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Review Paper

FROM STRING OF BEADS TO DOUBLE LINES: THE NUMBER LINE IN MATHEMATICS TEXTBOOKS (1961. TO 2020.)

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ABSTRACT

This study investigates the use of the number line in mathematics education, with particular emphasis on textbook materials. The number line is one of the most widely recommended didactic models across all grade levels. As a key visualization tool, it appears in multiple formats and contexts: representing whole numbers and fractions in early grades, and rational and integer numbers in later elementary education. The primary aim is to examine its presence and pedagogical role in supporting the conceptualization of numbers and arithmetic operations. The study employs qualitative content analysis of selected mathematics textbooks published in Bosnia and Hercegovina, Croatia, Serbia, and Slovenia from 1961 to 2020 and a modern time textbook from the USA, focusing on visual representations and contextual uses of the number line. By comparing formats and contexts, prevailing patterns in the use of the number line were identified. The findings indicate an increased presence of the number line in contemporary textbooks, together with a stronger emphasis on contextualized and conceptually oriented approaches to mathematics learning, including approaches associated with Realistic Mathematics Education (RME). The educational implications point to the need for further research on the impact of number-line representations on students' mathematical reasoning and computational skills.

Keywords: didactic model, mathematics textbooks, realistic mathematics education (RME), visualization.

INTRODUCTION

Mathematics textbooks play a crucial mediating role between curricular intentions and classroom practice, as they embody specific didactical models through which mathematical concepts are represented and communicated to learners. The choice of representations and task structures within textbooks shapes students' opportunities to construct meaning and develop conceptual understanding (Rezat & Strässer, 2012). Consequently, the analysis of textbook design provides valuable insight into the implicit pedagogical and epistemological assumptions underlying mathematics instruction (Fan et al., 2013).

Within the framework of RME, textbooks are designed to support concept formation through guided reinvention, starting from meaningful, experientially rich contexts that gradually lead students toward more formal mathematical models and abstractions. Tasks in RME textbooks typically progress from contextual or model-of situations to model-for reasoning, promoting vertical mathematization and encouraging learners to construct connections between representations and underlying structures (Gravemeijer, 1999). The sequencing and design of tasks thus reflect a didactical orientation toward fostering students' active mathematizing rather than reproducing ready-made algorithms (Van den Heuvel-Panhuizen, 2003).

Concept of number line

Grasping numerical relationships and developing orientation within the number system are key goals of primary mathematics education. The number line serves as a powerful visual representation supporting these processes (Schneider et al., 2018; Woods et al., 2017; Urff, 2025). Research highlights that number line estimation task's function both as diagnostic tools for assessing children's numerical understanding and as effective means for targeted instructional interventions (Gaidoschik, 2024).

From a cognitive perspective, the number line is closely related to the concept of the *mental number line*, a spatial representation underlying numerical processing. Its development is intrinsically linked to spatial abilities, which are recognized as an important predictor of mathematical achievement (LeFevre et al., 2013; Olkun et al., 2019). Within the framework of embodied cognition, numerical understanding is shaped through sensorimotor interaction with space, emphasizing the importance of physically and visually grounded representations (Way & Ginns, 2024).

Developing the concept of number and understanding the relationship between numbers are the foundation of mathematical education. During the last decades, in the didactics of mathematics, more and more attention is paid to the ways in which abstract numbers can be made visible, spatially representable and meaningful for students. In this context, the number lines occupy a prominent place as a model that connects concrete experience and abstract thinking. Starting with a series of beads and simple counting models, through an "empty" number line to more complex representations such as a double line, this model goes through different stages of development that reflect the progress of the student's understanding of number, size and operations.

Today, the number line is considered one of the key visual models in the teaching of mathematics, present in the curricula of almost all countries (Treffers, 1987; Van den Heuvel-Panhuizen, 2008). Its use allows students to perceive numbers not only as symbols, but also as positions on a continuum that can be spatially interpreted, which encourages the development of conceptual understanding, flexible calculation strategies and mathematical reasoning. The way the number line is presented in textbooks and teaching materials reflects the pedagogical paradigms of its time, from mechanical counting to modeling real situations in the spirit of realistic mathematics education (RME).

In this paper, we analyze how different representations of the number line, from traditional to contemporary formats, are present and pedagogically shaped in mathematics textbooks. The goal is to, through theoretical foundation and qualitative content analysis, look at their didactic function and contribution to the understanding of numbers and arithmetic operations.

Theoretical framework

Visualization is a key element of mathematical thinking and understanding (Arcavi, 2003; Presmeg, 2006). Students who develop the ability to "see" mathematical relationships through pictures, models and diagrams, move more easily between concrete and abstract levels of understanding (Milinković, 2024). The number line as a visual-spatial model enables such **multiple representations** (Duval, 1999), connecting the verbal, symbolic and graphic register.

Through the use of the number line, students develop:

- understanding **the relative relationships** between numbers (larger-smaller, closer-further),
- **the concept of linear measure** and continuum,

- **assessment** and **numerical reasoning,** that is, the ability to see numbers in the context of size and distance.

Research by Siegler and Opfer (2003) shows that the development of children's numerical representation moves from a logarithmic model (where the spaces between smaller numbers are wider) to a linear mental model, which can be systematically encouraged by using a number line. Students who often use the number line during classes show a deeper understanding of rational numbers, negative numbers and decimals (Opfer & Thompson, 2006; Booth & Siegler, 2008).

Number line as a didactic model

The number line occupies a central place among the didactic models used to develop the concept of number and arithmetic operations in primary school education. As a linear visual representation, it enables students to perceive numbers as positions on a continuum, and not only as symbols or quantities (Fuson, 2009; Van den Heuvel-Panhuizen, 2008). Such a spatial representation enables the intuitive observation of size relationships, order and differences between numbers.

Conceptually, the number line connects the quantitative ("how much") and ordinal ("in which order") aspects of number (Resnick et al., 1991). Its use makes it easier to understand that there are infinitely many other numbers between each whole number, which is crucial for establishing the continuity of the number system.

One of the most important applications of the number line is in the development of understanding of arithmetic operations (addition, subtraction, multiplication and division). Unlike traditional approaches that emphasize mechanical calculation, the number line allows students to see arithmetic operations as spatial transformations (shifts, jumps, segmentations). For example, addition can be represented as a shift to the right, subtraction as shifting to the left, while multiplying and sharing can be represented as repeated shifts or sharing intervals (Beishuizen, 1993; Klein et al., 2018). Such a visualization enables students to intuitively understand the structure of calculation operations and to develop flexible mental calculation strategies.

Studies show that students who learn operations using the empty number line develop reasonable estimation strategies and move more easily to formal algorithmic computation (Gravemeijer, 1999; Van den Heuvel-Panhuizen & Drijvers, 2014). In this way, the number line acts as a mediator between concrete

manipulation and abstract symbolism, which is especially important in early education.

In didactic practice, various formats of number lines can be found in textbooks, but some are more frequently used than others: a bead string, an open number line, an empty number line, closed number lines, a double number line. Gravemeijer points to three dominant formats of the number line (Gravemeijer, 1991): number sequence, "empty" number line and double number lines as most useful representations. Number line as a number sequence, consists of a straight line marked with numbers or dots positioned evenly. It is used in the initial stages of learning to represent whole numbers and develop counting skills. An "empty" number line representation is a simple straight line. It encourages students to independently determine reference points, which develops flexible and conceptual calculation. **Double number lines** allow for the parallel presentation of two numerical situations (e.g. fractions and decimal numbers), which encourages understanding of the relationships between different numerical systems. It can be used to find equivalent ratios. For example, the number line in Figure 1 shows that the ratio 15 : 3 is equivalent to 5 : 1. The context is mixing 15 spoons of flour with 3 eggs for a cake mixture. Two times bigger amount of this mix is made of 30 spoons and 6 eggs.

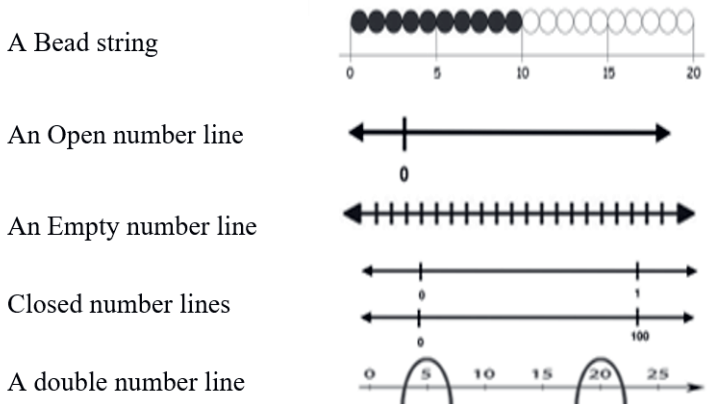


Figure 1. Variants of number line models

Empirical research shows that students who regularly use number line in class better understand the relationships between numbers and show more advanced calculation strategies (Siegler & Booth, 2004; Thompson & Saldanha, 2003).

Number line in the Realistic Mathematics Education (RME) framework

The theoretical framework of Realistic Mathematics Education (RME), developed in the Netherlands (Treffers, 1987; Gravemeijer, 1994), is based on the idea that mathematical knowledge is built through active modeling of real situations. In this approach, models are not ready-made representations, but "bridges" between everyday experience and formal mathematical expression.

In this sense, the number line is the prototypical modelof the RME approach. It enables the so-called horizontal mathematization– the transition from a real context (e.g., length of a path, temperature, money, debt) to a symbolic level, as well as vertical progression, the transition from an intuitive to an abstract understanding of numbers and operations (Gravemeijer, 1994; Treffers, 1991).

Through number line, students connect specific situations with symbolic records, which facilitates the process of mathematical modeling. Instead of passively adopting algorithms, they actively construct the meaning of numbers and calculation procedures. According to Van den Heuvel-Panhuizen (2008), the number line is the most important model for developing the concept of difference, distance and proportionality, which form the basis of many mathematical concepts.

Number lines in mathematics textbooks

Mathematics textbooks play a key role in conveying curricular ideas and shaping teaching practice. The analysis of textbooks provides insight into how theoretical principles are concretized in didactic material (Pepin & Haggarty, 2001). Number line can appear in textbooks in different forms: as a counting tool, as a contextual model in tasks, or as a graphic representation to explain arithmetic operations.

Previous analyzes of textbooks from different countries (Van den Heuvel-Panhuizen, 2008; Jones et al., 2012; Hong et al., 2021) show that the way the number line is presented reflects dominant didactic paradigms. In traditional materials, the number line is used predominantly as an aid to counting and labeling, while modern textbooks (in accordance with RME principles) emphasize the active use of the number line as a model for research and thinking. Such materials encourage students to set their own reference points, evaluate differences and visually interpret relationships, thus developing deeper mathematical competences - especially mathematical reasoning and conceptual

understanding. In this sense, the analysis of textbooks can serve as an important indicator of the alignment of teaching materials with modern educational approaches.

Starting from theoretical and empirical findings, number line can be seen as a central visual model of mathematics education, which unites cognitive, didactic and curricular aspects of learning mathematics. Its role goes beyond a mere graphic aid as it is a tool for thinking, a bridge between the real and the abstract, and a basis for the development of arithmetic and numerical reasoning. For this reason, the study of the way number lines are used and presented in textbooks is extremely important for understanding contemporary trends in mathematics teaching. *Table 1 summarizes the main types of tasks that engage students in using different number line formats across cognitive domains.*

Table 1. Types of tasks involving different formats of the number line

Type of Task	Typical Format of Number Line	Example Task (Primary Grades)	Mathematical / Cognitive Focus	Cognitive Demand
Locating numbers	Scaled number line with evenly spaced intervals or open number line with endpoints	Mark the number 7 on a number line from 0 to 10; place ½ on a line from 0 to 1	Understanding magnitude, counting sequences, early fraction sense	**Knowing** – identify and represent numbers on a given scale
Comparing and ordering	Number line with labeled reference points or parallel (double) number lines	Place 3, 5, and 8 on a number line and determine which is largest; compare ½ and ¼ on the line	Comparing magnitudes, using spatial representation to order numbers	**Applying** – use number line structure to compare and order
Estimation and approximation	Partially labeled number line with benchmark numbers	Place the number 6 on a line marked 0 and 10; estimate where ¾ lies between 0 and 1	Use of benchmarks, sense of relative size, proportional reasoning	**Reasoning** – estimate using reference points and intervals
Operations on numbers	Open number line with directional jumps (arrows) indicating movement	Show 5 + 3 as jumps on a number line; solve 9 – 6 by moving backward	Understanding addition and subtraction as movement along a line	**Applying** – model operations through spatial actions
Working with negative numbers	Bidirectional number line centered at 0 with equal intervals	Show the position of –2 and 3 on a number line; represent moving 2 steps left from 1	Understanding direction, zero as a reference point, relative position	**Applying / Reasoning** – interpret movement and distance
Fractions and decimals	Segmented number line partitioned into equal parts	Divide a line from 0 to 1 into four equal parts and mark ¼ and ¾; place 0.5 on the line	Partitioning the whole, equivalence of representations, density (intuitive level)	**Applying** – connect fractions and positions on the line

Measurement and scaling	Measurement number line with units (e.g., length, time) or simple double number line	Mark 3 hours on a timeline from 0 to 6;	Linking numbers to continuous quantities, unit iteration, early proportional thinking	**Reasoning** – connect numerical values with real-world measures
Algebraic and functional reasoning	Number line with marked intervals, rays, or highlighted sets of points	Show all numbers greater than 5 on a number line; represent "a number and 2 more" as a jump forward from any point	Generalization of numerical relationships, early functional thinking, interpreting unknowns	**Reasoning** – generalize patterns and interpret relationships

METHODOLOGY

This study examines the representational use of number lines in primary mathematics textbooks from a didactical perspective. A qualitative content analysis was conducted, focusing on textbook tasks that incorporate number line representations. The convenience sample consisted of nine mathematics textbooks from Bosnia and Herzegovina, Croatia, Serbia, Slovenia published between 1961 and 2020 and a contemporary textbook from the United States. The textbooks from the first four countries were included due to their shared curricular heritage up to 1990s. After that period, the newly formed countries developed their own curricular approaches. At present, we assume that countries worldwide are strongly influenced by international research findings, e.g. TIMSS findings, as well as shared practices among mathematics educators(Bartolini Bussi, Milinkovic, & Gooya, 2023). The U.S. textbook was selected as representative of reform-oriented mathematics materials influenced by Realistic Mathematics Education (RME) and related constructivist approaches that became increasingly influential in U.S. school mathematics at the beginning of the 21st century. Baker et al. (2010) pointed that the increasing emphasis on conceptual understanding and cognitively demanding problem solving can be identified in late-20th-century U.S. textbooks.

Each instance of a number line was analyzed according to the following attributes: format, mathematical domain, contextual use, task type, and cognitive demand.

RESULTS AND DISCUSSION

The analysis was conducted chronologically to trace the evolution of number line representations. Early textbooks (e.g., Prvanović, 1961) primarily used bead-string models to support counting sequences, reflecting a procedural and limited use of the number line.

In subsequent decades, the number line gradually assumed more diverse roles. By the 1970s, it appeared in tasks involving subtraction strategies and fraction comparison, indicating a shift toward conceptual use. Later textbooks increasingly incorporated open and empty number lines, supporting flexible calculation strategies and reasoning.

From the 1980s onward, contextualized uses emerged, linking number lines to real-world situations and measurement. Contemporary textbooks demonstrate a broader and more systematic use across domains, including operations, proportional reasoning, and algebraic thinking.

The oldest examined textbook was published in 1961 (Prvanović, 1961). The problems were „counting by number“,for example „counting by 5“, using a bead string representation of a number line (Figure 2).

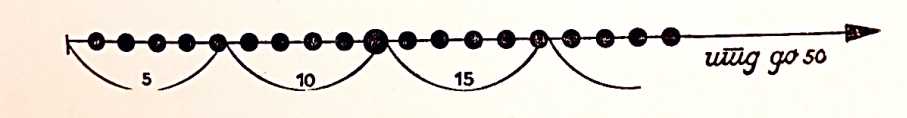
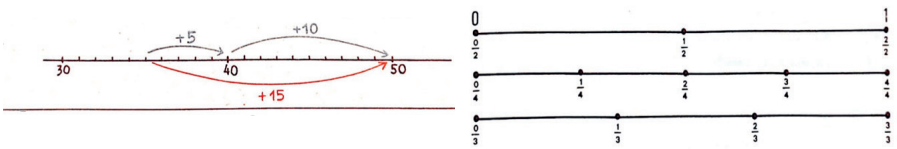


Figure 2. A String of beads (Prvanović, 1961, p. 51)

The task type is „locating numbers“ on the number line. The number line is evenly scaled. The focus is on the counting sequence. Cognitive domain is „knowing “. Pupils are asked to continue „drawing“ on a given scale and to read numbers: 5, 10, 15, ...

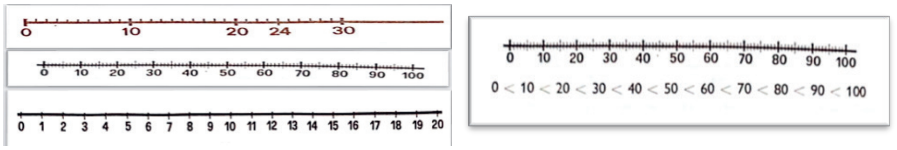
In a textbook published in 1976, the number line appeared in two problems. The first one is used in a lesson on a particular method of subtraction. An empty number line is used in the textbook (Figure 3a). It served as a visualization tool for “subtraction by adding the remainder to the numerator - by completing”. In the second case, pupils are asked to solve a problem in the domain of fractions. They compared fractions, visually represented on three parallel number-line segments (Figure 3b).



a) b)

Figure 3. a) Subtraction by adding the remainder to the numerator - by completing (Nikodijević & Nikodijević, 1976, p. 31; b) Comparing fractions on a number line (Nikodijević & Nikodijević, 1976, p. 119)

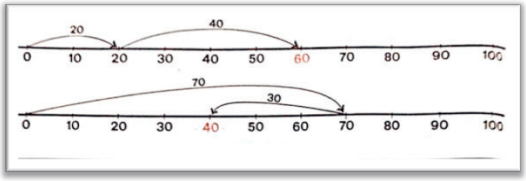
In the same decade, in another textbook the number line is used a visual aid for locating whole numbers, comparing numbers, and determining a sequence of numbers (Figure 4). In the same textbook closed number line is used to visually represent arithmetic operations of addition and subtraction.



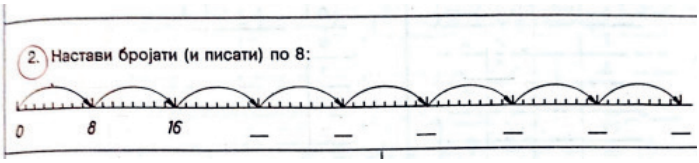
a) b)

Figure 4. a) Locating numbers, (Markovac & Benčić, 1977), b) Comparing whole numbers a number line (Markovac & Benčić, 1977)

Ten years later Gorušanin (1986) attempts to use potential of the number line for developing mathematical knowledge in different domains and problem situations. First, the author returns to the „counting by number"task using an empty number line. In the same textbook, for the first time we found a contextualized empty number line which can support r**easoning** by connecting numerical values with real-world measures.



a)



b)

Figure 5. Number line as a counting tool, a) (Markovac & Benčić, 1977), b) (Gorušanin, 1986)

Markovac and Benčič (1977), perform addition and subtraction on a number line (Figure 5a). Gorušanin (1986) uses a number line for "counting by 8" (Figure 5b) but also for locating whole numbers, comparing numbers, creating a sequence of numbers, adding and subtractoning numbers, and in the process of solving an inequality to represent the set of values for each the inequality is true (Figure 6).

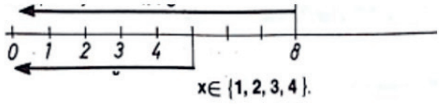


Figure 6. Using number line to represent the solution (Gorušanin, 1986, p. 22)

In the last decade of the 20th century Marjanović, Latković, and Nikodijević (1991) used number line for visually adding and subtracting whole numbers.

At the begining of the 21st century a math textbook written by Foresman (2004) explores the number line concept in comparing numbers, number rounding and estimation, determining „halfway number" and the nearest hundred (Figure 7, 2004).

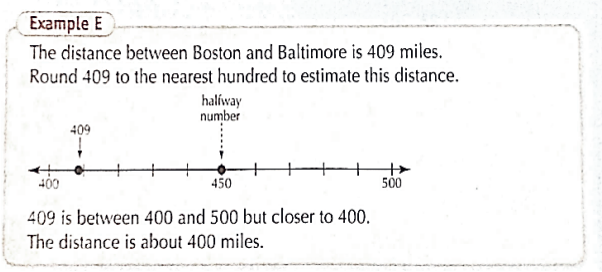


Figure 7. Rounding using a closed number line (Foresman, 2004)

Krnel, Hodnik Čadež, Kokalj, and Pristovnik (2011) used a nonstandard model of number line, consisting of boxes of numbers to visually represent an arithmetic operation (Figure 8a) but also for measuring and estimating length (Figure 8b).

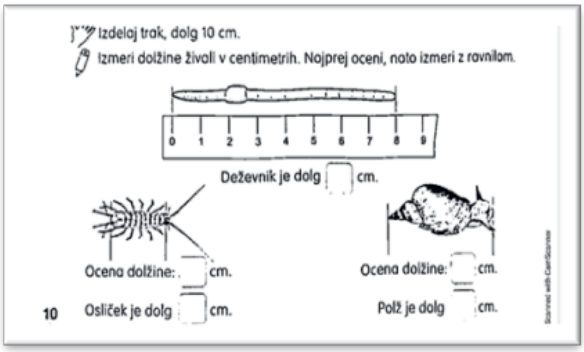


Figure 8. Calculating and estimating with a number line (Krnel, et al., 2011)

Milinković (2018, 2020) finds different ways to use number line to determine number sequence, a predecessor and a successor of a number, place of a number in the number sequence or to visualize arithmetic law (Figure 9).

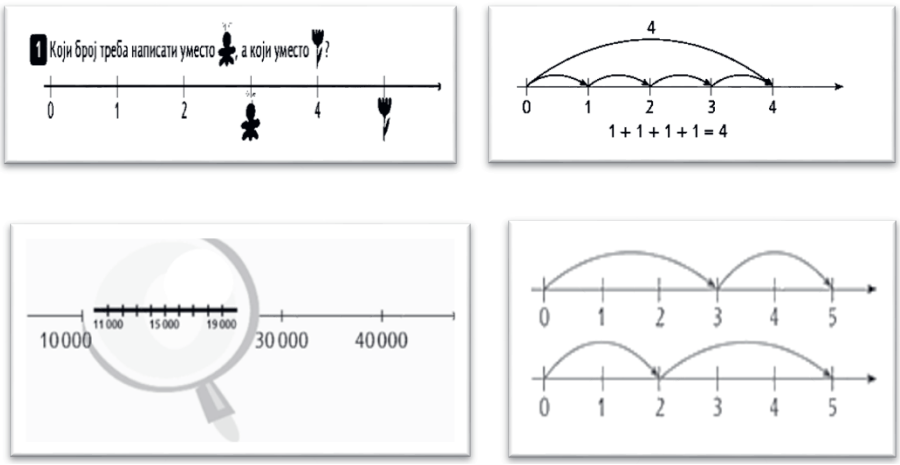


Figure 9. Knowing and reasoning with different models of number line (Milinković, 2018, 2020)

As we have shown number line models were given modest attention in earlier time, dominantly in counting. Over time number line models varied

and started to be investigated in different formats and appeared in various math domains.

Number line is employed in diverse contextual situations to represent: (I)Numbers (Whole numbers, Fractions, Rational numbers, Integers), (II) Arithmetic operations, (III)Relations, (IV) Measurement.

The following math procedures and activities are identified as linked to use of number line models: counting, comparing numbers, number sequence, place of a number in number sequence, comparing measures, estimating, rounding, adding, subtracting, multiplying numbers, solving an inequality, converting between measurement units, determining distance or length.

CONCLUSIONS AND DIDACTICAL IMPLICATIONS

This study examined the historical development of number line representations in mathematics textbooks, with particular attention to their pedagogical role. The findings indicate a clear progression from limited procedural use toward more conceptually oriented and versatile applications.

In the modern textbooks included in the sample, the number line is noticeably present as a didactical model supporting the development of numerical understanding, arithmetic operations, and mathematical reasoning. Its increasing alignment with Realistic Mathematics Education (RME) principles reflects a shift toward modeling, contextualization, and student-centered learning.

Future research should further investigate how different uses of the number line influence students' mathematical competencies, particularly in relation to reasoning, estimation, and flexible problem-solving strategies. Special attention should be given to task design, cognitive demand, and the role of visualization in supporting conceptual understanding.

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OD NIZA PERLI DO DVOSTRUKIH LINIJA: BROJEVNA PRAVA U MATEMATIČKIM UDŽBENICIMA (1961. – 2020.)

Ova studija istražuje upotrebu brojevnog pravca u obrazovanju iz matematike, s posebnim naglaskom na udžbeničke materijale. Brojevni pravac jedan je od najčešće preporučenih didaktičkih modela u svim razredima. Kao ključni alat za vizualizaciju, pojavljuje se u više formata i konteksta: predstavljanje cijelih brojeva i razlomaka u nižim razredima te racionalnih i cijelih brojeva u kasnijem osnovnoškolskom obrazovanju. Primarni cilj je ispitati njegovu prisutnost i pedagošku ulogu u podršci konceptualizaciji brojeva i aritmetičkih operacija. Studija koristi kvalitativnu analizu sadržaja odabranih udžbenika matematike, s naglaskom na vizualne prikaze i kontekstualnu upotrebu brojevnog pravca. Usporedbom formata i konteksta identificiraju se prevladavajući obrasci. Nalazi ukazuju na povećanu prisutnost i jaču usklađenost s principima realističnog matematičkog obrazovanja (RME). Obrazovne implikacije ističu potrebu za daljnjim istraživanjima o njegovom utjecaju na matematičko razmišljanje i računalne vještine učenika.

Ključne riječi: didaktički model, udžbenici matematike, realistično matematičko obrazovanje (RME), vizualizacija.