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Exploring Pre-Service Teachers' Handling of Student Errors in Fractions: A Case-Based Study

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Abstract

The purpose of this research is to explore the extent to which pre-service mathematics teachers handle students' errors in fraction multiplication and division within a teacher education course grounded in conceptual understanding and case-based pedagogy. A learning environment was purposefully designed to strengthen the connection between theory and practice using teaching cases. The findings show that case-based pedagogy helps pre-service teachers notice the underlying reasons for students' errors in fraction operations and handle these errors conceptually. The findings are discussed by focusing on common errors in learning fraction multiplication and division, the theory-practice connection, participants' attention to student errors, and the instructor's dual roles in supporting the learning of fractions.

Keywords: *conceptual understanding; mathematics learning; teacher decision making; theory-practice connection; student mistakes*

Introduction

Comprehension of fractions is essential for mathematical development (Braithwaite & Siegler, 2021; Jordan et al., 2017). However, understanding fractions and fraction arithmetic is challenging for students worldwide (OECD, 2014). We also know that students who have difficulty with fractions often show improvement only after years of instruction (Jordan et al., 2017). Therefore, rich learning opportunities are needed to establish a solid foundation in fractional understanding (Ismail et al., 2024).

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There are two major types of knowledge: conceptual and procedural knowledge (Hiebert & Lefevre, 1986). Conceptual knowledge refers to the comprehension of mathematical ideas, operations, and relations (National Research Council, 2001), while procedural knowledge involves knowledge of the formal language of mathematics and the ability to carry out algorithms (Hiebert & Lefevre, 1986). To illustrate, consider the operation $\frac{1}{2} \div \frac{1}{4} = ?$, the question of 'how many quarters are in a half' corresponds to conceptual understanding, whereas solving this division problem using the "flip and multiply" method corresponds to procedural understanding (Li & Kulm, 2008).

Operations with fractions are complex, and even pre-service teachers often struggle to develop a strong understanding of them (Lee, 2017; Ma, 1999; Olanoff et al., 2014). Some in-service and prospective teachers do not conceptually understand multiplication and division of fractions and rely on procedural methods (Copur-Gencturk, 2021; Erdem et al., 2015). Research consistently shows that teachers must have a deep mathematical understanding of the concepts they teach (Scheiner et al., 2019); otherwise, limitations in content knowledge are likely to affect students' learning negatively (Copur-Gencturk & Doleck, 2021). For this reason, teacher education programs should provide pre-service teachers with sufficient opportunities to develop a solid knowledge base in fraction multiplication and division (Olanoff et al., 2014). In addition, they should be given the time and structured opportunities to observe mathematics instruction and how students learn mathematics (Petersen & Treagust, 2014). However, there is a well-documented theory-practice gap in teacher education (Korthagen, 2017). Case-based pedagogy is a promising approach for bridging this gap (Helleve et al., 2023). Therefore, the present study employed case-based pedagogy to support pre-service mathematics teachers in analyzing and reflecting on middle school students' errors.

Case-based pedagogy

Case-based pedagogy uses teaching cases to support teachers' critical reflection on classroom events (Gunn et al., 2018). By engaging with teaching cases, pre-service teachers are allowed "to think like a teacher" (Shulman, 1992, p. 4) and to analyze representations of practice (Grossman et al., 2009) that illustrate the complexity of instructional settings and the interrelationships among student thinking, teacher actions, and teacher reasoning (Kavanagh et al., 2020). Cases help novice teachers develop professional ways of envisioning and understanding the work of teaching (Grossman et al., 2009). Prior research shows that teaching cases support the development of a range of cognitive skills in pre-service teachers, including problem-solving (Merseth, 2003), critical thinking and reasoning (Cochran-Smith & Zeichner, 2009). Cases also provide opportunities for pre-service teachers to explain how they would respond to classroom situations and to justify their decisions using theory (Ulvik et al., 2020), thereby serving as an effective bridge between theory and practice in teacher education (Ulvik et al., 2020).

Error management: in-service and pre-service teachers' responses to student errors

Correct answers and errors are two essential instances that require scrutiny in relation to student thinking (Larrain & Kaiser, 2019; Son, 2013). However, compared to correct answers, errors and their management appear to receive more attention in research on teacher noticing (Larrain & Kaiser, 2019; Santagata, 2004). As Son (2013, p. 50) emphasizes, the National Council of Teachers of Mathematics ([NCTM], 2000) regards attending to student mistakes appropriately as one of the primary responsibilities of mathematics teachers. It encourages viewing mathematical errors as "potential avenues for student learning" rather than "dead ends."

Regarding in-service teacher handling, Santagata's study (2004, p. 141) examined differences in how Italian and US teachers handle mistakes in real classroom settings and found that "mistakes are discussed publicly in Italy twice as often as in the United States." US teachers also displayed a more positive stance toward students when they made mistakes. In another study, Santagata (2005) also reported ten categories of teachers' mistake-handling strategies: giving correction, hinting to the same student, repeating the question to the same student, asking for the reason, hinting to another student, redirecting the question, picking the correct answer, asking the class, student initiative, and others. Tulis (2013) explored teachers' patterns of mistake handling through two main categories of teacher responses: maladaptive (e.g., redirecting the question to another student, ignoring the mistake, disappointment/hopelessness, criticizing the student, humiliating/laughing) and adaptive (e.g., correction by the student, discussion with the whole class, waiting, emphasizing the learning potential, impeding negative reactions from class). Tulis also reported, "more maladaptive patterns of mistake-handling activities in mathematics were found" (p. 56).

Regarding pre-service teachers, Son (2013) highlights the need for further research on their responses to student mistakes and how they handle them. Dofková and Kvintová's study (2019) with 212 pre-service primary school teachers revealed that error handling is rarely covered in university mathematics teaching courses. Another study (Son & Sinclair, 2010) investigated elementary pre-service teachers' responses to and interpretation of students' work about reflective symmetry. It was found that even though most pre-service teachers interpreted students' errors conceptually, while responding, they tried to cope with errors "by invoking procedural knowledge."

Regarding operations with fractions, related studies have shown that pre-service teachers' ability to identify, notice, and explain students' incorrect answers regarding fractions and fraction operations, as well as the underlying reasons for these errors, remains limited, algorithmic, and superficial (Son & Crespo, 2009; Lin, 2017; Coskun et al., 2023). Son and Crespo (2009), for instance, reveal that pre-service teachers' reasoning and reactions regarding a student's non-traditional strategy for dividing fractions were teacher-centered, meaning that the students' ideas were not sufficiently considered. Lin's (2017) study shows that pre-service teachers typically focus on the algorithmic aspects of fraction division errors; however, when provided with insights

into these errors and their underlying causes, their attention shifts from an algorithm-based to a conceptual approach. This clearly indicates that preservice teachers need purposeful intervention for handling students' ideas and errors.

Our literature review to date on in-service and pre-service teachers suggests that student error handling has generally been procedural (Shaughnessy et al., 2020). The need for teacher education to support students' conceptual understanding is consistently emphasized (e.g., Jansen et al., 2017). Building on the perspectives of Santagata (2004; 2005) and Tulis (2013), the present study extends this line of work by examining not only how pre-service teachers respond to student errors, but also the nature of the understanding that is foregrounded in these responses. Specifically, we analyze whether pre-service teachers' responses emphasize conceptual explanations, such as reasoning about units and quantities, or procedural interventions, such as describing and applying algorithms, thereby linking mistake-handling practices to underlying forms of mathematical understanding.

Although numerous studies have examined error management, the existing literature has not focused on students' error management within the context of case-based pedagogy, which justifies this study. We have focused on two ideas, the division of fractions and the multiplication of fractions, which are challenging for pre-service and novice teachers (Jansen et al., 2017; Son & Lee, 2016). So, this study aims to investigate the following research questions:

- How do pre-service mathematics teachers handle students' errors related to the multiplication of fractions within a teacher education course grounded in conceptual understanding and case-based pedagogy?
- How do pre-service mathematics teachers handle students' errors related to the division of fractions within a teacher education course grounded in conceptual understanding and case-based pedagogy?

Methodology

Setting, instruction, and cases

Data were collected from a teacher education program at a public university in Türkiye with 58 senior pre-service middle school mathematics teachers enrolled in a pedagogy course. The course aimed to enhance teaching for conceptual and procedural understanding and to link theory with practice. Participants had prior coursework in mathematics and pedagogy (e.g., calculus, algebra, and geometry). They were also introduced to conceptual and procedural understanding, including their definitions, relationships, and instructional support (Rittle-Johnson, 2017).

The research focuses on four weeks of instruction (12 hours), during which participants engaged in a range of instructional tasks related to teaching fraction multiplication and division. They reflected on the meanings of fraction multiplication and division, discussed a case on fraction division (Ma, 1999), and used area, length, and set models to visualize fractions. Participants reflected on common student errors, such

as confusing division by one-half with division by two and multiplication by one-half (Ma, 1999), and the belief that multiplication always increases a number (Siegler & Lortie-Forgues, 2015). They also discussed instructional strategies to address these misconceptions and teach fraction operations conceptually.

Theoretical knowledge in this study refers to the conceptual frameworks and instructional principles introduced in the course. The course syllabus and instructional activities explicitly introduced the frameworks, and the cases were designed to prompt their application in practice. Rather than assessing this knowledge directly, the analysis focused on how participants utilized these ideas in their written case analyses. Case-based pedagogy (Merseth, 2003) was used to support this process, with participants analyzing five instructor-developed cases on fraction operations: three on multiplication and two on division.

The cases were based on the first author's authentic classroom experiences and were designed to foreground students' conceptual errors. They were adopted as learning tools to engage pre-service teachers in analyzing student errors in fraction multiplication and division. Th

se activities to enhance understanding of theoretical concepts, such as conceptual and procedural understanding, and to encourage pre-service teachers to attend to students' reasoning rather than focusing solely on correctness.

For this study, two cases were selected for in-depth analysis due to their strong potential to link theory and practice (Darling-Hammond & Hammerness, 2002).

Case of fraction multiplication

The teacher wanted his middle school students to make sense of multiplication with fractions. After discussing the meaning of multiplication with whole numbers, the teacher introduced an operation requiring the multiplication of a fraction by a whole number, $2 \times \frac{1}{2}$. The students were asked to share the meaning of the operation and its product. During the whole group discussion, one of his students, Ayşe, said that the result of $2 \times \frac{1}{2}$ would be 5. She defended her claim by saying, "2 times two is 4, and 4 plus 1 makes 5." She produced an incorrect product of 5 rather than 1.

MQ1: If you were Ayşe's teacher, how would you help her fix her error?

MQ2: What questions would you ask her to guide her thinking?

Case of fraction division

After discussing the meaning of division with whole numbers, the teacher asked the students to elaborate on the meaning of $2 \div \frac{1}{2}$. The students supplied meaningful responses such as "how many one halves are in two wholes." Then, the teacher gave a more challenging operation, $\frac{1}{2} \div \frac{1}{4}$. Again, students produced different responses. For example, while one student responded, "The operation is asking for the quarter of one half, so the answer is one eighth," another student responded, "The answer is

16." Another one, on the other hand, replied, "I think it is 8." Thus, different responses show that students were unclear about the meaning of $\frac{1}{2} \div \frac{1}{4}$.

DQ1: If you were the teacher, how would you help students fix their errors?

DQ2: What questions would you ask them to guide their thinking?

Results

Data analysis

The analysis followed a hybrid deductive–inductive pathway to qualitative content analysis. Initially, major categories were formed deductively based on the literature on conceptual and procedural understanding (Copur-Gencturk, 2021; Hiebert & Lefevre, 1986; Lenz et al., 2020; Rittle-Johnson & Schneider, 2015; Sidney & Alibali, 2015). Then, categories not anticipated at the beginning were inductively formed.

Deductively, the analytic framework was grounded in established distinctions between conceptual and procedural understanding (Hiebert & Lefevre, 1986; Rittle-Johnson & Schneider, 2015). Categories reflecting conceptual understanding included responses engaging with mathematical ideas, operations, and relationships. On the other hand, Procedural categories involve responses focused on formal language and algorithmic execution. Each participant's complete response to a question was treated as a unit of analysis and examined for evidence of conceptual or procedural understanding, as well as for emerging themes.

The researchers' iterative examinations of the data, generated four themes: (A) Evidence for conceptual understanding, EFCU, (Hiebert & Lefevre, 1986; Rittle-Johnson & Schneider, 2015; Son & Sinclair, 2010), (B) Evidence for procedural understanding, EFPU, (Hiebert & Lefevre, 1986; Lenz et al., 2020), (C) Evidence for both conceptual and procedural understanding, (D) No evidence for either conceptual or procedural understanding.

While the distinctions among the categories are guided by a theoretical knowledge base, it is important to acknowledge that conceptual and procedural understanding are intertwined and not always mutually exclusive in practice (Lenz et al., 2020). Some responses reflected elements of both types of understanding and were coded accordingly. Therefore, our framework should be understood as an analytical lens for portraying the dominant features of responses rather than as an exact measure of participants' knowledge.

The themes consisted of categories (e.g., A1. Making sense of the operation), subcategories (e.g., A.1.1. Making connections), and codes (e.g., a. Making connections between whole number multiplication and fraction multiplication). The EFCU theme, for instance, consists of three categories of evidence for conceptual understanding: A1. Making sense of the operation (Clarke et al., 2008; Rittle-Johnson & Schneider, 2015), A2. Generating cognitive conflict, and A3. Eliciting student reasoning and thinking (Neumann, 2014). These categories consist of subcategories. The subcategories consist of codes, and their contents are closely related to the mathematics of the teaching

cases, including making connections between a) whole number multiplication and fraction multiplication, b) repeated addition and multiplication, and c) whole number division and fraction division.

For example, the subcategory "making connections between whole number and fraction multiplication" was formed inductively within the broader category "making sense of the operation." This subcategory refers to responses in which participants explained fraction multiplication by associating it with whole-number multiplication. Its inclusion reflects the role of analogical reasoning in supporting conceptual understanding (English, 2004). Here is a sample excerpt of the subcategory, "I would ask simple questions. For example, I would ask what 2×2 means. They would probably say that it means adding two 2s. In the same way here, they would think that they need to add twos." It was assigned to this specific subcategory because the participant explicitly linked fraction multiplication to whole-number multiplication.

Upon initial readings and data coding, the researchers realized that the participants' responses included instances of generating cognitive conflict in students' thinking (inductive coding). Some participants conceptually redirected the mathematical problem presented in the teaching case. While in some cases the entire response was coded to a single category, in others, only a single sentence or a word was coded to a category. Two mathematics education researchers with more than 15 years of experience in designing and coding qualitative research traditions developed the coding framework and coded the data. They analyzed the entire data set individually and compared their analyses to reconcile coding disagreements and improve coding quality (Campbell et al., 2013). While they disagreed on some coding categories, after rounds of discussion and re-coding, they finally reached an acceptable agreement level of over 80 % (Neuendorf, 2016). They reached an agreement after further discussions on all the analyzed responses.

Results about pre-service teachers' handling of student errors in fraction multiplication

Regarding MQ1, participants' responses exhibit conceptual features and supporting evidence (Table 1). While 66 % of the pre-service teachers' responses have 'evidence for conceptual understanding,' only 12 % have 'evidence for procedural understanding.' 5 % of responses exhibited both conceptual and procedural evidence, and 16 % had no evidence for either type of understanding. The data show that 71 % of the responses (66 % + 5 %) are conceptual.

Responses frequently reflected 'evidence for conceptual understanding', suggesting that participants focused on conceptual approaches when responding to student errors. Their responses indicate that they would help the student by drawing on conceptual perspectives, most commonly by enhancing understanding of the operation itself. In contrast, strategies that require more active engagement with students' thinking, such as generating cognitive conflict and eliciting student reasoning, appeared less

frequently. This pattern suggests that, although pre-service teachers demonstrate an inclination toward conceptual understanding, their approaches are relatively limited in scope and emphasize explanation over deeper diagnostic or dialogic engagement with students' ideas.

The most common code was connecting repeated addition and multiplication, indicating that participants frequently relied on connecting new topics to familiar whole number operations. For instance, one participant said, "I would show that 2 x $\frac{1}{2}$ (Two times one half) is equivalent to adding $\frac{1}{2}$ and $\frac{1}{2}$ and hence $2 x \frac{1}{2} = \frac{1}{2} + \frac{1}{2}$." Further, models such as shape and paper folding were employed to model fractions. This move indicates an effort to make an abstract mathematical idea more tangible and comprehensible to students. One participant said, "I would help Ayşe through concrete examples and ask her to think of the question of 'suppose we have two halves of an apple and how many apples these two halves would make.' The above strategies suggest that participants relied on connecting fraction operations to familiar arithmetic structures and supporting understanding through concrete representations.

Only a few participants mentioned generating cognitive conflict, suggesting they attempted to challenge students' existing conceptions. They tended to focus on distinguishing between fraction addition and fraction multiplication to highlight conceptual differences. Similarly, within the 'eliciting student reasoning and thinking' category, participants often suggested encouraging students to explain or reflect on their responses. These responses indicate that some participants attended to students' thinking; however, only a small number employed this strategy.

Responses to MQ2 showed a pattern similar to that reported for MQ1. Most responses reflected evidence for conceptual understanding, indicating that pre-service teachers continued to employ conceptual approaches when suggesting questions for the student. Given this similarity in distribution, a detailed analysis of subcategories was not repeated for MQ2.

Table 1
Results from the Analysis of Multiplication and Division Cases

	Multiplication		Division	
	MQ1	MQ2	DQ1	DQ2
Main Themes/categories/sub-categories/codes	n = 58	n = 58	n = 54	n = 54
A. Evidence for conceptual understanding (EFCU)	66 %	81 %	74 %	83 %
A.1.Making sense of the operation by	48 %	48 %	70 %	44 %
A.1.1. Making a connection	40 %	36 %	70 %	28 %
a. Between whole number and fraction multiplication	-	9 %	NA	NA
b. Between repeated addition and multiplication	38 %	24 %	NA	NA
c. Between whole number division and fraction division	NA	NA	NA	NA
d. Between fraction multiplication and fraction division	2 %	-	NA	NA
e. Among repeated subtraction (measurement division), whole number division, and fraction division	NA	NA	7 %	9 %

f. Between repeated subtraction (meas. div.) & fraction div.	NA	NA	61 %	15 %
g. Between addition and multiplication	-	1 (2 %)	NA	NA
h. Between real life and mathematics	5 %	5 %	7 %	%2
A.1.2. Modeling fractions	33 %	24 %	61 %	13 %
A.1.3. Redirecting the question conceptually	3 %	7 %	-	15 %
A.1.4. Posing follow-up questions to see if the student understands the operation conceptually	-	3 %	-	4 %
A.2. Generating cognitive conflict by	9 %	3 %	-	4 %
A.2.1. Differentiating operations with fractions by contrasting	7 %	-	-	%2
(I) fraction addition and fraction multiplication	7 %	2 %	NA	NA
(II) improper fractions and fraction multiplication	-	2 %	NA	NA
(III) fraction division and fraction multiplication	NA	NA	-	2 %
A.2.2. Working backward to spot the error	-	2 %	-	2 %
A.3. Eliciting student reasoning and thinking by	17 %	34 %	6 %	57 %
A.3.1. Asking students to reflect on the meaning of the operation	2 %	2 %	-	17 %
A.3.2. Asking the student to explain/reflect on his/her response	12 %	16 %	2 %	7 %
A.3.3. Asking to recall one's prerequisite knowledge	-	9 %	2 %	20 %
A.3.4. Asking the student to reflect on the difference between operations with fractions	3 %	12 %	2 %	19 %
(I) between division and multiplication	3 %	3 %	2 %	19 %
(II) between fraction addition and multiplication	-	3 %	NA	NA
(III) between improper fractions and fraction multiplication	-	5 %	NA	NA
B. Evidence for procedural understanding (EFPU)	12 %	2 %	6 %	-
B.1. Showing how to operate procedurally	12 %	-	6 %	6 %
B.2. Posing follow-up questions to see if the student understands the operation procedurally	2 %	2 %	-	-
C. Evidence for conceptual and procedural understanding	5 %	3 %	11 %	6 %
D. No evidence for either conceptual or procedural understanding	16 %	3 %	9 %	9 %
D.1. Stating the response without further details.	12 %	2 %	6 %	6 %
D.2. Summarize the content	3 %	-	4 %	-
D.3. Providing an incorrect explanation	-	2 %	-	4 %
Not Attempted	-	12 %	-	2 %
Not categorized	2 %	-	-	-

n = number of participants

Results about pre-service teachers' handling of student errors in fraction division

Regarding DQ1, Table 1 shows that 74 % of participants' responses have 'evidence for conceptual understanding' for handling students' wrong answers. When themes A (evidence for conceptual understanding) and C (evidence for both conceptual and procedural understanding) are considered together, the findings show that 85 % (74 % + 11 %) of the responses contain 'evidence for conceptual understanding'. It is therefore evident that most participants' treatment of students' errors is supported by conceptual evidence.

Responses within this theme focused primarily on making sense of the operation. Pre-service teachers most often suggested establishing connections between fraction division and familiar whole number operations, particularly through repeated subtraction. This indicates a tendency to formulate explanations based on prior knowledge, utilizing known arithmetic procedures to make fraction operations more comprehensible for students. For example, Participant #1 made a connection between whole number division and fraction division by saying, "For $12 \div 2$, you answer, how many times does two go into twelve? So, $\frac{1}{2} \div \frac{1}{4}$ means finding the number of $\frac{1}{4}$ in $\frac{1}{2}$" Thus, the participant related repeated subtraction and fraction division.

In addition, many participants employed representations such as shapes or paper folding to model fraction relationships. These responses indicate an effort to make abstract operations more accessible and to support students' conceptualization of fractions. For example, Participant #28 wrote, "I would use physical materials so that they can see visually and understand the operations." She also modeled one-half and one-fourth on two separate circles. This participant intends to model fractions with materials to enhance students' conceptual understanding of fraction division.

Findings regarding DQ2 indicate that pre-service teachers' responses provide conceptual evidence for responding to students' errors in fraction division cases, indicating that pre-service teachers employed conceptual approaches when suggesting questions for students. Specifically, more than half employed the 'eliciting student reasoning and thinking' strategy to formulate questions about student errors. For example, Participant #36 wrote, "I would ask, 'how did you find 16?' and 'Would you show it to me by modeling your answer?' So, I could explore where the student made an error and the misconception behind it." The questions are asked to elicit student thinking. The participant wanted to understand the roots of the student errors. Only a few responses were oriented toward a procedural perspective; thus, those responses were formulated to help students focus on how to operate procedurally.

Discussion

This study aimed to explore the extent to which pre-service teachers drew on theoretical concepts introduced in the course when responding to middle school students' errors based on student thinking. In addition, we designed a unique setting

to increase the likelihood of closing the gap between theory and practice in teaching mathematics by using teaching cases in which the university instructor was the case teacher. Finally, we discuss our findings by focusing on the errors often associated with learning fraction operations in light of this study, the theory-practice connection, attending to student errors, and the instructor's dual role in learning fractions.

In this research, we expected the participants to use their theoretical knowledge to describe and interpret middle school students' errors. We analyzed participants' responses to see if they related what they learned in their college course to actual teaching events. An in-depth analysis confirmed that our expectations were met. In participants' written responses, there was ample evidence of characteristics of conceptual understanding. The findings clearly show that many participants' responses had references to making sense of division or multiplication with fractions. In handling students' conceptual errors, participants' responses focused on helping students understand the meaning of the operations. To be more specific, they proposed instructional moves to help students connect mathematical operations and model fractions, so that errors can be recognized and corrected. Given that pre-service teachers are often reported to have their own difficulties with fractions (Jansen et al., 2017; Son & Lee, 2016), our findings have revealed that the case-based pedagogy, as employed in this study, provided opportunities for pre-service teachers to develop their own conceptual understanding and hence engage with student thinking and errors conceptually.

The instructor purposefully used research-based resources to connect theory and practice, drawing on literature on conceptual understanding (Hiebert & Carpenter, 1992), multiple representations (Ainsworth, 1999), noticing (Stockero et al., 2017), and conceptual error handling (Tulis, 2013). This approach was reflected in pre-service teachers' responses, which included making connections between repeated addition and multiplication, using models (e.g., shapes, paper cutting), and eliciting student reasoning through questioning. These patterns suggest that both the instructor and participants drew on research-informed practices (Dreyfus et al., 2021) when addressing student errors.

Although earlier research shows that teachers and pre-service teachers often handle errors procedurally rather than conceptually (e.g., Son & Sinclair, 2010; Tulis, 2013; Santagata, 2005), our findings show that participants predominantly adopted conceptual approaches. Their responses involved making connections, using models, and attending to students' reasoning, indicating an effort to interpret and address the underlying causes of errors.

Case-based pedagogy has been shown to connect theory and practice and support reflection on critical teaching instances (Gunn et al., 2018; Ulvik et al., 2020). In this study, participants analyzed authentic teaching cases created by their instructor, enabling them to engage with authentic instances of student thinking. The findings suggest that this approach helped participants notice key aspects of student errors and respond in conceptually oriented ways. Teacher candidates' use of fraction models, making

connections between addition and multiplication, and posing eliciting questions are all indicators of the case-based pedagogy.

Conclusion, limitations, and some implications

Our study has shown that the adopted case-based pedagogy has helped teacher candidates conceptualize student errors, indicating that the instructor successfully used mathematics education research as a resource (Dreyfus et al., 2021). However, like prior studies (Lin, 2017), this pattern cannot be attributed solely to the instructional method, as fraction reasoning itself may elicit both conceptual and procedural responses (Siegler & Lortie-Forgues, 2015). Thus, it remains unclear whether these outcomes reflect pedagogy or the nature of the mathematical topic, and further research is needed to clarify this issue.

The analytical framework developed in this study represents a methodological contribution and may serve as a tool for analyzing teachers' responses to student errors. Further research is needed to examine how operational this framework is. The absence of pre- and post-test measures of participants' development is a limitation of our study, and further research is clearly needed to examine the effect of an interventional study regarding error-handling practices. Investigating how pre-service teachers handle similar student errors in actual classroom settings should also be considered for further research.

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Disclosure of conflict of interest

There is no real or apparent conflict of interest that may have a direct bearing on the subject matter of the article.

Ethical considerations

This study includes human research participants. Ethical approval was obtained from the Science and Engineering Ethical Committee, Kocaeli University (Approval: 917846).

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Istraživanje načina na koji budući učitelji reagiraju na učenikove pogreške u razlomcima: studija temeljena na slučajevima

Sažetak

Svrha ovoga istraživanja jest istražiti u kojoj mjeri budući nastavnici matematike reagiraju na učenikove pogreške u množenju i dijeljenju razlomaka unutar kolegija za obrazovanje učitelja koji se temelji na konceptualnom razumijevanju i pedagogiji temeljenoj na slučajevima. Okružje za učenje namjerno je osmišljeno kako bi se ojačala povezanost između teorije i prakse korištenjem nastavnih slučajeva. Rezultati pokazuju da pedagogija temeljena na slučajevima pomaže budućim nastavnicima uočiti temeljne razloge učenikovih pogrešaka u operacijama s razlomcima te konceptualno pristupiti tim pogreškama. O nalazima se raspravlja usredotočujući se na česte pogreške u učenju množenja i dijeljenja razlomaka, na povezanost teorije i prakse, usmjerenost sudionika na učenikove pogreške te dvojne uloge nastavnika u podržavanju učenja razlomaka.

Ključne riječi: *donošenje nastavničkih odluka; konceptualno razumijevanje; povezanost teorije i prakse; učenikove pogreške; učenje matematike*

Uvod

Razumijevanje razlomaka ključno je za matematički razvoj (Braithwaite i Siegler, 2021; Jordan i sur., 2017). Međutim, razumijevanje razlomaka i računskih operacija s razlomcima predstavlja izazov učenicima diljem svijeta (OECD, 2014). Također znamo da učenici koji imaju poteškoća s razlomcima često pokazuju napredak tek nakon višegodišnje nastave (Jordan i sur., 2017). Stoga su potrebne bogate prilike za učenje kako bi se uspostavila čvrsta osnova za razumijevanje razlomaka (Ismail i sur., 2024).

Postoje dvije glavne vrste znanja: konceptualno i proceduralno znanje (Hiebert i Lefevre, 1986). Konceptualno znanje odnosi se na razumijevanje matematičkih ideja, operacija i odnosa (National Research Council, 2001), dok proceduralno znanje uključuje poznavanje formalnoga jezika matematike i sposobnost izvođenja algoritama (Hiebert i Lefevre, 1986). Primjerice, u operaciji $\frac{1}{2} \div \frac{1}{4} = ?$, pitanje „koliko četvrtina ima u jednoj polovini" odgovara konceptualnom razumijevanju, dok rješavanje ovoga zadatka metodom „okreni i množi" odgovara proceduralnom razumijevanju (Li i Kulm, 2008).

Operacije s razlomcima složene su, a čak i budući učitelji često imaju poteškoća u razvijanju čvrstoga razumijevanja tih operacija (Lee, 2017; Ma, 1999; Olanoff i sur., 2014). Neki učitelji i budući učitelji ne razumiju konceptualno množenje i dijeljenje razlomaka te se oslanjaju na proceduralne metode (Copur-Gencturk, 2021; Erdem i sur., 2015). Istraživanja dosljedno pokazuju da učitelji moraju imati duboko matematičko razumijevanje koncepata koje poučavaju (Scheiner i sur., 2019); u suprotnom će ograničenja u sadržajnom znanju vjerojatno negativno utjecati na učenje učenika (Copur-Gencturk i Doleck, 2021). Zbog toga bi programi obrazovanja učitelja trebali budućim učiteljima pružiti dovoljno prilika za razvoj čvrste baze znanja o množenju i dijeljenju razlomaka (Olanoff i sur., 2014). Osim toga, potrebno im je omogućiti vrijeme i strukturirane prilike za promatranje nastave matematike i načina na koji učenici uče matematiku (Petersen i Treagust, 2014). Međutim, u obrazovanju učitelja dobro je dokumentiran jaz između teorije i prakse (Korthagen, 2017). Pedagogija temeljena na slučajevima predstavlja obećavajući pristup za premošćivanje toga jaza (Helleve i sur., 2023). Stoga je u ovome istraživanju korištena pedagogija temeljena na slučajevima kako bi se podržalo buduće nastavnike matematike u analizi i promišljanju o pogreškama učenika viših razreda osnovne škole.

Pedagogija temeljena na slučajevima

Pedagogija temeljena na slučajevima koristi nastavne slučajeve kako bi podržala kritičku refleksiju učitelja o događajima u učionici (Gunn i sur., 2018). Sudjelovanjem u radu na nastavnim slučajevima budućim učiteljima omogućuje se „razmišljati poput učitelja" (Shulman, 1992, str. 4) te analizirati prikaze nastavne prakse (Grossman i sur., 2009) koji ilustriraju složenost nastavnih okružja i međusobne odnose između učenikova razmišljanja, nastavnikovih postupaka i nastavničkoga rasuđivanja (Kavanagh i sur., 2020). Slučajevi pomažu učiteljima početnicima razviti profesionalne načine zamišljanja i razumijevanja učiteljskoga rada (Grossman i sur., 2009). Dosadašnja istraživanja pokazuju da nastavni slučajevi podržavaju razvoj različitih kognitivnih vještina kod budućih učitelja, uključujući rješavanje problema (Merseth, 2003), kritičko mišljenje i rasuđivanje (Cochran-Smith i Zeichner, 2009). Slučajevi također pružaju prilike budućim učiteljima da objasne kako bi reagirali u situacijama iz učionice i da svoje odluke opravdaju teorijom (Ulvik i sur., 2020), čime služe kao učinkovit most između teorije i prakse u obrazovanju učitelja (Ulvik i sur., 2020).

Upravljanje pogreškama: reakcije učitelja i budućih učitelja na učenikove pogreške

Točni odgovori i pogreške dvije su ključne situacije koje zahtijevaju pažljivo razmatranje u odnosu na učenikovo razmišljanje (Larrain i Kaiser, 2019; Son, 2013). Međutim, u usporedbi s točnim odgovorima, pogreške i način njihova rješavanja dobivaju više pozornosti u istraživanjima nastavničkoga uočavanja (*teacher noticing*) (Larrain i

Kaiser, 2019; Santagata, 2004). Kao što Son (2013, str. 50) naglašava, Nacionalno vijeće učitelja matematike (National Council of Teachers of Mathematics [NCTM], 2000) smatra primjereno obraćanje pozornosti na učenikove pogreške jednom od glavnih odgovornosti nastavnika matematike. Pritom se matematičke pogreške promatraju kao „potencijalni putevi za učenikovo učenje", a ne kao „slijepe ulice".

U vezi s reakcijama učitelja u praksi, Santagatino istraživanje (2004, str. 141) ispitivalo je razlike između talijanskih i američkih učitelja u postupanju s pogreškama u stvarnim učioničkim situacijama te je pokazalo da se „o pogreškama javno raspravlja u Italiji dvostruko češće nego u Sjedinjenim Američkim Državama". Američki učitelji također su pokazivali pozitivniji stav prema učenicima kada bi pogriješili. U drugome istraživanju Santagata (2005) navodi deset kategorija strategija učiteljeva postupanja s pogreškama: davanje ispravka, davanje natuknice istom učeniku, ponavljanje pitanja istom učeniku, traženje razloga, davanje natuknice drugom učeniku, preusmjeravanje pitanja, odabir točnoga odgovora, postavljanje pitanja cijelom razredu, učenikova inicijativa i ostalo. Tulis (2013) je istraživao obrasce učiteljeva postupanja s pogreškama tijekom dvije glavne kategorije reakcija: maladaptivne (npr. preusmjeravanje pitanja drugom učeniku, ignoriranje pogreške, razočaranje/beznadnost, kritiziranje učenika, ponižavanje/ismijavanje) i adaptivne (npr. učenik ispravlja pogrešku, rasprava s cijelim razredom, čekanje, naglašavanje potencijala za učenje, sprječavanje negativnih reakcija razreda). Tulis je također izvijestio da su „u matematici pronađeni više maladaptivni obrasci postupanja s pogreškama" (str. 56).

Što se tiče budućih učitelja, Son (2013) ističe potrebu za većim brojem istraživanja o reakcijama budućih učitelja na učenikove pogreške i o načinima njihova postupanja. Istraživanje Dofkove i Kvintove (2019) provedeno s 212 budućih učitelja razredne nastave pokazalo je da se postupanje s pogreškama rijetko obrađuje na sveučilišnim kolegijima matematike. Drugo istraživanje (Son i Sinclair, 2010) proučavalo je reakcije i interpretacije rada učenika budućih učitelja osnovne škole u vezi s refleksivnom simetrijom. Utvrđeno je da, iako je većina budućih učitelja konceptualno interpretirala učenikove pogreške, pri reagiranju pokušavali su rješavati pogreške „pozivanjem na proceduralno znanje".

U vezi s operacijama s razlomcima, povezana istraživanja pokazala su da sposobnost budućih učitelja da identificiraju, uoče i objasne učenikove netočne odgovore povezane s razlomcima i operacijama s razlomcima, kao i temeljne razloge tih pogrešaka, ostaje ograničena, algoritamska i površna (Son i Crespo, 2009; Lin, 2017; Coskun i sur., 2023). Primjerice, Son i Crespo (2009) pokazuju da su rasuđivanje i reakcije budućih učitelja u vezi s netradicionalnom učenikovom strategijom dijeljenja razlomaka bili usmjereni na učitelja, što znači da učenikove ideje nisu bile dovoljno uzete u obzir. Linovo istraživanje (2017) pokazuje da se budući učitelji obično usredotočuju na algoritamske aspekte pogrešaka pri dijeljenju razlomaka; međutim, kada im se pruže uvidi o tim pogreškama i njihovim uzrocima, njihova se pažnja pomiče s

algoritamskoga na konceptualni pristup. To jasno pokazuje da budući učitelji trebaju svrhovite intervencije za rad s učenikovim idejama i pogreškama.

Naš pregled literature o učiteljima i budućim učiteljima sugerira da je postupanje s učenikovim pogreškama uglavnom bilo proceduralno (Shaughnessy i sur., 2020). Potreba da obrazovanje učitelja podrži konceptualno razumijevanje učenika stalno se naglašava (npr. Jansen i sur., 2017). Polazeći od perspektiva Santagate (2004; 2005) i Tulisa (2013), ovo istraživanje proširuje navedeni smjer rada ispitivanjem ne samo načina na koji budući učitelji reagiraju na učenikove pogreške, nego i prirode razumijevanja koje je naglašeno u tim reakcijama. Konkretno, analiziramo naglašavaju li reakcije budućih učitelja konceptualna objašnjenja, poput rasuđivanja o jedinicama i količinama, ili proceduralne intervencije, poput opisivanja i primjene algoritama, povezujući tako postupanje s pogreškama s temeljnim oblicima matematičkoga razumijevanja.

Iako su provedena brojna istraživanja o upravljanju pogreškama, postojeća literatura nije se usredotočila na upravljanje učenikovim pogreškama u kontekstu pedagogije temeljene na slučajevima što opravdava potrebu za ovim istraživanjem. Usredotočili smo se na dvije ideje — dijeljenje razlomaka i množenje razlomaka — koje predstavljaju izazov budućim i učiteljima početnicima (Jansen i sur., 2017; Son i Lee, 2016). Stoga se ovim istraživanjem nastoji odgovoriti na sljedeća istraživačka pitanja:

- Kako budući nastavnici matematike reagiraju na učenikove pogreške povezane s množenjem razlomaka unutar kolegija za obrazovanje učitelja koji se temelji na konceptualnom razumijevanju i pedagogiji temeljenoj na slučajevima?
- Kako budući nastavnici matematike reagiraju na učenikove pogreške povezane s dijeljenjem razlomaka unutar kolegija za obrazovanje učitelja koji se temelji na konceptualnom razumijevanju i pedagogiji temeljenoj na slučajevima?

Metodologija

Okružje, nastava i slučajevi

Podatci su prikupljeni u programu obrazovanja učitelja na javnom sveučilištu u Turskoj, s 58 studenata završne godine studija za buduće nastavnike matematike u višim razredima osnovne škole, upisanih na pedagoški kolegij. Cilj kolegija bio je unaprijediti poučavanje za konceptualno i proceduralno razumijevanje te povezati teoriju i praksu. Sudionici su prethodno pohađali kolegije iz matematike i pedagogije (npr. analiza, algebra i geometrija). Također su bili upoznati s konceptualnim i proceduralnim razumijevanjem, uključujući njihove definicije, odnose i nastavnu podršku (Rittle-Johnson, 2017).

Istraživanje se usredotočuje na četiri tjedna nastave (12 sati), tijekom kojih su sudionici sudjelovali u različitim nastavnim aktivnostima povezanima s poučavanjem množenja i dijeljenja razlomaka. Razmišljali su o značenjima množenja i dijeljenja razlomaka, raspravljali o slučaju dijeljenja razlomaka (Ma, 1999) te koristili modele površine, duljine i skupova za vizualizaciju razlomaka. Sudionici su promišljali o čestim

učenikovim pogreškama, poput zamjene dijeljenja s jednom polovinom i dijeljenja s dva te množenja s jednom polovinom (Ma, 1999), kao i o uvjerenju da množenje uvijek povećava broj (Siegler i Lortie-Forgues, 2015). Također su raspravljali o nastavnim strategijama za rješavanje tih pogrešnih predodžbi i konceptualno poučavanje operacija s razlomcima.

Teorijsko znanje u ovome istraživanju odnosi se na konceptualne okvire i nastavna načela predstavljena tijekom kolegija. Nastavni plan i nastavne aktivnosti izričito su uvodili te okvire, a slučajevi su bili osmišljeni kako bi potaknuli njihovu primjenu u praksi. Umjesto izravnoga vrednovanja toga znanja, analiza se usredotočila na koji su se način sudionici koristili tim idejama u svojim pisanim analizama slučajeva. Pedagogija temeljena na slučajevima (Merseth, 2003) korištena je za podršku tome procesu, pri čemu su sudionici analizirali pet slučajeva o operacijama s razlomcima koje je razvio nastavnik: tri o množenju i dva o dijeljenju.

Slučajevi su se temeljili na autentičnim iskustvima prvoga autora iz učionice i bili su osmišljeni kako bi istaknuli učenikove konceptualne pogreške. Korišteni su kao alati za učenje kako bi se buduće učitelje uključilo u analizu učenikovih pogrešaka u množenju i dijeljenju razlomaka. Slučajevi su integrirani u aktivnosti kolegija radi boljega razumijevanja teorijskih koncepata, poput konceptualnoga i proceduralnoga razumijevanja te kako bi se buduće učitelje potaknulo da obraćaju pozornost na učenikovo rasuđivanje, a ne samo na točnost odgovora.

Za ovo istraživanje odabrana su dva slučaja za detaljnu analizu zbog njihovoga snažnog potencijala povezivanja teorije i prakse (Darling-Hammond i Hammerness, 2002).

Slučaj množenja razlomaka

Nastavnik je želio da njegovi učenici viših razreda osnovne škole razumiju značenje množenja razlomaka. Nakon rasprave o značenju množenja cijelih brojeva, nastavnik je uveo operaciju koja zahtijeva množenje razlomka cijelim brojem, $2 \times \frac{1}{2}$. Učenici su trebali objasniti značenje operacije i njezina rezultata. Tijekom rasprave u cijelome razredu jedna učenica, Ayşe, rekla je da će rezultat $2 \times \frac{1}{2}$ biti 5. Svoju je tvrdnju branila riječima: „2 puta 2 je 4, a 4 plus 1 je 5." Time je dobila netočan rezultat 5 umjesto 1.

MQ1**:** Da ste Ayşin učitelj, kako biste joj pomogli ispraviti pogrešku?

MQ2**:** Koja biste joj pitanja postavili kako biste usmjerili njezino razmišljanje?

Slučaj dijeljenja razlomaka

Nakon rasprave o značenju dijeljenja cijelih brojeva, nastavnik je zamolio učenike da objasne značenje operacije $2 \div \frac{1}{2}$. Učenici su dali smislene odgovore poput: „Koliko polovina ima u dva cijela." Zatim je nastavnik zadao zahtjevniju operaciju, $\frac{1}{2} \div \frac{1}{4}$. Učenici su ponovno dali različite odgovore. Primjerice, jedan učenik odgovorio je: „Operacija traži četvrtinu jedne polovine, pa je odgovor jedna osmina", dok je drugi odgovorio: „Odgovor je 16." Treći je rekao: „Mislim da je 8." Različiti odgovori pokazali su da učenicima nije bilo jasno značenje operacije $\frac{1}{2} \div \frac{1}{4}$.

DQ1: Da ste vi učitelj, kako biste pomogli učenicima ispraviti njihove pogreške?
DQ2: Koja biste im pitanja postavili kako biste usmjerili njihovo razmišljanje?

Rezultati

Analiza podataka

Analiza je slijedila hibridni deduktivno–induktivni pristup kvalitativnoj analizi sadržaja. U početku su glavne kategorije oblikovane deduktivno, na temelju literature o konceptualnom i proceduralnom razumijevanju (Copur-Gencturk, 2021; Hiebert i Lefevre, 1986; Lenz i sur., 2020; Rittle-Johnson i Schneider, 2015; Sidney i Alibali, 2015). Zatim su induktivno oblikovane kategorije koje nisu bile predviđene na početku.

Deduktivno-analitički okvir bio je utemeljen na ustaljenim razlikama između konceptualnoga i proceduralnoga razumijevanja (Hiebert i Lefevre, 1986; Rittle-Johnson i Schneider, 2015). Kategorije koje odražavaju konceptualno razumijevanje uključivale su odgovore koji se bave matematičkim idejama, operacijama i odnosima. S druge strane, proceduralne kategorije uključivale su odgovore usmjerene na formalni jezik i algoritamsko izvođenje. Cjeloviti odgovor svakoga sudionika na pojedino pitanje smatrao se jedinicom analize te je ispitan u pogledu dokaza konceptualnoga ili proceduralnoga razumijevanja, kao i novonastalih tema.

Iterativnim pregledima podataka istraživači su oblikovali četiri teme: (A) dokazi konceptualnoga razumijevanja, EFCU, (Hiebert i Lefevre, 1986; Rittle-Johnson i Schneider, 2015; Son i Sinclair, 2010), (B) dokazi proceduralnoga razumijevanja, EFPU, (Hiebert i Lefevre, 1986; Lenz i sur., 2020), (C) dokazi i konceptualnoga i proceduralnoga razumijevanja, (D) bez dokaza konceptualnoga ili proceduralnoga razumijevanja.

Iako su razlike među kategorijama vođene teorijskom bazom znanja, važno je priznati da su konceptualno i proceduralno razumijevanje međusobno isprepleteni i da u praksi nisu uvijek međusobno isključivi (Lenz i sur., 2020). Neki su odgovori odražavali elemente obiju vrsta razumijevanja te su u skladu s tim kodirani. Stoga naš okvir treba razumjeti kao analitičku leću za prikaz dominantnih obilježja odgovora, a ne kao preciznu mjeru znanja sudionika.

Teme su se sastojale od kategorija (npr. A1. Osmišljavanje operacije), potkategorija (npr. A.1.1. Uspostavljanje poveznice) i kodova (npr. a. Uspostavljanje poveznica između množenja cijelih brojeva i množenja razlomaka). Tema EFCU, primjerice, sastoji se od triju kategorija dokaza konceptualnoga razumijevanja: A1. Osmišljavanje operacije (Clarke i sur., 2008; Rittle-Johnson i Schneider, 2015), A2. Stvaranje kognitivnoga konflikta i A3. Poticanje učenikova rasuđivanja i razmišljanja (Neumann, 2014). Te se kategorije sastoje od potkategorija. Potkategorije se sastoje od kodova, a njihov je sadržaj usko povezan s matematikom nastavnih slučajeva, uključujući uspostavljanje poveznica između a) množenja cijelih brojeva i množenja razlomaka, b) ponovljenoga zbrajanja i množenja te c) dijeljenja cijelih brojeva i dijeljenja razlomaka.

Primjerice, potkategorija „uspostavljanje poveznica između množenja cijelih brojeva

i množenja razlomaka" oblikovana je induktivno unutar šire kategorije „osmišljavanje operacije". Ova se potkategorija odnosi na odgovore u kojima su sudionici objašnjavali množenje razlomaka povezujući ga s množenjem cijelih brojeva. Njezino uključivanje odražava ulogu analogijskoga rasuđivanja u podržavanju konceptualnoga razumijevanja (English, 2004). Slijedi primjer izvatka iz ove potkategorije: „Postavio/postavila bih jednostavna pitanja. Primjerice, pitao/pitala bih što znači 2×2. Vjerojatno bi rekli da to znači zbrojiti dvije dvojke. Na isti način, ovdje bi pomislili da trebaju zbrojiti dvije $\frac{1}{2}$". Ovaj je odgovor dodijeljen toj specifičnoj potkategoriji jer je sudionik izričito povezao množenje razlomaka s množenjem cijelih brojeva.

Nakon početnih čitanja i kodiranja podataka, istraživači su uočili da su odgovori sudionika uključivali primjere stvaranja kognitivnoga konflikta u učenikovu razmišljanju (induktivno kodiranje). Neki su sudionici konceptualno preusmjerili matematički problem predstavljen u nastavnom slučaju. Dok je u nekim slučajevima cijeli odgovor bio kodiran jednom kategorijom, u drugim je slučajevima samo jedna rečenica ili riječ bila kodirana unutar određene kategorije. Dva istraživača u području obrazovanja matematike s više od 15 godina iskustva u osmišljavanju i kodiranju kvalitativnih istraživačkih tradicija, razvila su kodni okvir i kodirala podatke. Cijeli skup podataka analizirali su pojedinačno te su usporedili svoje analize kako bi uskladili razlike putem pregovora o neslaganjima u kodiranju i poboljšali kvalitetu kodiranja (Campbell i sur., 2013). Iako se nisu slagali oko nekih kategorija kodiranja, nakon više krugova rasprava i ponovnoga kodiranja konačno su postigli prihvatljivu razinu slaganja veću od 80 % (Neuendorf, 2016). Nakon daljnjih rasprava postigli su suglasnost o svim analiziranim odgovorima.

Rezultati o načinu na koji budući učitelji reagiraju na učenikove pogreške u množenju razlomaka

U vezi s MQ1, odgovori sudionika pokazuju konceptualna obilježja i popratne dokaze (Tablica 1). Dok 66 % odgovora budućih učitelja sadrži „dokaze konceptualnoga razumijevanja", samo 12 % sadrži „dokaze proceduralnoga razumijevanja". Pet posto odgovora istodobno je pokazalo konceptualne i proceduralne dokaze, a 16 % odgovora nije sadržavalo dokaze ni za jednu vrstu razumijevanja. Podatci pokazuju da je 71 % (66 % + 5 %) odgovora konceptualno usmjereno.

Odgovori su često odražavali „dokaze konceptualnoga razumijevanja", što sugerira da su se sudionici usredotočili na konceptualne pristupe pri reagiranju na učenikove pogreške. Njihovi odgovori ukazuju na to da bi učeniku pomogli oslanjajući se na konceptualne perspektive, najčešće unaprjeđivanjem razumijevanja same operacije. Nasuprot tome, strategije koje zahtijevaju aktivnije uključivanje u učenikovo razmišljanje, poput stvaranja kognitivnoga konflikta i poticanja učenikova rasuđivanja i razmišljanja, pojavljivale su se rjeđe. Ovaj obrazac sugerira da, iako budući učitelji pokazuju sklonost konceptualnom razumijevanju, njihovi su pristupi relativno ograničenoga opsega te naglašavaju objašnjavanje više nego dublje dijagnostičko ili dijaloško uključivanje u učenikove ideje.

Najčešći kod bio je povezivanje ponovljenoga zbrajanja i množenja, što pokazuje da su se sudionici često oslanjali na povezivanje novih tema s poznatim operacijama s cijelim brojevima. Primjerice, jedan je sudionik rekao: „Pokazao/pokazala bih da je $2 \times \frac{1}{2}$ (dva puta jedna polovina) ekvivalentno zbrajanju $\frac{1}{2}$ i $\frac{1}{2}$ te stoga $2 \times \frac{1}{2} = \frac{1}{2} + \frac{1}{2}$" Nadalje, za modeliranje razlomaka korišteni su modeli poput oblika i presavijanja papira. Ovaj postupak ukazuje na nastojanje da se apstraktna matematička ideja učini opipljivijom i razumljivijom učenicima. Jedan je sudionik rekao: „Pomogao/pomogla bih Ayşe pomoću konkretnih primjera i zamolio/zamolila je da razmisli o pitanju: 'Pretpostavimo da imamo dvije polovice jabuke, koliko bi jabuka činile te dvije polovice?'" Navedene strategije sugeriraju da su se sudionici oslanjali na povezivanje operacija s razlomcima s poznatim aritmetičkim strukturama te na podržavanje razumijevanja putem konkretnih prikaza.

Samo se nekoliko sudionika osvrnulo na stvaranje kognitivnoga konflikta, što sugerira da su pokušali izazvati postojeće učenikove predodžbe. Bili su skloni usredotočiti se na razlikovanje zbrajanja razlomaka i množenja razlomaka kako bi naglasili konceptualne razlike. Slično tome, unutar kategorije „poticanje učenikova rasuđivanja i razmišljanja", sudionici su često predlagali poticanje učenika da objasne ili promisle o svojim odgovorima. Ovi odgovori pokazuju da su neki sudionici obraćali pozornost na učenikovo razmišljanje, međutim, samo je mali broj sudionika koristio ovu strategiju.

Odgovori na MQ2 pokazali su obrazac sličan onome zabilježenom za MQ1. Većina odgovora odražavala je dokaze konceptualnoga razumijevanja, što pokazuje da su budući učitelji nastavili koristiti konceptualne pristupe pri predlaganju pitanja za učenika. S obzirom na ovu sličnost u raspodjeli, detaljna analiza potkategorija nije ponovljena za MQ2.

Tablica 1

Rezultati o načinu na koji budući učitelji reagiraju na učenikove pogreške u dijeljenju razlomaka

U vezi s DQ1, Tablica 1 pokazuje da 74 % odgovora sudionika sadrži „dokaze konceptualnoga razumijevanja" pri reagiranju na učenikove netočne odgovore. Kada se tema A (dokazi konceptualnoga razumijevanja) i tema C (dokazi i konceptualnoga i proceduralnoga razumijevanja) razmotre zajedno, nalazi pokazuju da 85 % (74 % + 11 %) odgovora sadrži „dokaze konceptualnoga razumijevanja". Stoga je očito da je način na koji većina sudionika pristupa učenikovim pogreškama sadrži konceptualne dokaze.

Odgovori unutar ove teme uglavnom su bili usmjereni na osmišljavanje operacije. Budući učitelji najčešće su predlagali uspostavljanje poveznica između dijeljenja razlomaka i poznatih operacija s cijelim brojevima, osobito putem ponovljenoga oduzimanja. To ukazuje na sklonost oblikovanju objašnjenja temeljenih na prethodnom znanju, koristeći poznate aritmetičke postupke kako bi operacije s razlomcima bile razumljivije učenicima. Primjerice, Sudionik #1 uspostavio je poveznicu između dijeljenja cijelih brojeva i dijeljenja razlomaka rekavši: „Za $12 \div 2$ odgovarate na pitanje koliko puta dva

stane u dvanaest. Dakle, $\frac{1}{2} \div \frac{1}{4}$. znači pronaći broj četvrtina u jednoj polovini." Tako je sudionik povezao ponovljeno oduzimanje i dijeljenje razlomaka.

Osim toga, mnogi su sudionici koristili prikaze poput oblika ili presavijanja papira kako bi modelirali odnose među razlomcima. Ovi odgovori ukazuju na nastojanje da se apstraktne operacije učine pristupačnijima te da se podrži učenikova konceptualizacija razlomaka. Primjerice, Sudionica #28 napisala je: „Koristila bih konkretne materijale kako bi mogli vizualno vidjeti i razumjeti operacije." Također je modelirala jednu polovinu i jednu četvrtinu na dva odvojena kruga. Ova sudionica namjerava modelirati razlomke pomoću materijala kako bi unaprijedila učenikovo konceptualno razumijevanje dijeljenja razlomaka.

U nalazima vezanima uz DQ2 uočeno je da odgovori budućih učitelja sadrže konceptualne dokaze pri reagiranju na pogreške učenika u slučajevima dijeljenja razlomaka, što pokazuje da su budući učitelji koristili konceptualne pristupe pri predlaganju pitanja za učenika. Konkretno, više od polovice sudionika koristilo je strategiju „poticanja učenikova rasuđivanja i razmišljanja" pri oblikovanju pitanja vezanih uz učenikove pogreške. Primjerice, Sudionik #36 napisao je: „Pitao/pitala bih: 'Kako si dobio/dobila 16?' i 'Bi li mi mogao/mogla pokazati modeliranjem svoj odgovor?' Tako bih mogao/mogla istražiti gdje je učenik pogriješio i koja se pogrešna predodžba nalazi iza toga." Pitanja su postavljena kako bi potaknula učenikovo razmišljanje. Sudionik je želio razumjeti korijene učenikovih pogrešaka. Samo je nekoliko odgovora bilo usmjereno prema proceduralnoj perspektivi, stoga su ti odgovori bili formulirani kako bi pomogli učenicima usredotočiti se na proceduralno izvođenje operacija.

Diskusija

Cilj ovoga istraživanja bio je istražiti u kojoj su se mjeri budući učitelji oslanjali na teorijske koncepte predstavljene tijekom kolegija kada su odgovarali na pogreške učenika viših razreda osnovne škole na temelju učenikova razmišljanja. Osim toga, osmislili smo jedinstveno okružje kako bismo povećali vjerojatnost premošćivanja jaza između teorije i prakse u poučavanju matematike korištenjem nastavnih slučajeva u kojima je sveučilišni nastavnik bio nastavnik u slučaju. Konačno, raspravljamo o našim nalazima usredotočujući se na pogreške koje se često povezuju s učenjem operacija s razlomcima u svjetlu ovoga istraživanja, povezanost teorije i prakse, obraćanje pozornosti na učenikove pogreške te nastavnika s dvostrukim ulogama u učenju razlomaka.

U ovome istraživanju očekivali smo da će sudionici koristiti svoje teorijsko znanje kako bi opisali i interpretirali pogreške učenika viših razreda osnovne škole. Analizirali smo odgovore sudionika kako bismo vidjeli povezuju li ono što su naučili tijekom sveučilišnoga kolegija sa stvarnim nastavnim događajima. Dubinska analiza pokazala je da su naša očekivanja ispunjena. U pisanim odgovorima sudionika pronađeno je mnoštvo dokaza koji pokazuju obilježja konceptualnoga razumijevanja. Nalazi jasno pokazuju da su mnogi odgovori sudionika sadržavali upućivanja na osmišljavanje dijeljenja ili množenja razlomaka. Kako bi konceptualno pristupili učenikovim

pogreškama, odgovori sudionika bili su usmjereni na pomaganje učenicima da razumiju značenje operacija. Točnije, predlagali su nastavne postupke koji bi učenicima pomogli uspostaviti poveznice među matematičkim operacijama i modelirati razlomke kako bi se pogreške mogle prepoznati i ispraviti. S obzirom na to da se često navodi kako budući učitelji i sami imaju poteškoće s razlomcima (Jansen i sur., 2017; Son i Lee, 2016), naši su nalazi pokazali da je pedagogija temeljena na slučajevima, kako je primijenjena u ovome istraživanju, pružila budućim učiteljima prilike za konceptualni razvoj vlastitoga razumijevanja te posljedično konceptualno bavljenje učenikovim razmišljanjem i pogreškama.

Nastavnik je namjerno koristio izvore utemeljene na istraživanjima kako bi povezao teoriju i praksu, oslanjajući se na literaturu o konceptualnom razumijevanju (Hiebert i Carpenter, 1992), višestrukim prikazima (Ainsworth, 1999), uočavanju (Stockero i sur., 2017) i konceptualnom postupanju s pogreškama (Tulis, 2013). Ovaj se pristup odrazio u odgovorima budućih učitelja, koji su uključivali uspostavljanje poveznica između ponovljenoga zbrajanja i množenja, korištenje modela (npr. oblika, rezanja papira) te poticanje učenikova rasuđivanja putem pitanja. Ovi obrasci sugeriraju da su se i nastavnik i sudionici oslanjali na praksu informiranu istraživanjima (Dreyfus i sur., 2021) u radu s učenikovim pogreškama.

Iako ranija istraživanja pokazuju da učitelji i budući učitelji često pristupaju pogreškama proceduralno, a ne konceptualno (npr. Son i Sinclair, 2010; Tulis, 2013; Santagata, 2005), naši nalazi pokazuju da su sudionici pretežno usvajali konceptualne pristupe. Njihovi odgovori uključivali su uspostavljanje poveznica, korištenje modela i obraćanje pozornosti na učenikovo rasuđivanje, što ukazuje na nastojanje da se interpretiraju i adresiraju temeljni uzroci pogrešaka.

Pokazalo se da pedagogija temeljena na slučajevima povezuje teoriju i praksu te podržava refleksiju o ključnim nastavnim situacijama (Gunn i sur., 2018; Ulvik i sur., 2020). U ovome su istraživanju sudionici analizirali autentične nastavne slučajeve koje je oblikovao njihov nastavnik, što im je omogućilo uključivanje u autentične primjere učenikova razmišljanja. Nalazi sugeriraju da je ovaj pristup podržao sudionike u uočavanju ključnih aspekata učenikovih pogrešaka i odgovaranju na konceptualno usmjerene načine. Korištenje modela razlomaka među budućim učiteljima, uspostavljanje poveznica između zbrajanja i množenja te postavljanje pitanja koja potiču rasuđivanje pokazatelji su pedagogije temeljene na slučajevima.

Zaključak, ograničenja i implikacije

Naše je istraživanje pokazalo da je primijenjena pedagogija temeljena na slučajevima pomogla budućim učiteljima da konceptualno pristupe učenikovim pogreškama, što znači da je nastavnik uspješno koristio istraživanja iz područja obrazovanja matematike kao resurs (Dreyfus i sur., 2021). Međutim, kao i u prethodnim istraživanjima (Lin, 2017), ovaj se obrazac ne može pripisati isključivo nastavnoj metodi, budući da samo rasuđivanje o razlomcima može potaknuti i konceptualne i proceduralne odgovore

(Siegler i Lortie-Forgues, 2015). Stoga ostaje nejasno odražavaju li ovi ishodi pedagogiju ili prirodu matematičke teme te su potrebna buduća istraživanja kako bi se ovo pitanje dodatno razjasnilo.

Analitički okvir razvijen u ovome istraživanju predstavlja metodološki doprinos i može poslužiti kao alat za analizu reakcija učitelja na učenikove pogreške. Potrebna su daljnja istraživanja kako bi se ispitalo koliko je ovaj okvir operativan. Izostanak mjerenja razvoja sudionika prije i nakon intervencije predstavlja ograničenje našega istraživanja te su jasno potrebna daljnja istraživanja kako bi se ispitao učinak intervencijskih studija vezanih uz praksu postupanja s pogreškama. Također bi trebalo razmotriti istraživanje načina na koji budući učitelji reagiraju na slične učenikove pogreške u stvarnim učioničkim okružjima.

Napomena

Autori nisu primili financijsku potporu za istraživanje, autorstvo i/ili objavu ovoga članka. Ne postoje financijski ili osobni odnosi s drugim osobama ili organizacijama koji bi mogli neprimjereno utjecati na ovaj rad.

Izjava o sukobu interesa

Ne postoji stvarni ni prividni sukob interesa koji bi mogao imati izravan utjecaj na predmet članka.

Etička razmatranja

Ovo istraživanje uključuje ljudske sudionike. Etičko odobrenje dalo je Etičko povjerenstvo za znanost i inženjerstvo Sveučilišta Kocaeli (Odobrenje: 917846).