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Restricted VAR Hedging with the Presence of Multiple Breaks

Chien-Liang Chiu*

Jer-Shiou Chiou**

Pei-Shan Wu***

Abstract: Distinct from the existing literatures that most of them focussed on the case of a single change on issues related to structural change. This study addresses the practical advantage of hedging ratio when time varying structural breakings are considered. Data used in this study include daily observations of spot prices of WTI (Cushing, Oklahoma FOB), U.S. crude oil production, and futures closing prices of NYMEX over the period of 2002/1/2 ~ 2005/7/26. We compare on out-of-sample hedging effectiveness of this structural break with restricted VAR hedging model against standard VAR hedge model. It has been found that there are four structural breaks. And the improvement in hedging performance is clearly presented. Smaller hedging of a futures position can therefore reduce the investors cost extensively.

Keywords: multiple structural breaks, hedging performance

JEL Classification: G14, G22

Introduction

Most commodity trading theorists have taken the hedger as a trader who desires insurance against the price risks he faces. Regardless the motive of portfolio managers, the relation between the futures and their stock portfolio has constantly attracted attention. Whenever the relation is seemingly to have changed, the holdings in futures and portfolio may changed accordingly. In order to minimize the portfolio's variance, hedging is commonly undertaken to reduce the risk of holding a portfolio of

* Chien-Liang Chiu is at Graduate Institute of Money, Banking and Finance, TamKang University, Taipei, Taiwan.
** Jer-Shiou Chiou is at the Department of Finance and Banking, Shih-Chien University, Taipei, Taiwan.
*** Pei-Shan Wu is at the Department of Finance, Ching-Yun University, Jung-Li City, Taiwan.

risky assets. In the late 1970's and early 1980's, rolling window methodology had been commonly adopted to exploit the strategy of dynamic hedging. In the late of the 1980's, hedging models that account for time-varying covariance had been noticed and studied. However, apart from time-varying distortions, the occasional change in policy and environment has the potential to significantly and persistently change the spot-futures price relationship. If the relationship is subject to these structural changes, the rolling window and Exponentially Weighted Least Squares (EWLS) models may all inappropriate. In accounting for structural breaks, Kalman filter and Markov regime switching models can be used to capture structural breaks in the hedge ratio, while Bai and Perron (1998, 2003) propose a least squares estimation procedure to test for multiple breaks.

The purpose of this paper is to determine if and when structural changes occurred between the spot and futures markets by analysing their price relationship. Since the effectiveness of constant hedge ratio performances relative to alternative hedging strategies may have altered due to the financial crisis and the subsequent impact this has on hedging behaviour, different hedging strategies will be compared in an emerging and dynamic market. In this study, the comparison between with and without the consideration of structural breaks will be conducted. The estimates derived from these models are then used to calculate an alternate hedge ratio. Different from the existing literatures that most of them focussed on the case of a single change on issues related to structural change, the problem of multiple structural changes has been considered in this study. Data used in this study include daily observations of spot prices of WTI (Cushing, Oklahoma FOB), U.S. crude oil production, and futures closing prices of NYMEX over the period of 2002/1/2 ~ 2005/7/26.

Multiple Structure Changes

In applying on real life data, most time series models are experiencing structural instability. Consequently, the estimation and inference without knowing this fact will lead to inconsistent estimator and thus unreliable results. During the last decade, both the statistics and econometrics literature contain a great deal of studies in the theory of identification, estimation and testing of structural breaks, and brought these theories into practice; Papell, Murray and Ghiblawi (2000), Rapach and Wohar (2004). And the problem of multiple structural changes are receiving increasingly attention, Garcia and Perron (1996), Liu, Wu and Zidek (1997), Lumsdaine and Papell (1997), and among others. While Bai and Perron (1998) considered multiple structural changes in a linear model under a general framework but allows a subset of the parameters not to change.

Consider the following multiple linear regression with m breaks ($m+1$ regimes):

$$f_t = \alpha_j + u_t \quad t = T_{j-1} + 1, \dots, T_j \tag{1}$$

for $j = 1, \dots, m+1$. f_t is the observed dependent variable (futures index) at time t; α_j ($j = 1, \dots, m+1$) is the corresponding vector of coefficients; u_t is the disturbance at time t. The indices ($T_1, \dots, T_m$), or the break points, are explicitly treated as unknown ($T_0 = 0$ and $T_{m+1} = T$ are used). The unknown regression coefficients along with the break points are estimated, when T observations on f_t are available. The variance of u_t needs not be constant. Indeed, breaks in variance are permitted provided they occur at the same dates as the breaks in the parameters of the regression.

The linear regression in (1) can be expressed in matrix form as

$$F = A + U \tag{2}$$

where $F = (f_1, \dots, f_T)'$, $A = (\alpha_1, \dots, \alpha_T)'$, and $U = (u_1, \dots, u_T)'$. We denote the true value of a parameter with a 0 superscript. In particular, $A^0 = (\alpha_1^{0'}, \dots, \alpha_{m+1}^{0'})'$ and $(T_1^0, \dots, T_m^0)$ are used to denote, respectively, the true values of the parameters 'A' and the true break points. The data-generating process is assumed to be

$$F = A^0 + U \tag{3}$$

The method of estimation considered is that based on the least-squares principle. For each m-partition ($T_1, \dots, T_m$), the associated least-squares estimates of α_j are obtained by minimizing the sum of squared residuals

$$(F - A)'(F - A) = \sum_{i=1}^{m+1} \sum_{t=T_{i-1}+1}^{T_i} \left[f_t - \alpha_t \right]^2 \tag{4}$$

Let $\hat{\alpha}(\{T_j\})$ denote the estimates based on the given m-partition $(T_1, \dots, T_m)$ denoted $\{T_j\}$. Substituting these in the objective function and denoting the resulting sum of squared residuals as $S_T(T_1, \dots, T_m)$, the estimated break points $(\hat{T}_1, \dots, \hat{T}_m)$ are such that $(\hat{T}_1, \dots, \hat{T}_m) = arg\ min_{T_1, \dots, T_m} S_T(T_1, \dots, T_m)$, where the minimization is taken over all partitions $(T_1, \dots, T_m)$ such that $T_i - T_{i-1} \geq q$. Thus the break-point estimators are global minimizers of the objective function. The regression parameter estimates are the estimates associated with the m-partition $\{T_j\}$, i.e. $\hat{\alpha} = \hat{\alpha}(\{\hat{T}_j\})$. Since, the break points are discrete parameters and can only take a finite number of values, they can be estimated by a grid search.

Hedging with VAR and Structure Changes

The evidence on performance of the estimated conditional optimal hedge ratio in commodity and financial futures markets in terms of risk reduction is mixed. Despite the substantial interests in the theory and practice of hedging, several important issues of optimal hedging are repeatedly ignored in practice of hedge design. Usually, in minimizing the risks, an appropriate hedge ratio can be found by regressing realized price changes on the futures contracts. Choosing hedge ratios in this way suffers from some shortcomings. Not only most research on hedging has disregarded both the long-run cointegrating relationship between financial assets and the dynamic nature of the distributions of the assets, but also omitted the joint distribution of cash and futures price which may change substantially over time, therefore the hedge ratio may be estimated incorrectly, Alizadeh (2004).

Sims (1980) utilized Vector Autoregression (VAR) model to construct a dynamic model. The model need not consider the causality relationship among the variables and no prior theory is needed either. As the appropriate lagged terms have been determined, the model can involve all the information from the variables. A typical restricted VAR model can be written as:

$$S_t = \alpha_s + \varepsilon_{St} \tag{5}$$

$$F_t = \alpha_f + \varepsilon_{Ft} \tag{6}$$

where $\begin{bmatrix} \varepsilon_{St} \\ \varepsilon_{Ft} \end{bmatrix} \Big| \Omega_{t-1} \sim N(0, H_t)$, ε_{St} and ε_{Ft} are independent bivariate random variables, $Var(\varepsilon_{st}) = \sigma_{ss}$, $Var(\varepsilon_{ft}) = \sigma_{ff}$ and $Cov(\varepsilon_{st}, \varepsilon_{ft}) = \sigma_{sf}$. The hedge ratio is therefore:

$$hr = Cov(\varepsilon_{St}, \varepsilon_{Ft} | \Omega_{t-1}) / Var(\varepsilon_{Ft} | \Omega_{t-1}) = \frac{\sigma_{Sf}}{\sigma_F^2} \tag{7}$$

where Ω_{t-1} is an information collection set which includes all the applicable information at time t-1. This hedge ratio is exactly the same as the hedge ratio obtained from OLS model.

However, it had been known that there are over-differencing the data and obscuring the long-run relationship between S_t and F_t in applying OLS. This leads to a downward bias in *hr*. In addition, because the risk in spot and futures markets is assumed constant over time, the minimum risk hedge ratio will be the same regardless of when the hedging is undertaken. As Bollerslev (1990) or Kroner and Sultan (1991) had shown, the risk-minimizing hedge ratio is actually time varying. Thus a conventional model cannot produce risk-minimizing hedge ratio.

In this study, after taking the possible structural breaks into account, the modified hedge model is therefore rewritten as:

$$S_t = \alpha_{10} + \alpha_{11} D_1 + \alpha_{12} D_2 + \alpha_{13} D_3 + \alpha_{14} D_4 + \alpha_{15} D_5 + \varepsilon_{St} \quad (8)$$

$$F_t = \alpha_{20} + \alpha_{21} D_1 + \alpha_{22} D_2 + \alpha_{23} D_3 + \alpha_{24} D_4 + \alpha_{25} D_5 + \varepsilon_{Ft} \quad (9)$$

where D_1, D_2, D_3, D_4, D_5 are the segments which created by the structural breaks. The hedge ratio is therefore switched as

$$hr_{VAR} = \frac{Cov(\varepsilon_{S,t}, \varepsilon_{F,t})}{Var(\varepsilon_{F,t})}. \quad (10)$$

The Data and the Empirical Results

The Data Description

Data used in this study include daily observations of spot prices of WTI (Cushing, Oklahoma FOB), U.S. crude oil production, and futures closing prices of NYMEX over the periods of 2002/1/2 ~ 2005/7/26. All data used are available through the U.S. Department of Energy[1].

Table 1 contains fundamental statistics for the spot price of WTI and futures price of WTI. Since index futures are the derivatives of the stock market index, statistics for the spot and futures markets would be closely correlated. From Table 1 we find that the means are indeed almost similar, but the futures market fluctuated a little more than the spot did. Both the spot and futures markets present fat tails. And through the Jarque-Beta normality test, we find that all the underlying indices reject the normality hypothesis for both.

Table 1: Basic Statistics for the Underlying Variables

statistics / variables	Samples	Mean	Standard Variation	Skew	Kurtosis	Min	Max	J.B.
Spot price of WTI	891	36.02	10.12	0.6445***	-0.6037***	18.02	61.24	75.2210***
Futures price ofWTI	891	35.99	10.17	0.6541***	-0.6019***	17.97	61.28	76.9884***

Note: 1. *, **, *** represents 10%, 5%, 1% significant level respectively. 2. J.B. represents the statistics of Jarque-Beta normality test.

The Empirical Results

In finding the multiple structural breaks, the futures prices of WTI crude oil is applied. It has been found that there are four structural breaks; those are on 2002/08/09, 2003/10/09, 2004/04/26, and 2005/01/13. In Figure 1, the vertical axial represents the futures prices of WTI crude oil, while the horizontal axial represents the sample period.

Benet (1992) studied foreign currency futures and suggested that using out-of-samples or ex-ante to evaluate hedge effectiveness would be more meaningful for investors. Hence, we take Benet's suggestion in evaluating the hedging performance. The estimated time expansion is 250 days, which we start on 2002/1/2 and end with 2005/7/26, the technique of a moving window is adopted. Figure 2 takes 250 days' time expansion, 10 days' moving window as an example.

Figure 1: The futures prices of WTI crude oil and the structural breaks

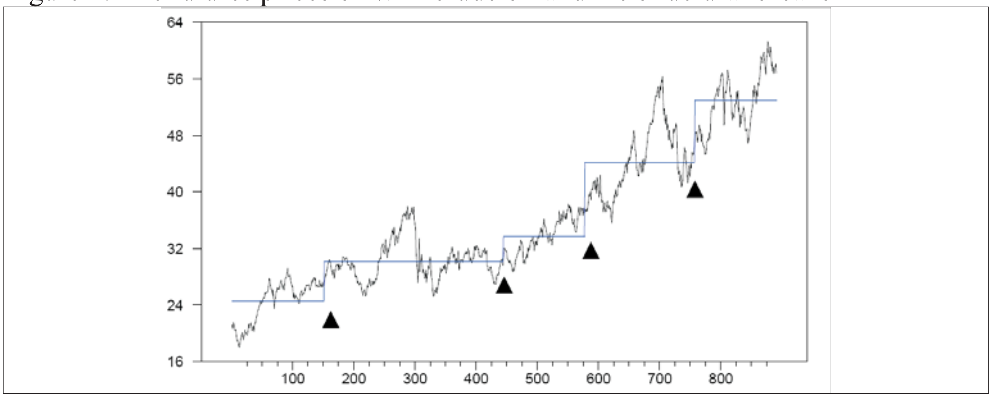
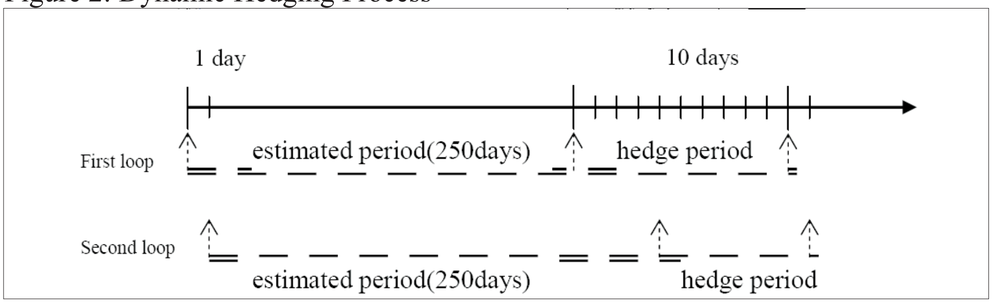


Figure 2: Dynamic Hedging Process



In order to obtain the out-of-sample empirical results, we use the latest information to estimate the next period's hedge ratio. Therefore, the entire hedging ratio we derived is a dynamic process instead of being a constant hedging ratio. Table

2 presents the hedging ratio and hedging performance. It is found that the hedge ratio is greater than one in standard restricted VAR hedge model, while the structural break hedge model has the ratio less than one. This implies that it is not necessary to take a 100% hedging of a futures position if structural breaks are considered. This can reduce the hedging cost for investors.

Table 2: The Hedging Ratio and the Hedging Performance

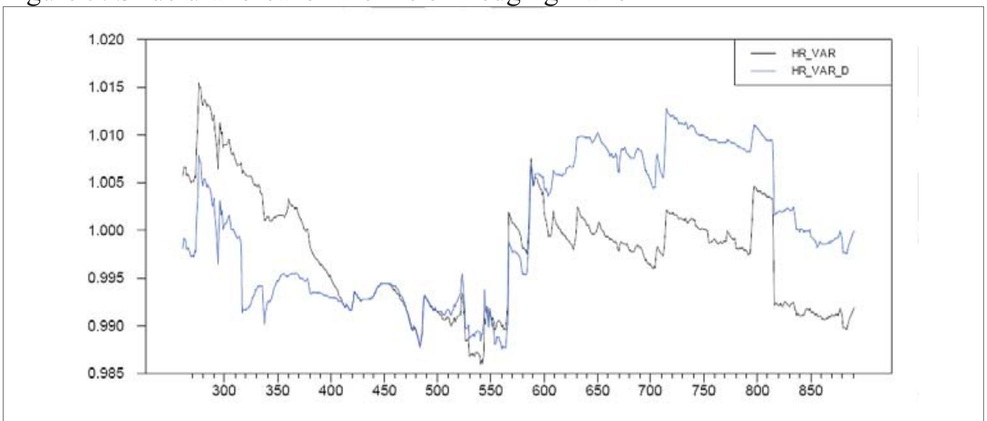
Statistics / Model	Hedge ratio	Hedge performance
Standard VAR hedge model	1.000023	0.86705
Structural break hedge model	0.997687	0.86763

Note: the hedge ratio and hedge performance (HEI) are represented as:

$$HE = \frac{Var(U) - Var(H)}{Var(U)} = \frac{\sigma_u^2 - \sigma_h^2}{\sigma_u^2}, HEI = \frac{\sum_{j=1}^{M} HE^{(j)}}{M}$$

In Table 2 we also found that the structural break hedge model has a better HEI performance, where the HEI is obtained by the method of a moving window. The result implies that a better optimal hedging ratio is not only time varying but also structural breaking, and is better than the fixed hedging ratio model which is obtained by the traditional regression.

Figure 3: Structural breaks in terms of Hedging Ratio



Conclusion

For the last decade, various hedging strategies have been rapidly grown because of development in techniques for measuring and managing financial risk. A number of literatures indicated that the unconditional distribution of financial assets is not only characterized by non-normal, fat tailed and high peaks but also influenced by a number of stylized facts. The relationship between the futures returns and their stock portfolio returns are therefore attracts portfolio manager's attention. If the relation had changed, the re-balance of holdings in futures-portfolio is thus worth of reconsidering.

This study addresses the computational advantage of the hedging ratio under the consideration of dynamic hedging algorithm which takes time varying and structural breakings into account without taking any further causality consideration among influential factors. Since in reality imperfect hedge are more likely being seen, the assumption of unconditional joint distribution of portfolio and futures returns is stable may not be applicable, Bai and Perron (2003) method is therefore applied in this study to determine the multiple breakings. We compare on out-of-sample hedging effectiveness of this structural break with restricted VAR hedging model against standard VAR hedge model, the improvement in hedging performance is clearly found. It has also found that it is not necessary to take a 100% hedging of a futures position if structural breaks are considered. The hedging cost for investors can therefore reduced extensively.

NOTE

[1] http://www.eia.doe.gov/emeu/international/petroleu.html

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