method is implicit, unconditionally stable and capable of dealing with nonlinearities.

At the inlet node the pressure or the inflow can be prescribed. In all simulations, the initial conditions were $Q(x,0)=0$, $A(x,0)=A_0$ and $p(x,0)=p_0$. The integration time should be long enough to achieve cycle-to-cycle periodicity, and the last cycle is examined.

## Results and discussion

Fig. 3 schematically shows the examined network. All data relevant to this problem are provided in the supplement material [6]. In the performed simulation each segment was divided into a number of elements (total number of elements was 431), and integration time step was 1 ms. Fig. 4 shows the comparison of the measured and calculated pressure and flow (for the case of elastic and viscoelastic wall), and Table 1 shows the percentage root mean square (RMS) errors of the calculated results with respect to the experimental data of the developed method and the range of these errors from the six other methods examined in [6].

In the case of elastic wall, most of the MOC errors are very similar in size to errors from six numerical schemes (see Table 1). In the case of viscoelastic wall, the pressure and flow RMS errors are slightly reduced, and a reduction in peak values of pressure and flow rate is achieved (that is closer to experimental data) because of damping high frequency oscillations.

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# Estimation of Velocity Profile and Pulsatile Wall Shear Stress in Arteries Using the Measured Maximal Velocity

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## Introduction

Blood flow in the arterial tree is essentially pulsatile. Time variation of the maximal velocity at certain points of the arterial tree may be obtained non-invasively and used for tuning of 1D model parameters. In 1D models it is convenient to assume a nearly flat velocity profile (blood flow rate is calculated as the product of maximal velocity and area) and Hagen-Poiseuille friction model derived from a steady state flow condition. The goal of this work was to estimate how good these two assumptions are. We used the measured maximal velocity and the Womersley quasi two dimensional (Q2D) solution of pulsatile flow in a rigid pipe [1] to define the velocity profile at the observed cross-section. From this velocity profile, the actual flow rate (or mean axial velocity) and wall shear stress were calculated.

## Womersley solution of Q2D flow

We observed an incompressible fluid flow in a rigid circular pipe of radius $R$ under a harmonic pressure gradient defined as:

$$\frac{\mathrm{d}p}{\mathrm{d}x} = S\sin(\omega t) + C\cos(\omega t), \qquad (1)$$

where $p(x,t)$ is pressure, $x$ is space coordinate along the pipe axis, $t$ is time and $\omega$ is circular frequency. If we assume an axial velocity $u(r,t)$, the linear momentum equation is:

$$\rho\frac{\partial u}{\partial t} = \mu\left(\frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2}\right) - \frac{\mathrm{d}p}{\mathrm{d}x}, \qquad (2)$$

where $\rho$ is fluid density, $\mu$ is viscosity and $r$ is radial coordinate. Eq. (2) is solved in the frequency domain, where the pressure gradient and axial velocity can be represented as real parts of:

$$\frac{\mathrm{d}p}{\mathrm{d}x} = \mathcal{R}e\left\{-i\hat{P}e^{i\omega t}\right\},\quad u = \mathcal{R}e\left\{-i\hat{U}(r)e^{i\omega t}\right\}, \qquad (3)$$

where $\hat{P} = S + iC$, and $\hat{U} = S_u + iC_u$ are complex amplitudes. The solutions for axial velocity, flow rate $Q = \int_0^R 2\pi u r\mathrm{d}r$ and wall shear stress $\tau_w$ are:

$$u = \mathcal{R}e\left\{\frac{\hat{P}}{\rho\omega}\left[1 - \frac{J_0(\Lambda y)}{J_0(\Lambda)}\right]e^{i\omega t}\right\}, \qquad (4)$$

$$Q = \mathcal{R}e\left\{ i\frac{2\pi\hat{P}\mu}{\rho^2\omega^2}\left[ -\frac{\mathrm{Wo}^2 i}{2} - \frac{\Lambda J_1(\Lambda)}{J_0(\Lambda)} \right] e^{i\omega t} \right\}, \quad (5)$$

$$\tau_w = \mathcal{R}e\left\{ \hat{P}\sqrt{\frac{\mu}{\rho\omega}}\frac{J_1(\Lambda)}{J_0(\Lambda)} e^{i\left(\omega t + \frac{3\pi}{4}\right)} \right\}, \quad (6)$$

where $\mathrm{Wo} = R\sqrt{\rho\omega/\mu}$ is Womersley number, $y = r/R$, $\Lambda = \mathrm{Wo}\exp(3\pi i/4)$, $J_0$ and $J_1$ are Bessel functions of the first kind and of zero and first order, respectively.

## Method

The maximal velocity is defined by Eq. (4) for $y = 0$. We start from the measured maximal velocity $u^{\mathrm{M}}_{\max}(t)$ during one heart period $T$ which may be decomposed into Fourier series:

$$u_{\max}(t) = \sum_{n=0}^{N}\left( S_u^n \sin(n\omega_0 t) + C_u^n \cos(n\omega_0 t) \right), \quad (7)$$

where $\omega_0 = 2\pi/T$, and $N$ is the number of harmonics. Using Eq. (4) for $u_{\max}$, it is possible to calculate the complex pressure gradient amplitudes at each frequency $\omega = n\omega_0$, and after that to calculate $Q(t)$ and $\tau_w(t)$ from Eqs. (5) and (6) (as a sum of contributions of all $N$ harmonics). The obtained average velocity $u_{\mathrm{avg}} = Q(t)/(R^2\pi)$ will be compared with the measured maximal velocity, and obtained $\tau_w(t)$ with $\tau_{w,S} = 4\mu Q/(R^3\pi)$ obtained from the Hagen-Poiseuille formula.

## Results

Simultaneous measurements of pressure gradient and axial velocity $u^{\mathrm{M}}(y,t)$ at $y =$ (0; 0.5; 0.75 and 0.95) in the femoral artery of a dog are given in [2]. We use $u^{\mathrm{M}}_{\max}(t) = u^{\mathrm{M}}(0,t)$ to calculate pressure gradient and axial velocity at the other measurement locations. The assumed Womersley number associated with the heart frequency is $\mathrm{Wo} = 3.8$. To check the applicability of the proposed method, we compared measured and reconstructed values depicted in Fig. 1.

Our results indicate that the proposed method could be used to estimate the axial velocity profile on the basis of the measured maximal velocity. Fig. 2 shows the time variation of the measured maximal velocity and obtained mean axial velocity, as well as the wall shear stress $\tau_w$ and $\tau_{w,S}$.

## Remarks

- The method proposed for the calculation of the axial velocity using the measured maximal velocity requires the assumption of the Wo number associated with heart frequency.
- The discrepancy of maximal and mean axial velocity is high. Because of that, the estimation of blood flow rate in blood vessels by using the maximal velocity is poor.
- The proposed method offers a good estimation of the mean velocity and wall shear stress.

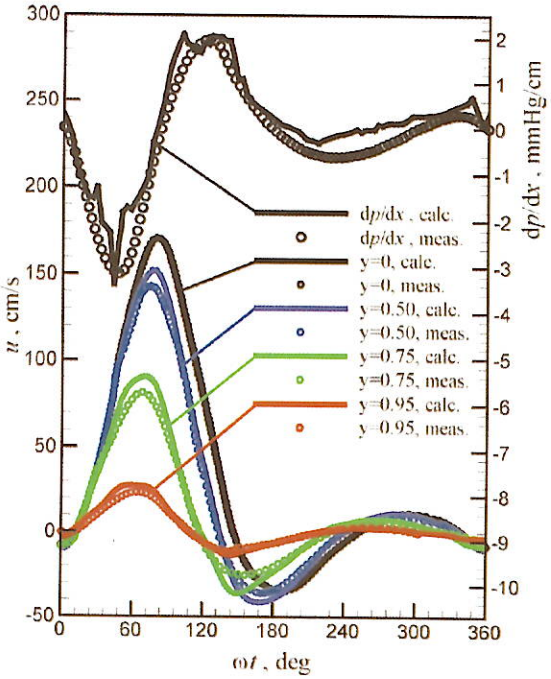


**Fig. 1.** Measured and calculated pressure gradient and axial velocity at different radii

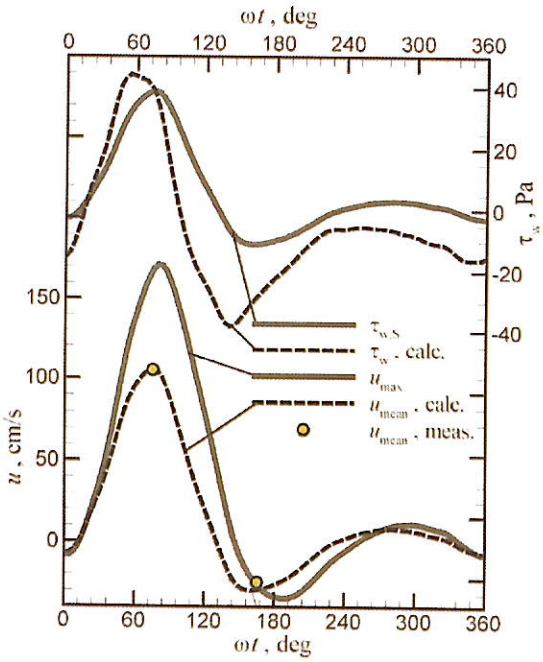


**Fig. 2.** Comparison between two wall shear stresses and measured maximal velocity with calculated mean axial velocity. Two points denote the measured values of maximal and minimal mean axial velocity

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