panel B in Fig. 1) $V_{\mathrm{L0}}=\alpha A_{\mathrm{av}}\sqrt{4A_{\mathrm{av}}/\pi}$, where α is the model parameter. During the second opening phase the orifice increases from zero to A_{av}, and leaflets sweep an additional volume (see panel C in Fig. 1) $V_{\mathrm{L1}}=\beta A_{\mathrm{av}}\sqrt{4A_{\mathrm{av}}/\pi}$, where β is the model parameter. In this phase $Q_{\mathrm{L}}=(1-A/A_{\mathrm{av}})Q$, where A is the orifice area which is related to V_{L} as $A=\left[(V_{\mathrm{L}}-V_{\mathrm{L0}})/V_{\mathrm{L1}}\right]^{2}$, see [1]. During these two phases the inertance coefficient is defined as $M=(2-A/A_{\mathrm{av}})M_0$, where $M_0=\rho L/A_{\mathrm{av}}$ and L is inertance length. After the flow has reached its maximum, the slow leaflets closing phase starts with $Q_{\mathrm{L}}=Q-Q_{\max}A/A_{\mathrm{av}}$. During this phase V_{L} decreases from $V_{\mathrm{L0}}+V_{\mathrm{L1}}$ to V_{L0} (see panels D and E in Fig. 1), and at a certain moment Q becomes negative. For negative Q the inertance coefficients is defined as $M=\delta M_0$, where δ is model parameter. At the end of this phase (for $V_{\mathrm{L}}=V_{\mathrm{L0}}$, see panel E in Fig. 1) leaflets coapt and equation (1) does not hold anymore. Measurements suggest that in the last rapid closing phase (after the leaflets coapt) the leaflets behave as a dumped oscillating system defined by

$$\frac{\mathrm{d}^2Q}{\mathrm{d}t^2}+2\xi\frac{\mathrm{d}Q}{\mathrm{d}t}+(\omega^2+\xi^2)Q=0, \qquad (3)$$

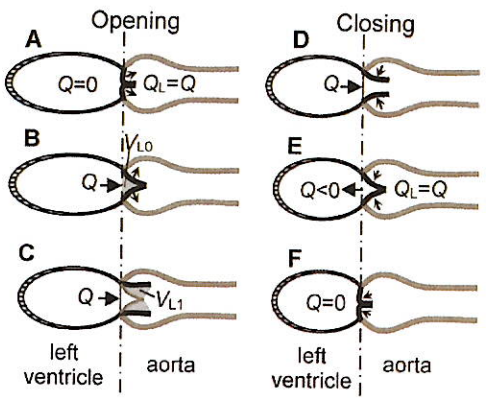


Fig. 1. Scheme of aortic valve opening (panels A to C) and closing (panels D to F)

where ξ and ω are constant parameters. During this phase $Q_{\mathrm{L}}=Q$, and V_{L} should decrease from V_{L0} to zero. If we introduce another parameter $\gamma=\xi/\omega$, parameters ξ and ω are uniquely defined by V_{L0} (or α) and γ.

For the given measured left ventricular and arterial pressures and the set of model parameters: A_{av}, L, K, α, β, γ and δ, the set of equation (1) or (3) and (2) is solved numerically by the fourth order Runge-Kutta method.

Results and conclusions

The proposed model was applied to the measured data in humans and pigs. The measured left ventricle and arterial pressure was used as input, and the calculated aortic flow was compared with the measured ones in Figs. 2 and 3.

There is a very good agreement of the model results with the measured ones, with model parameters in physiological range. Time varying aortic valve orifice A shows two closing phases: a slow one followed by a rapid one as it

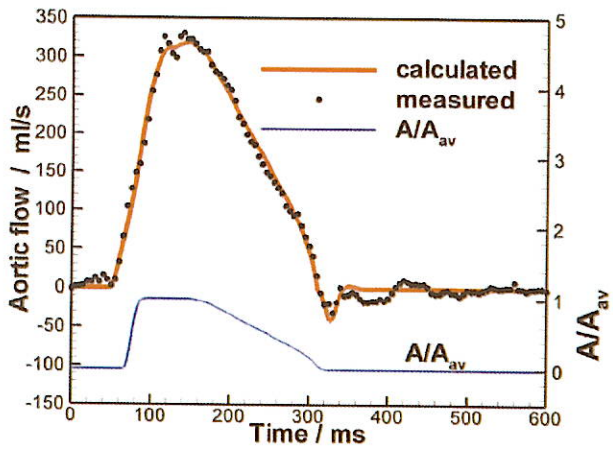


Fig. 2. Model results and measured data [2] in human (model data were $A_{\mathrm{av}}=2.9$ cm^2, $L=3$ cm, $K=1$, $\alpha=0.15$, $\beta=0.4$, $\gamma=0.8$, $\delta=10$)

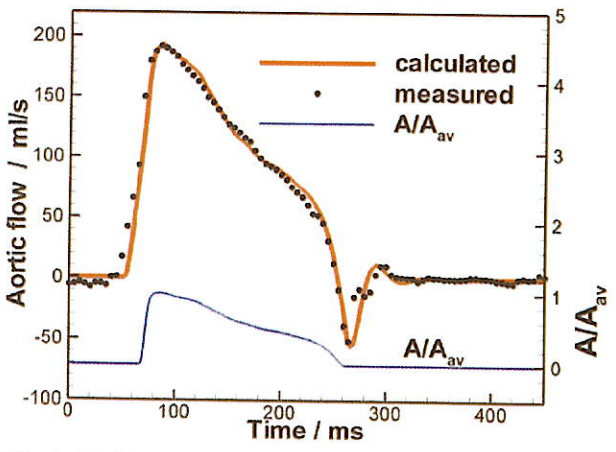


Fig. 3. Model results and measured data [3] in pig (model data were $A_{\mathrm{av}}=1.6$ cm^2, $L=1$ cm, $K=1$, $\alpha=0.5$, $\beta=0.6$, $\gamma=0.5$, $\delta=3$)

is in experimental observations. The model indicates that the leaflets coapt before the maximal back-flow occurs.

References

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