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Preliminary Communication

COMPARISON OF TECHNICAL AND PRACTICAL ACTION RESEARCH IN IMPROVING MATHEMATICS TEACHING

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ABSTRACT

The aim of this paper is to compare individual practical and collaborative technical action research in mathematics teaching and analyse their contributions to changes in teaching practice and teachers' professional development within the Croatian context. The study was conducted as a qualitative analytical-comparative examination of two papers, using action research typologies (technical, practical, and emancipatory) and levels of participation (individual and collaborative dimensions) as the analytical framework. The results indicate that both models promote changes in teaching, albeit through different mechanisms: the individual practical model operates through the reflective transformation of professional identity and teaching approaches, while the collaborative technical model facilitates the improvement of practice, more systematic evaluation of student achievement, and strengthened professional cooperation. Key factors for success include reflexivity, strong theoretical grounding, collaboration, and institutional support. The paper classifies and compares the two models, providing a conceptual framework for understanding professional and pedagogical change in mathematics teaching.

Keywords: action research, collaborative learning, innovations in mathematics teaching, reflective practice, teacher professional development

INTRODUCTION

In contemporary educational systems, there is a growing need for teaching approaches that move beyond the transmission model of instruction and foster active, reflective, and problem-oriented learning. In this context, particular emphasis is placed on teachers' professional development, which should not rely solely on external forms of professional training, but rather on the systematic inquiry into and critical examination of one's own practice (Bognar, 2016). One approach that bridges theory and practice and enables immediate change in the classroom is action research. Through a cyclical process of planning, acting, observing, and reflecting, teachers become reflective practitioners who develop their practice on the basis of empirical insights and professional collaboration (Kemmis et al., 2014). In this sense, action research is not merely a methodological framework, but also a form of professional engagement aimed at improving the quality of teaching and learning.

Despite the growing interest in action research within Croatian educational literature, empirical studies that systematically compare different models of its implementation in mathematics teaching remain limited. In particular, there is a lack of analytical-comparative research that, drawing on theoretical typologies of action research, examines their specific professional and pedagogical effects within the Croatian context.

Building on this premise, the paper presents a theoretically grounded comparative analysis of two previously published action research studies conducted in Croatian primary schools. The analysis focuses on the typological classification of the action research models, the level of reflexivity and collaboration involved, and the scope of changes achieved in teaching practice. In doing so, the paper contributes to a more systematic understanding of the potential of action research within the Croatian educational system and offers a conceptual framework for future applications and research.

The original contribution of this paper is primarily conceptual and analytical: it offers a theoretically grounded typological framework for comparing distinct models of action research in mathematics education, which is largely absent from both the Croatian and the broader international literature. Although the paper does not present new empirical data, its analytical synthesis — grounded in established theoretical typologies and applied to documented cases — constitutes a valuable scholarly contribution. The findings are intended to serve as a conceptual foundation for future empirical research, including systematic or

scoping reviews that would enable broader comparative analysis across a larger number of cases.

Accordingly, the study strives to address the following research questions:

1. What types of action research are represented in the analysed examples?
2. To what extent do the analysed studies embody the core characteristics of action research (cyclicality, reflexivity, collaboration, and change in practice)?
3. What similarities and differences can be identified in their professional and instructional outcomes?
4. Which dimensions of change in mathematics teaching emerge from the analysed action research models?

Theoretical Framework – Action Research as a Tool for Professional Development

Action research is a research approach in which participants seek to improve their own practice through systematic and critical reflection, striving for greater rationality, sustainability, and fairness (Kemmis et al., 2014). Unlike traditional empirical research, which often remains at the level of descriptive or explanatory models, action research is oriented toward transforming concrete practice within real-life contexts. It therefore integrates both research and transformative dimensions.

The contemporary understanding of action research is grounded in Lewin's (1946) concept of a spiral cycle comprising planning, acting, observing, and reflecting. This cyclical structure enables the continuous adaptation of interventions based on systematically collected and analysed data, with reflection serving as the central mechanism of professional learning. A key characteristic of action research is its flexibility: the research design evolves in response to the needs and circumstances of practice rather than adhering to a rigid, predetermined structure (Mužić, 2004).

In the educational context, action research is particularly significant as a form of professional development. It positions teachers as reflective practitioners who, through the investigation of their own practice, develop professional autonomy and the capacity to make pedagogical decisions grounded in critical reflection of their practice and its outcomes (Bognar, 2006; 2016). In this way, the gap between theoretical models and everyday teaching practice is narrowed,

and teachers become active agents in the development of their professional competences.

Different typologies of action research enable a more precise understanding of its aims and scope. According to Carr and Kemmis (1986), three types can be distinguished: technical, practical, and emancipatory (critical) action research. The technical model focuses on improving the effectiveness of predefined goals and often involves the implementation of theoretically grounded interventions whose effects are empirically evaluated. The practical model emphasises reflective understanding and the critical examination of the goals and values that guide practice, whereas emancipatory action research seeks to transform the broader institutional and social context of education, highlighting the autonomous role of practitioners in all phases of the research process.

An additional level of differentiation is offered by Torbert's (2005) classification into individual (first-person), collaborative (second-person), and organisational (third-person) action research. The individual model focuses on the professional practice of a single teacher (Coghlan & Brannick, 2005; McNiff, 2016), whereas the collaborative model involves collective reflection and the joint planning of change within a professional community. The organisational model extends beyond the level of the individual classroom to encompass broader institutional processes. A specific form is collaborative action research, which is grounded in partnership between researchers and practitioners, as well as in the co-construction of knowledge (Somekh, 2006).

In mathematics teaching, action research has been recognised as a framework that facilitates the integration of theoretically grounded didactic models with concrete classroom situations. Research indicates that the systematic implementation of reflective cycles and the empirical monitoring of student progress can enhance students' understanding of mathematical concepts, foster the development of higher-order cognitive processes, and support teachers' professional growth (Dolapcioglu & Doğanay, 2020; Asmira et al., 2021; Bohara, 2024).

Meta-analytical evidence further confirms that reflective practice and collaborative professional learning constitute key determinants of successful action research in mathematics education (Özturan Ecemiş et al., 2024).

In this paper, the concept of substantive change in teaching refers to transformations that extend beyond isolated methodological adjustments and involve the reconfiguration of the roles of teachers and students, the organisation of learning processes, and the understanding of mathematics as an active, problem-oriented, and contextually situated domain of knowledge. In this

regard, action research is conceptualised as a framework capable of generating and sustaining such changes through iterative cycles of action and reflection.

For the purposes of this analysis, substantive change is operationalised through the following observable indicators: (1) a measurable shift in the distribution of classroom activity from teacher-led exposition towards student-centred engagement (as evidenced, for example, by COPUS protocol data); (2) a qualitative transformation in the types of tasks employed, from reproductive and procedural tasks towards exploratory, open-ended, and problem-oriented activities; (3) changes in the teacher's pedagogical role, from knowledge transmitter to facilitator of inquiry; and (4) qualitative shifts in teacher reflective discourse, moving from technical problem-solving towards critical examination of pedagogical values and assumptions.

Although action research has been theoretically examined in Croatian educational literature (Bognar, 2006; 2016), empirical comparative analyses of different models of its implementation in mathematics teaching remain limited. In particular, there is a lack of studies that compare different types of action research and examine their professional and instructional effects.

The theoretical framework outlined above therefore provides the foundation for a comparative analysis of two action research studies conducted in Croatian primary schools. The analysis focuses on their typological classification, level of reflexivity, degree of collaborative engagement, and the scope of the changes generated in teaching practice.

It should be noted, however, that action research is not without its limitations. Scholars have pointed to challenges related to the subjective nature of self-reflection, which may introduce researcher bias and compromise the objectivity of the findings (Carr & Kemmis, 1986). Furthermore, the context-specific and small-scale character of most action research studies limits the generalisability of their results beyond the immediate research setting. Methodological rigour may also be difficult to maintain consistently across all phases of the cyclical process, particularly in school contexts where time and institutional support are constrained (Kemmis et al., 2014).

Acknowledging these limitations does not diminish the value of action research; rather, it points to the importance of methodological transparency and the use of multiple data sources to strengthen the credibility of findings.

Methodological Framework for the Comparative Analysis

The paper presents a qualitative analytical-comparative study of two published action research projects in mathematics teaching. A secondary interpretative analysis was carried out, focusing on the theoretical classification and comparison of the models. The criteria used as the analytical matrix were derived from the theoretical framework of action research (Carr & Kemmis, 1986; Torbert, 2005):

- type of action research (technical, practical, emancipatory),
- level of participation (individual, collaborative, organisational),
- cyclicality and reflexivity of the process,
- scope of change in teaching practice,
- teachers' professional development,
- effects on students.

Throughout this paper, the two studies are characterised along two complementary dimensions: the typological dimension (technical vs. practical action research, following Carr & Kemmis, 1986) and the participatory dimension (individual vs. collaborative action research, following Torbert, 2005). These two dimensions are analytically distinct but interrelated: the first concerns the epistemological orientation and goals of the research, while the second concerns the social structure of its implementation. Both dimensions are therefore used consistently in the comparative analysis.

The two studies were selected on the basis of the following criteria: (1) both were conducted in Croatian primary school settings, thus sharing the same institutional and curricular context; (2) they represent two typologically distinct categories of action research — practical-individual and technical-collaborative — enabling a theoretically meaningful contrast; and (3) both are sufficiently documented in peer-reviewed publications to allow for systematic secondary analysis. While these two cases cannot be considered statistically representative of all action research conducted in Croatian mathematics education, they are theoretically purposeful: their contrasting characteristics are precisely what makes them analytically valuable for typological comparison (Flyvbjerg, 2006).

The data were analysed through an interpretative comparison to identify similarities and differences in the conceptual, organisational, methodological, and pedagogical characteristics of the two studies. The objective was not to evaluate the quality of the research, but to provide a theoretical typology and assess the extent of changes in practice, thereby contributing to a deeper understanding of

the application and potential of different models of action research within the Croatian context. The analysis was conducted in three successive stages. In the first stage, each study was independently reviewed and its key characteristics were extracted according to the six analytical criteria listed above. In the second stage, the extracted characteristics were organised into a comparative matrix to enable systematic juxtaposition of the two cases across all analytical dimensions. In the third stage, an interpretative synthesis was carried out to identify patterns, similarities, and divergences, drawing on the theoretical frameworks of Carr and Kemmis (1986) and Torbert (2005) as interpretative lenses. No formal coding scheme was applied; instead, the analysis relied on theoretically guided close reading and structured comparison, which is consistent with qualitative secondary analysis in educational research (Heaton, 2004).

Case 1: Individual Practical Action Research

As an example of practical, individual action research grounded in systematic reflection on changes implemented in one's own teaching, two studies published by Mužar Horvat may be highlighted. The primary aim of both research projects was to design and implement substantive changes in early mathematics education.

The first action research study (Mužar Horvat, 2023) was conducted during the second semester of the 2018/2019 school year in a third-grade class, whereas the second study (Mužar Horvat & Bognar, 2024) was carried out during the 2021/2022 school year in a combined second- and third-grade primary school classroom.

The specificity of the first action research study lies in the fact that the teacher introduced changes aimed at improving feedback to students, drawing on a conventional review of relevant literature. In contrast, the second action research study began with a systematic literature review conducted according to predefined criteria for the selection and analysis of sources, with the aim of identifying the key characteristics of effective early mathematics teaching (Mužar Horvat, 2024).

In addition to the features identified through this review, the study was grounded in the principles of the Montessori model and the Singapore model of mathematics instruction (Jukić Matić et al., 2024), which were subsequently implemented into classroom practice in a systematic and planned manner.

The methodological approach included the systematic video documentation and analysis of classroom activities using the COPUS (Classroom Observation

Protocol for Undergraduate STEM) protocol (Smith et al., 2013), the maintenance of a research reflective journal, the analysis of students' work and evaluation sheets, and the conduct of interviews with students.

The quality and credibility of the reflective process were further strengthened through structured feedback (Hemmeter et al., 2011) provided by critical friends with experience in teaching and conducting action research. Their advice, constructive comments, and reflective insights contributed to the planning, implementation, and evaluation of the change process.

The lessons were structured according to the stages proposed by Bognar and Matijević (2005): agreement, implementation, and evaluation. At the beginning of each lesson, instructional objectives were defined collaboratively with the students, and activities aimed at assessing their prior knowledge were conducted.

During the implementation stage, emphasis was placed on problem-solving activities, play-based learning, collaborative learning, and the use of concrete materials and technology. In the evaluation stage, particular attention was devoted to feedback and (self-)assessment.

Throughout each lesson, multiple activities reflecting different characteristics of effective mathematics teaching were incorporated, with at least three such characteristics typically represented within a single session.

An analysis of classroom video recordings collected during the first and second action research studies, conducted using the COPUS protocol (Figure 1), revealed a shift from predominantly traditional instruction - characterised by teacher-dominated activity - towards a more student-centred approach.

In the initial recordings, students spent 27% of the time listening to teacher exposition. During the action research process, this proportion decreased to 17% in the second grade and 19% in the third grade. Cognitive engagement accounted for 18% in the initial recordings; over the course of the study, it increased to 26% in the second grade, while reaching 13% in the third grade.

Independent activities constituted 26% of classroom time in the second grade and 40% in the third grade. When combined with students' responses, overall student activity reached 68% in the second grade and 64% in the third grade. Throughout the action research process, students posed more questions, with the teacher playing a key role in fostering this form of participation.

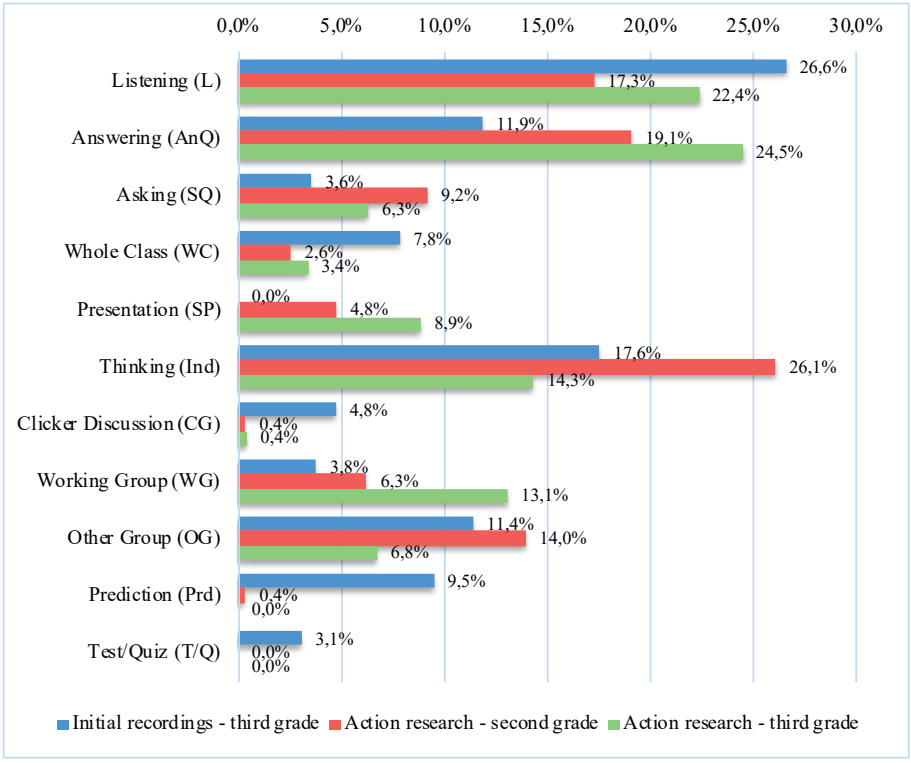


Figure 1. Analysis of student activities in the initial recordings and during the action research (Mužar Horvat & Bognar, 2024, p. 55)

In summary, students were given opportunities for active learning by linking a problem-based approach with real-life examples, as well as using collaborative learning, play-based activities, concrete materials, and technology. Questionnaires administered at the beginning and end of the study indicated increased student satisfaction with the instructional process. In the final interviews, students reported that, as a result of the changes, mathematics had become more interesting to them. The findings also indicate increased student self-confidence and a deeper understanding of abstract mathematical concepts.

It is important to emphasise that the teacher fully assumed the role of an action researcher, autonomously initiating and systematically investigating changes in her own practice based on informed action (McNiff, 2013). This informed action was evident in the deliberate planning and implementation of changes grounded in a previously conducted systematic review of the literature. The transformation of professional identity was a shift from the role

of a practitioner who implements externally defined curricula towards that of a reflective practitioner-researcher who autonomously identifies problems in her own practice, formulates research questions, generates evidence-based insights, and revises her pedagogical approach accordingly (McNiff, 2016). Concretely, this transformation was evident in the teacher's growing capacity for systematic self-reflection, her use of structured observation tools (COPUS), and her engagement with critical friends as a mechanism for external validation of her interpretations.

The teacher also independently collected and analysed data, and considered feedback from critical friends, which enabled her to critically examine her own practice. The changes affected not only instructional methods but also the teacher's theoretical understanding of teaching.

Although the aforementioned study demonstrates that an individual teacher's initiative can lead to significant changes in teaching practice and to the adoption of the role of teacher–action researcher, the question remains whether similar changes and effects can be achieved through a collaborative approach. In such an approach, several teachers, with the support of an external expert, jointly design, implement and evaluate an instructional intervention.

This approach is illustrated in the following study, which is based on collaborative action research in the teaching of geometry.

Case 2: Collaborative Technical Action Research

The study (Antunović, 2024) aimed to improve geometry instruction in the upper grades of primary school and to foster the development of students' geometric thinking through a collaborative action research process. It was conducted with seventh-grade students during the second semester of the 2021/2022 school year, over a period of approximately three and a half months.

Methodologically, the study was designed as mixed-method research, integrating a quasi-experimental design with elements of collaborative action research (CAR). The quasi-experimental component enabled a comparison of student achievement between the experimental and control groups, thereby examining the effectiveness of structured instructional interventions in relation to conventional practice (Creswell & Creswell, 2018; Ivankova & Wingo, 2018). Concurrently, the action research component focused on transforming teaching practice through iterative cycles of joint planning, implementation, reflection, and adaptation of instructional activities.

The study involved four mathematics teachers and 88 seventh-grade students. In the experimental group, the teachers participated in the CAR process and implemented a programme aimed at developing geometric thinking, while in the control group, instruction followed conventional teaching methods. Only students who participated in both measurement phases were included in the analysis, ensuring the comparability of the results.

The theoretical foundation of the interventions was based on the Van Hiele model of the development of geometric thinking (van Hiele, 1986) and Duval's framework of geometrical figure apprehension (Duval, 1995; 2017). The instructional activities were designed to support a gradual transition from visual recognition of shapes to an analytical and reasoned understanding of relationships among geometric objects, as well as to foster the coordination of different forms of representation (visual, symbolic, and verbal). The aim was to avoid purely procedural learning and to promote the development of discursive and deductive thinking.

The quantitative analysis was based on a pre-test and a post-test assessing levels of geometric thinking. The initial testing indicated that there were no statistically significant differences between the groups. After the implementation of the programme, the experimental group demonstrated statistically significant progress compared to the control group. More than half the students progressed by at least one level of geometric thinking, with improvements observed in their ability to visualise, coordinate different representations, and engage in reasoned argumentation. The qualitative component of the study included classroom observations, teacher interviews, questionnaires, analysis of students' work, and research field notes. The analyses indicated that students' initial difficulties were related to insufficient visualisation skills and limited understanding of dimensional relationships, while teachers expressed a need for deeper theoretical understanding of the development of geometric thinking. These findings formed the basis for adjusting the instructional interventions.

During the implementation of the programme, a gradual shift in instructional approaches was observed: from the reproduction of textbook tasks to the design of exploratory activities and open-ended problems requiring the explanation of procedures and mathematical communication. Teachers increasingly encouraged students to justify their reasoning and to connect different representations, while activities involving the manipulation of geometric figures facilitated the transition from concrete operations to abstract reasoning. The teachers' statements further confirm the transformation of instructional practice. One teacher emphasised that she had reduced her reliance on technical tasks in favour of activities that

promote the manipulation of shapes and the exploration of relationships ("I have even done many of these tasks without concrete measures, and I think that is great"), while another highlighted the increased engagement of students ("There was no passiveness"). It was also observed that the physical manipulation of geometric figures contributed to a clearer understanding and differentiation of concepts such as area and perimeter. The combination of activities focused on analysing geometric figures – implemented through individual, pair, and group work – demonstrated that seventh-grade students are capable of following elements of formal proof and solving tasks typically considered more demanding (Figure 2).

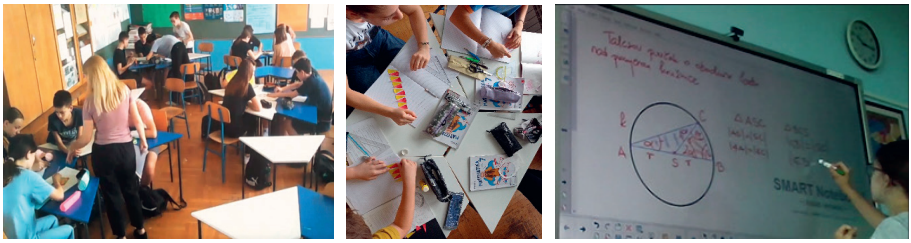


Figure 2. Illustration of student activities

Through cycles of joint planning and reflection, a professional learning community was formed in which teachers gradually assumed the role of co-researchers of their own practice. The researcher acted as a facilitator of professional dialogue, while the teachers developed greater autonomy in designing instructional interventions.

By integrating quantitative and qualitative findings, it was established that students' progress resulted not merely from the introduction of new instructional materials, but primarily from changes in teaching practices arising from a reflective and collaborative action research process. The study can therefore be typologically classified as collaborative (second-person) action research with pronounced technical elements, as it combines empirical outcome measurement with the transformation of professional practice.

Comparative Analysis

The comparison of practical and technical action research was based on the following criteria: type of research, level of participation, cyclicality and reflexivity, scope of change, teachers' professional development, and effects on

students. The analysis did not aim to evaluate effectiveness, but to understand how different models generate changes in teaching practice.

Type and Conceptual Orientation

The first study can be classified within the practical model of action research, as it gradually reconceptualises teaching practice by integrating research-based characteristics of high-quality early mathematics instruction, along with principles from the Montessori and Singapore models. This process was supported by the teacher's ongoing self-reflection and feedback from critical friends.

In contrast, the second study involved an external facilitator who brought together a group of teachers and guided them towards instructional changes based on the theories of van Hiele and Duval, thereby emphasising its technical dimension.

Level of Participation and Professional Dynamics

According to Torbert's typology, the first study can be classified as individual (first-person) action research, where professional change occurs at the level of a single teacher, supported by reflective input from critical friends. Although this model allows a high degree of autonomy and flexibility, the scope of change is limited to one classroom. Nevertheless, the study represents a valuable example of good practice and provides a foundation for further research on professional development.

The second study exhibits characteristics of collaborative (second-person) action research, where professional learning is achieved through team-based analysis, joint planning, and the dialogical construction of knowledge within a professional community. This approach generates collective knowledge and enhances the sustainability of change, as well as its potential institutional impact, although the changes remain linked to the context of the participating team and the level of organisational support. The key difference between the two models lies in the nature of professional responsibility - individual in the first case and distributed in the second.

Cyclicality and Reflexivity

Both studies follow Lewin's cyclical pattern of planning, acting, observing, and reflecting, yet they differ in the structure of the reflective process.

In the first study, reflection is predominantly qualitative and introspective, focused on analysing specific classroom situations and the teacher's own professional practice. In the second study, reflection is methodologically formalised and integrates both quantitative and qualitative data, thereby acquiring a collective and analytically structured dimension.

It may be concluded that the first study emphasises the depth of personal reflection, whereas the second highlights the breadth and methodological rigour of the reflective process. However, such a model requires substantial temporal, organisational, and institutional resources, which may limit its applicability in contexts with lower levels of structural support.

Scope of Change in Teaching Practice

Elements of substantive change are present in both studies; however, they differ in their nature and focus.

In the first study, change is reflected in the transformation of instructional dynamics: students become active participants, problem-based and collaborative learning move to the forefront, and the teacher redefines her professional identity.

In the second study, change is directed towards the development of geometric thinking. The interventions are theoretically structured and deliberately designed to promote higher levels of understanding, alongside the adaptation of instructional strategies. The key difference lies in the level of focus: the first study transforms the general instructional paradigm, whereas the second reconstructs the teaching of a specific mathematical domain.

Teachers' Professional Development

In the practical and individual model, professional development manifests through the strengthening of the teacher's professional autonomy, self-confidence, reflexivity, and research competences. The change is highly personalised and oriented towards the transformation of professional identity, specifically through assuming the role of an action researcher.

In the collaborative model, professional development has a collective dimension: teachers function as a professional learning community, and the shared theoretical framework deepens their understanding of students' cognitive processes. Although individual transformation is less explicitly emphasised, professional development is more institutionally stable. Accordingly, the practical and individual model can be described as a transformation of professional

identity, whereas the collaborative model represents a transformation of professional culture.

Effects on Students

Both studies report positive effects; however, they differ in the type and nature of the evidence provided.

In the first study, emphasis is placed on students' motivation, engagement, and collaboration, with outcomes that are predominantly qualitative and related to the learning experience and changes in classroom climate.

In the second study, statistically significant gains in levels of geometric thinking were identified, with outcomes that are measurable and cognitively operationalised.

The difference lies in the type of evidence: the first study offers an in-depth insight into the pedagogical and relational dimensions of change, whereas the second provides empirically measurable indicators of cognitive progress. However, this approach carries the risk of reducing the complexity of learning to quantifiable cognitive outcomes, potentially overlooking affective, motivational, and social aspects of the educational process.

The comparative analysis indicates that practical and technical models of action research represent distinct yet complementary pathways of professional and instructional change. The practical and individual model demonstrates that a teacher's reflective practice, grounded in relevant scientific and professional literature, can lead to a profound transformation of instructional dynamics and professional identity. The technical and collaborative model shows that structured cooperation and a clearly articulated theoretical foundation can result in empirically measurable outcomes and the development of collective professional knowledge.

Rather than assuming a fixed sequence between practical and technical action research, the relationship between these models can be understood as context-dependent and potentially bidirectional. While practical action research emphasises the development of teachers' reflective autonomy, technical models may provide an important foundation by introducing structured interventions and measurable outcomes. In this sense, technical action research can precede and support the development of more autonomous, practice-oriented inquiry, as suggested by Carr and Kemmis (1986).

The diversity of approaches thus represents a strength of action research as a flexible and contextually adaptable framework for the substantive transformation of mathematics teaching.

Table 1. Analytical comparison of the individual and collaborative action research models

Analytical dimension	Study 1 (Individual practical action research)	Study 2 (Collaborative technical action research)
Type of action research	Practical model; change arises from an independently conducted systematic literature review, based on predefined criteria for the selection and analysis of sources, accompanied by a continuous process of self-reflection.	Combination of technical and collaborative models; a theoretically structured intervention.
Level of participation (according to Torbert)	First-Person (Individual)	Second-person (Collaborative)
Epistemological approach	Systematic literature review conducted according to predefined criteria for the selection and analysis of sources	Theory as the structuring framework of the intervention
Methodological structure	Predominantly qualitative; emphasis on continuous self-reflection	Mixed methods; quasi-experimental design and qualitative analysis
Nature of reflection	Introspective and personalised	Collective and analytically structured
Scope of change	Transformation of instructional dynamics and the teacher's professional identity	Systematic reconfiguration of teaching in a specific mathematical domain
Professional development	Strengthening of individual autonomy, reflexivity, and research competences	Development of a professional community and collective knowledge
Student learning outcomes	Increased achievement, motivation, engagement, and collaboration	Statistically significant improvement in geometric thinking
Potential for sustainability	Dependent on the individual	Greater institutional stability through collaboration

DISCUSSION: THE ROLE OF ACTION RESEARCH IN TEACHING AND LEARNING

The results of the two analysed studies confirm that action research is a powerful mechanism for change in mathematics teaching. Whether conducted individually or collaboratively, it positions teachers as active creators and evaluators of their own practice, rather than merely implementers of the curriculum.

The comparative analysis supports the central assumption of the theoretical framework that action research is a transformative approach in which inquiry

and change occur simultaneously (Kemmis et al., 2014). In both cases, the transformation of teaching practice was the primary objective rather than a by-product of the research, thereby reaffirming Lewin's conception of the cyclical process of planning, acting, observing, and reflecting as a key mechanism of professional learning.

Teachers' Professional Development through Reflection and Collaboration

The practical model of action research is characterised by a practice-oriented approach in which a systematic literature review, in synergy with reflective examination of one's own practice, becomes the foundation of professional development. Such a methodologically grounded examination of pedagogical decisions fosters the development of professional competences, self-confidence, innovativeness, and flexibility, and contributes to the continuous reconstruction of professional identity, which entails assuming the role of an action researcher (Bognar, 2006, 2016; McNiff, 2016).

The technical model emphasises the cooperative dimension of action research, whereby the involvement of multiple teachers and the facilitator role of the researcher enable the development of a professional learning community. Reflection acquires a collective and analytically structured dimension, and teachers are affirmed as theoretically grounded practitioners capable of planning, analysis, and research-informed instruction (Bonner, 2006; Schoenfeld, 2019; Torbert, 2005; Wright, 2021).

Both models share reflection as the key driver of professional development, yet they differ in the degree of formalisation and scientific support. Together, they suggest that technical and practical dimensions can be understood as complementary aspects of professional development rather than fixed phases.

Impact on Students: Engagement, Understanding, and Motivation

The analysis indicates that action research has a direct impact on students. In the practical model, affective and motivational shifts are particularly evident—greater engagement, increased collaboration, and a more positive attitude towards mathematics. In the technical model, cognitively operationalised outcomes are emphasised, including the development of higher-order thinking skills such as problem-solving, visualisation, logical reasoning, and reflection.

Teachers' practical experiences further confirm these effects: students demonstrate a better conceptual understanding through the manipulation of materials, show increased activity and self-confidence, and solve tasks more successfully without relying on concrete models. These shifts suggest that changes in teaching practice grounded in reflection and inquiry lead to richer, deeper, and more meaningful learning.

In practical implementation, teachers commented: "When they physically manipulate shapes, they better understand concepts like area and perimeter." "There was no passiveness… I will do something because maybe I can do it too." "I have even done a lot of these tasks without concrete measures… and I think that's great." These reflections further confirm that when teachers transform their practice on the basis of reflection and research, students' learning experiences become more engaging, profound, and meaningful.

Theoretical Foundations and the Significance of the Research Approach

Differences between the models are also evident in their relationship to theory. In the practical study, theoretical grounding emerged from a systematic literature review conducted according to predefined criteria. Based on the identified characteristics of high-quality early mathematics instruction, foundational principles were formulated and subsequently implemented in a planned manner through cycles of action research. Nevertheless, the resulting changes cannot be attributed solely to the literature review; rather, they were co-constructed through the teacher's self-reflection, experiential knowledge, and autonomous pedagogical decision-making within the action research process.

In the technical model, theory (Duval, 1995; 2017; van Hiele, 1986) directly structured the planning, implementation, and evaluation of the interventions. Such integration enabled the systematic promotion of the transition from visual to analytical and deductive levels of geometric thinking, resulting in statistically significant cognitive gains.

The application of theoretical models within technical collaborative action research also ensured empirical validation and reflective evaluation of task effectiveness, thereby integrating quantitative elements without compromising the transformative core of the action research approach (Antunović, 2024; Creswell & Creswell, 2018; Ivankova & Wingo, 2018).

Structural and Institutional Preconditions for Wider Implementation

For the sustainable implementation of action research, it is necessary to:

1. Provide systematic support for teachers—including time for research, mentoring, access to relevant literature, and institutional incentives;
2. Foster collaboration between researchers, higher education institutions, and schools (e.g., through collaborative action research models);
3. Develop professional learning communities—promoting peer learning, joint projects, and structured reflection.

CONCLUSION

The comparative analysis of the two action research studies demonstrates that both practical and technical models can generate significant and sustainable changes in mathematics teaching, albeit through different mechanisms of professional learning.

The paper provides a theoretically grounded typologisation and a systematic comparison of the models within the Croatian context, offering an analytical framework for understanding changes in teaching practice, including transformations in teacher and student roles as well as in the organisation of learning.

The findings highlight the complementarity of the two models: the practical model fosters personal reflection, professional autonomy, and innovation, while the technical model enables stronger theoretical structuring, collective learning, and broader institutional impact. Rather than assuming a fixed developmental sequence between them, their relationship can be understood as context-dependent and potentially bidirectional. In this sense, technical action research may provide a structured foundation for practice-oriented inquiry, while practical action research enables deeper internalisation and autonomous transformation of teaching practice (Kemmis et al., 2014).

These findings open important directions for future research, particularly through systematic or scoping reviews that would allow for a broader and more comprehensive understanding of different action research models.

The analysis further suggests that educational systems should recognise and formalise action research as a legitimate professional activity of teachers; promote structured collaborative action research (CAR) programmes that

enable the application of theory in real classroom contexts; and support teachers in developing as practitioner-researchers, as any form of systematic reflection and evaluation contributes to higher-quality teaching (Betts et al., 2017; Bruce et al., 2011; Kostos & Shin, 2010).

Finally, given the limited number of empirical comparative studies on action research typologies, both in Croatia and internationally, this study represents an initial step towards a more systematic understanding of their distinctive contributions and potential in mathematics education.

REFERENCES

1. Antunović B. (2024). *Development of geometrical thinking in primary school through cognitive apprehension: doctoral dissertation* (Doctoral dissertation, Univerza v Ljubljani, Pedagoška fakulteta).
2. Asmira, A., Rusli, R., & Sabri, S. (2021). Improving students' interest and achievement in mathematics learning through problem-based learning model. In *Proceedings of the International Conference on Educational Studies in Mathematics (ICoESM 2021)* (Advances in Social Science, Education and Humanities Research, Vol. 611, pp. 303–307). Atlantis Press.
3. Betts, P., McLarty, M., & Dickson, K. (2017). An Action Research Project by Teacher Candidates and Their Instructor into Using Math Inquiry: Learning about Relations between Theory and Practice. *Networks: An Online Journal for Teacher Research, 19(1)*, 4 https://doi.org/10.4148/2470-6353.1011.
4. Bognar, B. (2006). Kako procijeniti kvalitetu akcijskog istraživanja. *Metodički ogledi*, *13*(1), 49-68. [How to Evaluate the Quality of Action Research**.** *Methodical Reviews (Metodički ogledi), 13(1), 49–68.*]
5. Bognar, B. (2016). Kako do suštinskih promjena u obrazovnom sustavu? U R. Jukić, K. Bogatić, S. Gazibara, S. Pejaković, S. Simel i A. Nagy Varga (Ur.), *Globalne i lokalne perspektive pedagogije* (str. 324–334). Filozofski fakultet Osijek. [How to Achieve Fundamental Changes in the Educational System? In R. Jukić, K. Bogatić, S. Gazibara, S. Pejaković, S. Simel, & A. Nagy Varga (Eds.), *Global and Local Perspectives of Pedagogy* (pp. 324–334). Faculty of Humanities and Social Sciences, Osijek]
6. Bognar, L. & Matijević, M. (2005). *Didaktika*. [*Didactics*] Školska knjiga.
7. Bohara, P. S. (2024). Supporting teachers in implementing project-based learning in teaching secondary mathematics*:* An action research. *Advances in Mobile Learning Educational Research, 4*(2), 1122–1134. https://doi.org/10.25082/AMLER.2024.02.007
8. Bonner, P. J. (2006). Transformation of teacher attitude and approach to math instruction through collaborative action research. *Teacher Education Quarterly*, *33*(3), 27-44.
9. Bruce, C. D., Flynn, T., & Stagg-Peterson, S. (2011). Examining what we mean by collaboration in collaborative action research: A cross-case analysis. *Educational Action Research*, *19*(4), 433-452. https://doi.org/10.1080/09650792.2011.625667
10. Carr, W., & Kemmis, S. (1986). *Becoming critical: Education, knowledge and action research*. Routledge/Falmer.
11. Coghlan, D., & Brannick, T. (2005). *Doing action research in your own organization*. SAGE.
12. Creswell, J. W., & Creswell, J. D. (2018). *Research design: Qualitative, quantitative, and mixed methods approaches*. SAGE
13. Dolapcioglu, S., & Doğanay, A. (2020). Development of critical thinking in mathematics classes via authentic learning: An action research. *International Journal of Mathematical Education in Science and Technology*. https://doi.org/10.1080/0020739X.2020.1819573
14. Duval, R. (1995). Geometrical pictures: Kinds of representation and specific processing. In R. Sutherland, & J. Mason (Eds.), *Exploiting mental imagery with computers in mathematics education* (pp. 142–157). Springer. https://doi.org/10.1007/978-3-642-57771-0_10

15. Duval, R. (2017). *Understanding the mathematical way of thinking: The registers of semiotic representations.* Springer International Publishing. https://doi.org/10.1007/978-3-319-56910-9

16. Flyvbjerg, B. (2006). Five misunderstandings about case-study research. *Qualitative Inquiry, 12*(2), 219–245. https://doi.org/10.1177/1077800405284363

17. Heaton, J. (2004). *Reworking qualitative data*. SAGE.

18. Hemmeter, M. L., Snyder, P., Kinder, K., & Artman, K. (2011). Impact of performance feedback delivered via electronic mail on preschool teachers' use of descriptive praise. *Early Childhood Research Quarterly*, *26*(1), 96–109. https://doi.org/10.1016/j.ecresq.2010.05.004

19. Ivankova, N., & Wingo, N. (2018). Applying mixed methods in action research: Methodological potentials and advantages. *American Behavioural Scientist*, *62*(7), 978–997. https://doi.org/10.1177/0002764218772673

20. Jukić Matić, L., Mužar Horvat, S., & Bognar, B. (2024). Usporedba Montessori i singapurskog modela početne nastave matematike. *Nova prisutnost, 22*(1), 203–216. [A Comparison of the Montessori and Singapore Approaches to Early Mathematics Education.] *Nova prisutnost, 22*(1), 203–216

21. Kemmis, S., McTaggart, R., & Nixon, R. (2014). *The action research planner: Doing critical participatory action research*. Springer. https://doi.org/10.1007/978-981-4560-67-2

22. Kostos, K., & Shin, E. (2010). Using math journals to enhance second graders' communication of mathematical thinking. *Early Childhood Education Journal, 38*(3), 223–231. https://doi.org/10.1007/s10643-010-0390-4

23. Lewin, K. (1946). Action research and minority problems. *Journal of Social Issues*, *2*(4), 34–46. https://doi.org/10.1111/j.1540-4560.1946.tb02295.x

24. McNiff, J. (2013). *Action research: Principles and practice* (3rd ed.). Routledge.

25. McNiff, J. (2016). *Writing up your action research project*. SAGE.

26. Mužar Horvat, S. (2023). Povratne informacije učenicima u početnoj nastavi matematike. [Feedback to Students in Early Mathematics Instruction.] *Metodički ogledi, 30*(2), 239–264. https://doi.org/10.21464/mo.30.2.1

27. Mužar Horvat, S. (2024). Značajke učinkovite početne nastave matematike: Sustavni pregled literature. *Marsonia: Časopis za društvena i humanistička istraživanja, 3*(1), 55–68. https://hrcak.srce.hr/file/463790 [Features of effective elementary mathematics instruction: A systematic literature review]. *Marsonia: journal of social and humanities research*, *3*(1), 55–68. https://hrcak.srce.hr/320935

28. Mužar Horvat, S., & Bognar, B. (2024). Achieving Essential Changes in the Elementary Teaching of Mathematics. *Mathematics teaching-research journal*, *16*(5), 40-65.

29. Mužić, V. (2004), *Uvod u metodologiju istraživanja odgoja i obrazovanja,* Zagreb: Educa. [Introduction to the methodology of education research] Educa.

30. Özturan Ecemiş, Ü., Aramış Barış, Z. F., & Kula Ünver, S. (2024). Examination of Action Research Articles in the Field of Mathematics Education in Türkiye. *Instructional Technology and Lifelong Learning*, *5*(2), 300-325. https://doi.org/10.52911/itall.1544320

31. Schoenfeld, A. H. (2019). What makes for powerful classrooms, and how can we support teachers in creating them? A story of research and practice productively intertwined. In G. Kaiser, & N.

Presmeg, (Eds.), *Compendium for early career researchers in mathematics education. ICME-13 Monographs* (pp. 495-510). Springer. https://doi.org/10.1007/978-3-030-15636-7_26

32. Smith, M. K., Jones, F. H. M., Gilbert, S. L., & Wieman, C. E. (2013). The Classroom Observation Protocol for Undergraduate STEM (COPUS): A new instrument to characterise university STEM instruction. *CBE—Life Sciences Education, 12*(4), 618–627. https://doi.org/10.1187/cbe.13-08-0154
33. Somekh, B. (2006). *Action research: A methodology for change and development*. Open University Press.
34. Torbert, W. R. (2005). *Action inquiry: The secret of timely and transforming leadership*. Berrett-Koehler Publishers.
35. van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education*. Academic Press, Inc.
36. Wright, P. (2021). Transforming mathematics classroom practice through participatory action research. *Journal of Mathematics Teacher Education*, *24*(2), 155-177. https://doi.org/10.1007/s10857-019-09452-1

USPOREDBA TEHNIČKOG I PRAKTIČNOG AKCIJSKOG ISTRAŽIVANJA U UNAPREĐENJU NASTAVE MATEMATIKE

Cilj rada je usporediti individualno praktično i suradničko tehničko akcijsko istraživanje u nastavi matematike te analizirati njihov doprinos promjeni nastavne prakse i profesionalnom razvoju učitelja u hrvatskom kontekstu. Istraživanje je provedeno kao kvalitativna analitičko-komparativna studija dvaju radova, koristeći tipologije akcijskog istraživanja (tehničko, praktično i emancipacijsko) te razine sudjelovanja (individualna i suradnička dimenzija) kao analitički okvir. Rezultati pokazuju da oba modela potiču promjene u nastavi, ali različitim mehanizmima: praktično-individualni model kroz refleksivnu promjenu profesionalnog identiteta i pristupa podučavanju, a tehnički-kolaborativni kroz unapređenje prakse, sustavniju evaluaciju postignuća i jačanje suradnje. Ključ uspješnosti čine refleksivnost, teorijska utemeljenost, suradnja i institucionalna podrška. Rad tipologizira i uspoređuje dva modela te nudi konceptualni okvir za razumijevanje profesionalnih i pedagoških promjena u nastavi matematike.

Ključne riječi: akcijsko istraživanje, inovacije u nastavi matematike, profesionalni razvoj učitelja, refleksivna praksa, suradničko učenje