Izračunavanje vrijednosti $\operatorname{tg}\frac{5\pi}{24}$

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U ovom članku prikazat ćemo deset različitih načina određivanja vrijednosti $\operatorname{tg}\frac{5\pi}{24}$, tj. $\operatorname{tg} 37°30'$, vodeći se poznatom činjenicom da je bolje jedan matematički zadatak riješiti na dva ili više načina nego riješiti desetine zadataka na jedan te isti način. Pri tome koristit ćemo razne tvrdnje iz algebre, geometrije i trigonometrije.

Zadatak. Pokažimo da je

$$\operatorname{tg}\frac{5\pi}{24} = \sqrt{6}+\sqrt{5}-\sqrt{2}-2.$$

***Rješenje* 1.** Korištenjem formula za tangens zbroja

$$\operatorname{tg}(\alpha+\beta) = \frac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{1-\operatorname{tg}\alpha\operatorname{tg}\beta} \qquad \text{i} \qquad \frac{5\pi}{12} = \frac{\pi}{4}+\frac{\pi}{6}$$

dobijemo

$$\operatorname{tg}\frac{5\pi}{12} = \frac{\operatorname{tg}\frac{\pi}{4}+\operatorname{tg}\frac{\pi}{6}}{1-\operatorname{tg}\frac{\pi}{4}\operatorname{tg}\frac{\pi}{6}} = \frac{1+\frac{\sqrt{3}}{3}}{1-\frac{\sqrt{3}}{3}} = 2+\sqrt{3}.$$

Nadalje, korištenjem formule $\cos\alpha = \dfrac{1}{\sqrt{1+\operatorname{tg}^2\alpha}}$, imamo

$$\cos\frac{5\pi}{12} = \frac{1}{\sqrt{1+(2+\sqrt{3})^2}} = \frac{1}{2\sqrt{2+\sqrt{3}}}.$$

Sada iz formule $\operatorname{tg}\frac{\alpha}{2} = \sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}$ redom dobivamo

$$\operatorname{tg}\frac{5\pi}{24} = \sqrt{\frac{1-\frac{1}{2\sqrt{2+\sqrt{3}}}}{1+\frac{1}{2\sqrt{2+\sqrt{3}}}}} = \sqrt{\frac{2\sqrt{2+\sqrt{3}}-1}{2\sqrt{2+\sqrt{3}}+1}} = \sqrt{\frac{(2\sqrt{2+\sqrt{3}}-1)^2}{4(2+\sqrt{3})-1}}$$

$$= \frac{2\sqrt{2+\sqrt{3}}-1}{\sqrt{7+4\sqrt{3}}} = \frac{\sqrt{2}\sqrt{(\sqrt{3}+1)^2}-1}{2+\sqrt{3}} = \frac{\sqrt{2}(\sqrt{3}+1)-1}{2+\sqrt{3}}$$

$$= \frac{\sqrt{6}+\sqrt{2}-1}{2+\sqrt{3}}\cdot\frac{2-\sqrt{3}}{2-\sqrt{3}} = \sqrt{6}+\sqrt{3}-\sqrt{2}-2.$$

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Rješenje 2. Korištenjem formule za tangens razlike

$$\operatorname{tg}(\alpha-\beta)=\frac{\operatorname{tg}\alpha-\operatorname{tg}\beta}{1+\operatorname{tg}\alpha\operatorname{tg}\beta} \qquad \text{i} \qquad \frac{\pi}{4}-\frac{\pi}{24}=\frac{5\pi}{24},$$

dobivamo

$$\operatorname{tg}\frac{5\pi}{24}=\operatorname{tg}\left(\frac{\pi}{4}-\frac{\pi}{24}\right)=\frac{\operatorname{tg}\frac{\pi}{4}-\operatorname{tg}\frac{\pi}{24}}{1+\operatorname{tg}\frac{\pi}{4}\operatorname{tg}\frac{\pi}{24}},$$

tj.

$$\operatorname{tg}\frac{5\pi}{24}=\frac{1-\operatorname{tg}\frac{\pi}{24}}{1+\operatorname{tg}\frac{\pi}{24}}. \tag{1}$$

Kako je

$$\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\cdot\frac{2\sin\frac{x}{2}}{2\sin\frac{x}{2}}=\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}},$$

odnosno

$$\operatorname{tg}\frac{x}{2}=\frac{1-\cos x}{\sin x},$$

imamo

$$\operatorname{tg}\frac{\pi}{24}=\frac{1-\cos\frac{\pi}{12}}{\sin\frac{\pi}{12}}. \tag{2}$$

Kako je

$$\cos\frac{\pi}{12}=\cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right)=\cos\frac{\pi}{4}\cos\frac{\pi}{6}+\sin\frac{\pi}{4}\sin\frac{\pi}{6}=\frac{1}{4}(\sqrt{6}+\sqrt{2}),$$
$$\sin\frac{\pi}{12}=\sin\left(\frac{\pi}{4}-\frac{\pi}{6}\right)=\sin\frac{\pi}{4}\cos\frac{\pi}{6}-\cos\frac{\pi}{4}\sin\frac{\pi}{6}=\frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right)$$

iz (2) slijedi

$$\operatorname{tg}\frac{\pi}{24}=\sqrt{6}-\sqrt{3}+\sqrt{2}-2. \tag{3}$$

Iz jednakosti (1) i (3) je

$$\operatorname{tg}\frac{5\pi}{24}=\sqrt{6}+\sqrt{3}-\sqrt{2}-2.$$

Zadatak 2. Koristeći formule za tanges razlike dva kuta,

$$\operatorname{tg}\frac{x}{2}=\frac{1-\cos x}{\sin x} \qquad \text{i} \qquad \frac{5\pi}{24}=\frac{\pi}{3}-\frac{\pi}{8},$$

izračunaj $\operatorname{tg}\dfrac{5\pi}{24}$.

Rješenje 3. Imamo redom:

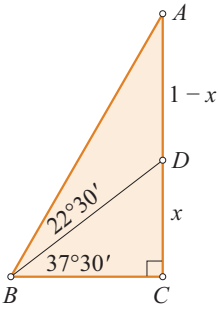
$$\operatorname{tg}\frac{5\pi}{24}=\frac{\sin\dfrac{5\pi}{24}}{\cos\dfrac{5\pi}{24}}=\frac{\cos\left(\dfrac{\pi}{2}-\dfrac{5\pi}{24}\right)}{\cos\dfrac{5\pi}{24}}=\frac{\cos\dfrac{7\pi}{24}}{\cos\dfrac{5\pi}{24}}.$$

Poslije proširivanja posljednjeg razlomka s $2\sin\dfrac{\pi}{24}$ i primjene formule $\sin\alpha-\sin\beta=2\sin\dfrac{\alpha-\beta}{2}\cos\dfrac{\alpha+\beta}{2}$ dobijemo

$$\operatorname{tg}\frac{5\pi}{24}=\frac{2\cos\dfrac{7\pi}{24}\sin\dfrac{\pi}{24}}{2\cos\dfrac{5\pi}{24}\sin\dfrac{\pi}{24}}=\frac{\sin\dfrac{\pi}{3}-\sin\dfrac{\pi}{4}}{\sin\dfrac{\pi}{4}-\sin\dfrac{\pi}{6}}=\frac{\dfrac{\sqrt{3}}{2}-\dfrac{\sqrt{2}}{2}}{\dfrac{\sqrt{2}}{2}-\dfrac{1}{2}},$$

tj. $\operatorname{tg}\dfrac{5\pi}{24}=\sqrt{6}+\sqrt{3}-\sqrt{2}-2.$

Rješenje 4. Promatrajmo pravokutni trokut ABC $(\sphericalangle BCA=90^\circ)$ kod kojeg je $\sphericalangle BAC=30^\circ$ i $|AC|=1$.



Slika 1.

Nasuprot kuta od 30° u pravokutnom trokutu je kateta koja je 2 puta kraća od hipotenuze, što znači da je $|BC|=\dfrac{1}{2}|AB|$. Primjenom Pitagorinog poučka na pravokutan trokut ABC imamo

$$|AB|^2=|BC|^2+|AC|^2=\left(\frac{1}{2}|AB|\right)^2+1^2,$$

odakle je $|AB|=\dfrac{2}{\sqrt{3}}$. Zbog toga je $|BC|=\dfrac{1}{\sqrt{3}}$.

Na kateti $\overline{AC}$ zadanog trokuta odredimo točku D tako da je $|\sphericalangle CBD|=37^\circ30'$, te $|\sphericalangle ABD|=22^\circ30'$. Ako s x označimo duljinu $|CD|$, odavde je $|AD|=1-x$. Primjenom poučka o sinusima na trokut ABD dobivamo $\dfrac{1-x}{\sin 22^\circ30'}=\dfrac{|BD|}{\sin 30^\circ}$. Odavde, $\sin 22^\circ30'$ zbog $\sin 22^\circ30'=\sqrt{\dfrac{1-\cos 45^\circ}{2}}=\dfrac{1}{2}\sqrt{2-\sqrt{2}}$ imamo

$$|BD|=\frac{1-x}{\sqrt{2-\sqrt{2}}}. \tag{4}$$

Korištenjem Pitagorinog poučka na pravokutni trokut BCD je $|BD|^2=|BC|^2+|CD|^2$, tj.

$$|BD|^2=\frac{1}{3}+x^2. \tag{5}$$

Iz jednakosti (4) i (5) slijedi

$$\left(\frac{1-x}{\sqrt{2-\sqrt{2}}}\right)^2 = \frac{1}{3} + x^2,$$

tj.

$$x^2 - 2(\sqrt{2}+1)x + \frac{1}{3}(\sqrt{2}+1)^2 = 0.$$

Rješavanjem ove kvadratne jednadžbe dobijemo:

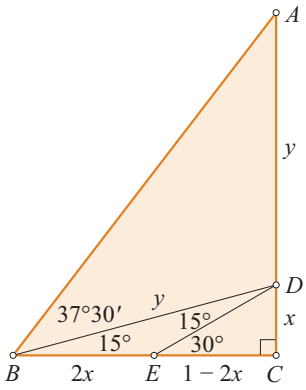
$$x = \frac{\sqrt{2}+1}{\sqrt{3}}(\sqrt{3}-\sqrt{2}),$$

$$\operatorname{tg}\frac{5\pi}{24} = \operatorname{tg} 37°30' = \frac{|CD|}{|BC|} = \frac{\frac{1}{\sqrt{3}}(\sqrt{2}+1)(\sqrt{3}-\sqrt{2})}{\frac{1}{\sqrt{3}}}, \qquad \text{tj.}$$

$$\operatorname{tg}\frac{5\pi}{24} = \sqrt{6}+\sqrt{3}-\sqrt{2}-2.$$

Rješenje 5. U trokutu ABC je $|BC| = 1$, $|\sphericalangle BAC| = 37°30'$ i $|\sphericalangle BCA| = 90°$.

Na stranici $\overline{AC}$ je točka D tako da $|\sphericalangle DBE| = 15°$. Odavde je $|\sphericalangle DBA| = 37°30'$. Trokut ABD je jednakokračni, što znači da je $|BD| = |AD| = y$. Na stranici $\overline{BC}$ je točka E tako da je $|\sphericalangle BDE| = 15°$. Trokut BED je jednakokračni, pa je $|BE| = |ED| = 2x$ i $|CE| = |BC| - |BE| = 1 - 2x$. Kako je u trokutu CDE, $|\sphericalangle DEC| = 30°$, to je $\frac{1}{2} = \sin 30° = |CD| : 2x$. Odavle slijedi $|CD| = x$. Primjenom Pitagorinog poučka na pravokutni trokut CDE dobivamo: $(2x)^2 = x^2 + (1-2x)^2$, odakle je ($x = 2 - \sqrt{3}$ jer mora biti $x < 1$). Trokut BDC je pravokutni, pa je $y^2 = 1^2 + (2-\sqrt{3})^2$, tj. $y = 2\sqrt{2-\sqrt{3}}$, tj. $y = \sqrt{6} - \sqrt{2}$. Sada je, iz trokuta ABC,



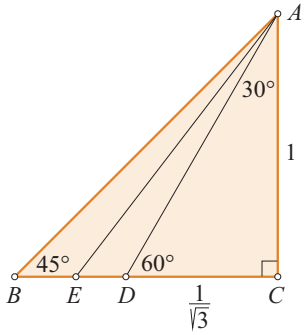
Slika 2.

$$\operatorname{tg}\frac{5\pi}{24} = \operatorname{tg} 37°30' = \frac{|BC|}{|AC|} = \frac{1}{x+y} = \frac{1}{2-\sqrt{3}+\sqrt{6}-\sqrt{2}}$$
$$= \frac{1}{(\sqrt{6}-\sqrt{2})+(2-\sqrt{3})} \cdot \frac{(\sqrt{6}-\sqrt{2})-(2-\sqrt{3})}{(\sqrt{6}-\sqrt{2})-(2-\sqrt{3})}$$
$$= \frac{\sqrt{6}-\sqrt{2}-2+\sqrt{3}}{(\sqrt{6}-\sqrt{2})^2-(2-\sqrt{3})^2},$$

a odavde

$$\operatorname{tg}\frac{5\pi}{24} = \sqrt{6}+\sqrt{3}-\sqrt{2}-2.$$

Rješenje 6. Konstruirajmo $|\sphericalangle CAD| = 30^\circ$ u jednakokračnom pravokutnom trokutu ABC ($|AB|$ je duljina hipotenuze), tako da točka D pripada kateti $\overline{BC}$.



Slika 3.

Neka je AE simetrala kuta $|\sphericalangle DAB| = 45^\circ - 30^\circ = 15^\circ$. Tada je $|\sphericalangle CAE| = 37^\circ 30' = \frac{5\pi}{24}$. Kako je $|AC| = |BC| = 1$, $\operatorname{tg}\frac{5\pi}{24} = |EC|$. Kako je $|AD| = 2|CD|$ i $|AC| = 1$, imamo $|CD|^2 = (2|CD|)^2 - |AC|^2$, odnosno $3|CD|^2 = 1$, odakle je $|CD| = \frac{1}{\sqrt{3}}$. Radi toga je $|AD| = \frac{2}{\sqrt{3}}$. Iz poučka o simetrali unutarnjeg kuta trokuta ABD je $|BE| : |ED| = |AB| : |AD|$, odakle zbog $|AB| = \sqrt{2}$ i $|BE| + |ED| = 1 - \frac{1}{\sqrt{3}}$, imamo

$$\left(1 - \frac{1}{\sqrt{3}} - |ED|\right) : |ED| = \sqrt{2} : \frac{2}{\sqrt{3}}, \quad \text{tj.} \quad |ED| = \frac{1}{3}(3\sqrt{6} + 2\sqrt{3} - 3\sqrt{2} - 6).$$

Sada je $|EC| = |ED| + |DC| = \sqrt{6} + \sqrt{3} - \sqrt{2} - 2$, to jest

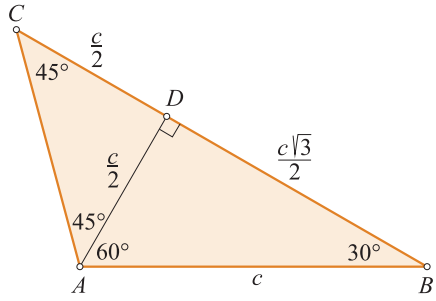
$$\operatorname{tg}\frac{5\pi}{24} = \sqrt{6} + \sqrt{3} - \sqrt{2} - 2.$$

Rješenje 7. Neka je u trokutu ABC $\alpha = 105^\circ$ i $\beta = 30^\circ$.

Na stranici $\overline{BC}$ odredimo točku D tako da bude $|\sphericalangle CAD| = 45^\circ$. Tada je trokut ADC jednakokračan i trokut ABD je polovina jednakostraničnog trokuta stranice duljine c. Stoga je $|AD| = |CD|$ i $|AD| = \frac{1}{2}|AB| = \frac{1}{2}c$. Primjenom Pitagorinog poučka na pravokutne trokute ABD i ADC, imamo:

$$b = |AC| = \frac{c\sqrt{2}}{2} \quad \text{i} \quad |BD| = \frac{c\sqrt{3}}{2}$$

pa je $a = |BC| = \frac{c}{2}(\sqrt{3} + 1)$.



Slika 4.

Iz Molvajdovih formula dobijemo:

$$\frac{a-b}{c} = \frac{\sin\frac{\alpha-\beta}{2}}{\cos\frac{\gamma}{2}} \quad \text{i} \quad \frac{a+b}{c} = \frac{\cos\frac{\alpha-\beta}{2}}{\sin\frac{\gamma}{2}}.$$

Dijeljenjem ovih jednakosti, zbog $\gamma = 180^\circ - (\alpha + \beta)$ i $\operatorname{ctg}(180^\circ - x) = \operatorname{tg} x$ imamo:

$$\frac{a-b}{a+b} = \frac{\operatorname{tg}\frac{\alpha-\beta}{2}}{\operatorname{ctg}\frac{\gamma}{2}} = \frac{\operatorname{tg}\frac{\alpha-\beta}{2}}{\operatorname{ctg}\left(90^\circ - \frac{\alpha+\beta}{2}\right)} = \frac{\operatorname{tg}\frac{\alpha-\beta}{2}}{\operatorname{tg}\frac{\alpha+\beta}{2}},$$

tj.

$$\operatorname{tg}\frac{\alpha-\beta}{2}=\frac{a-b}{a+b}\operatorname{tg}\frac{\alpha+\beta}{2}.$$

Kako je $a=\frac{c}{2}(\sqrt{3}+1)$, $b=\frac{c\sqrt{2}}{2}$, $\alpha=105^\circ$ i $\beta=30^\circ$, imamo

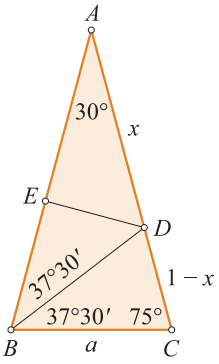
$$\operatorname{tg}\frac{105^\circ-30^\circ}{2}=\frac{\frac{c}{2}(\sqrt{3}+1)-\frac{c\sqrt{2}}{2}}{\frac{c}{2}(\sqrt{3}+1)+\frac{c\sqrt{2}}{2}}\operatorname{tg}\frac{105^\circ+30^\circ}{2}=\frac{\sqrt{3}-\sqrt{2}+1}{\sqrt{3}+\sqrt{2}+1}\operatorname{tg}\frac{135^\circ}{2}.$$

Odavde je zbog

$$\operatorname{tg}\frac{135^\circ}{2}=\sqrt{\frac{1-\cos 135^\circ}{1+\cos 135^\circ}}=\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}}}=\sqrt{2}+1,$$

$$\operatorname{tg}\frac{5\pi}{24}=\operatorname{tg}37^\circ30'=\frac{\sqrt{3}-\sqrt{2}+1}{\sqrt{3}+\sqrt{2}+1}(\sqrt{2}+1)=\frac{\sqrt{6}+\sqrt{3}-1}{\sqrt{3}+\sqrt{2}+1}$$
$$=\sqrt{6}+\sqrt{3}-\sqrt{2}-2.$$

Rješenje 8. Nacrtajmo jednakokračni trokut ABC kod kojeg je $|AB|=|AC|=1$ i $|\sphericalangle BAC|=30^\circ$.



Slika 5.

Konstruirajmo smetralu kuta $|\sphericalangle ABC|=75^\circ$, a njen presjek s AC neka je točka D. Uvedimo oznake $|BC|=a$ i $|AD|=x$. Na osnovu poučka o simetrali unutarnjeg kuta trokuta imamo $|AD|:|DC|=|AB|:|BC|$ ili $x:(1-x)=1:a$. Odavde dobivamo $x=\frac{1}{a+1}$. Nožište okomice iz točke D na stranicu $\overline{AB}$ obilježimo s E. U pravokutnom trokutu AED je $|AD|=x=\frac{1}{a+1}$ i $|DE|=\frac{1}{2}|AD|$, pa je $|DE|=\frac{1}{2(a+1)}$. Primjenom Pitagorinog poučka na taj trokut imamo:

$$|AE|^2=\left(\frac{1}{a+1}\right)^2-\left(\frac{1}{2(a+1)}\right)^2 \quad \text{ili} \quad |AE|=\frac{\sqrt{3}}{2(a+1)}.$$

Zbog toga je

$$|BE|=|AB|-|AE|=1-\frac{\sqrt{3}}{2(a+1)}=\frac{2a+2-\sqrt{3}}{2(a+1)}.$$

Na osnovu poučka o sinusima primijenjenog na trokut ABC dobivamo $\frac{a}{\sin 30^\circ}=\frac{1}{\sin 75^\circ}$.

Odavde, zbog $\sin 75^\circ=\sin(90^\circ-15^\circ)=\cos 15^\circ$ i $\cos 15^\circ=\sqrt{\frac{1+\cos 30^\circ}{2}}=$

$\frac{1}{2}\sqrt{2+\sqrt{3}} = \frac{1}{4}(\sqrt{6}+\sqrt{2})$ je

$$a = \frac{\sin 30^\circ}{\cos 15^\circ} = \frac{2}{\sqrt{6}+\sqrt{2}} = \frac{1}{2}(\sqrt{6}-\sqrt{2}).$$

U pravokutnom trokutu BDE je

$$\operatorname{tg} 37^\circ 30' = \operatorname{tg}\frac{5\pi}{24} = \frac{|DE|}{|BE|} = \frac{1}{2a+2-\sqrt{3}} = \frac{1}{\sqrt{6}-\sqrt{2}+2-\sqrt{3}},$$

a odavde je, zbog $(\sqrt{6}-\sqrt{2}+2-\sqrt{3})(\sqrt{6}+\sqrt{3}-\sqrt{2}-2) = 1$,

$$\operatorname{tg}\frac{5\pi}{24} = \sqrt{6}+\sqrt{3}-\sqrt{2}-2.$$

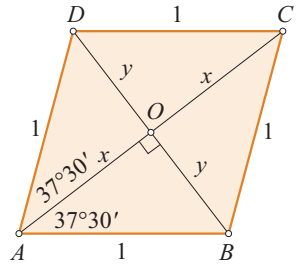
Rješenje 9. Neka je četverokut $ABCD$ romb stranice duljine 1 i šiljastog kuta 75°.

Obilježimo duljine njegovih dijagonala $\overline{AC}$ i $\overline{BD}$ redom s $2x$ i $2y$. Na osnovu kosinusovog poučka primijenjenog na jednakokračni trokut ABC imamo

$$(2x)^2 = 1^2 + 1^2 - 2\cdot 1\cdot 1\cdot \cos 105^\circ,$$

odakle je zbog

$$\begin{aligned}\cos 105^\circ &= \cos(90^\circ + 15^\circ) = -\sin 15^\circ \\ &= -\sqrt{\frac{1-\cos 30^\circ}{2}} = -\frac{1}{4}(\sqrt{6}-\sqrt{2}),\\ x^2 &= \frac{1}{8}(4+\sqrt{6}-\sqrt{2}).\end{aligned}$$



Slika 6.

Primjenom Pitagorinog poučka na pravokutni trokut ABO (O je točka presjeka dijagonala) dobijemo

$$y^2 = 1 - x^2 = \frac{1}{8}(4-\sqrt{6}+\sqrt{2}).$$

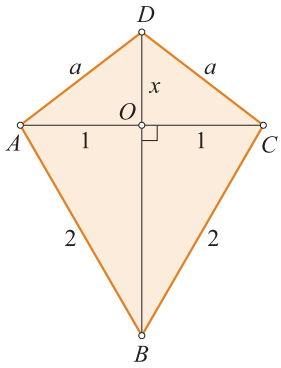
Kako je

$$\begin{aligned}\frac{y^2}{x^2} &= \frac{4-\sqrt{6}+\sqrt{2}}{4+\sqrt{6}-\sqrt{2}} = \frac{4-(\sqrt{6}-\sqrt{2})}{4+(\sqrt{6}-\sqrt{2})}\cdot\frac{4-(\sqrt{6}-\sqrt{2})}{4-(\sqrt{6}-\sqrt{2})} = \frac{6-\sqrt{3}-2\sqrt{6}+2\sqrt{2}}{2+\sqrt{3}}\\ &= \frac{6-\sqrt{3}-2\sqrt{6}+2\sqrt{2}}{2+\sqrt{3}}\cdot\frac{2-\sqrt{3}}{2-\sqrt{3}}\\ &= 15+10\sqrt{2}-8\sqrt{3}-6\sqrt{6} = (\sqrt{6}+\sqrt{3}-\sqrt{2}-2)^2.\end{aligned}$$

odakle je $\frac{y}{x} = \sqrt{6}+\sqrt{3}-\sqrt{2}-2$. Konačno,

$$\operatorname{tg} 37^\circ 30' = \operatorname{tg}\frac{5\pi}{24} = \frac{y}{x} = \sqrt{6}+\sqrt{3}-\sqrt{2}-2.$$

***Rješenje* 10.** Zadan je deltoid $ABCD$: $|AB| = |BC| = 2$, $|\sphericalangle ADC| = 75^\circ$ i $|\sphericalangle ABC| = 60^\circ$.



Slika 7.

Neka je $|AD| = |DC| = a$ i $|DO| = x$ (O je točka presjeka dijagonala). Kako je $|\sphericalangle ABO| = 30^\circ$, u pravokutnom trokutu ABO je $|AO| = \frac{1}{2}|AB| = 1$ pa je $|AC| = 2$. Iz kosinusovog poučka primijenjenog na jednakokračni trokut ACD imamo $2^2 = 2a^2 - 2a^2 \cos 75^\circ$, a odavde, zbog $\cos 75^\circ = \cos(90^\circ - 15^\circ) = \sin 15^\circ$ i $\sin 15^\circ = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ slijedi $a^2 = \dfrac{8}{4 - \sqrt{6} + \sqrt{2}}$. Primjenom Pitagorinog poučka na pravokutni trokut AOD imamo:

$$|OD|^2 = x^2 = \frac{8}{4 - \sqrt{6} + \sqrt{2}} - 1 = \frac{4 + \sqrt{6} - \sqrt{2}}{4 - \sqrt{6} + \sqrt{2}}.$$

Konačno je iz trokuta AOD

$$\operatorname{tg} 37^\circ 30' = \operatorname{tg} \frac{5\pi}{24} = \frac{1}{x} = \sqrt{\frac{4 - \sqrt{6} + \sqrt{2}}{4 + \sqrt{6} - \sqrt{2}}} = \sqrt{6} + \sqrt{3} - \sqrt{2} - 2.$$