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The Estimation of Indian Railway Cost Function

G. Alivelu*

Abstract: Analysis of the cost structure of Indian Railways yields reasonable estimates of technical change, scale and substitution analysis. Technological advancements reduced fuel and labour services and increased capital services. Price elasticity of demand indicates that the own price elasticities of the inputs have negative sign. The cross price elasticities of demand are positive between labour and capital and so is the case with capital and fuel. The elasticity of substitution between the labour and fuel indicate complementarity and the elasticity between the labour and capital shows substitutability. The results show that fuel and capital are substitutes.

Keywords: railroads and other surface transportation, industrialization, manufacturing and service industries, choice of technology

JEL Classification: C01, C22, L92, O14

Introduction

Indian Railways' network spreads all over the country from North to South and from East to West. For the past one and a half century, the Indian Railways (IR) has been the principal mode of transport in India. The journey of Railways in Indian Sub-continent started modestly in 1853 with 34 km. From that modest level, it has grown into a gigantic proportion. Today, it is a strong network of 63,221 route km. On an average, it is moving 1.5 million tonnes of freight and 14 million passengers per day. In the last 150 years, the growth of IR is closely linked with the economic, agricultural and industrial development of the nation (Indian Railways Year Book 2003-04).

In recent times, huge investments have been embarked upon for technological up-gradation of IR. The main objective of this paper is to analyse the nature and rapidity of technological change on IR. This is done with the help of estimation of a cost function. Given this outline, an attempt is made to investigate substitution

* G. Alivelu is at the Centre for Economic and Social Studies, NO Campus, Begumpet, Hyderabad, India.

possibilities among the three factors of production, such as, labour, fuel and capital inputs.

Review of literature on the cost functions on railways is presented in section two of this paper. Section three specifies the econometric approach to measure the technical change. The methodology involved in the transcendental logarithmic (translog) model is discussed in section four. The estimates of the translog cost function and the results relating to the tests of various hypotheses about the model are discussed in section five. Further, this section also explores the effect of technical change and the substitution possibilities among the various factors of production are analysed in this section. The summary of the findings is presented in the last section of the chapter.

Review of Literature

Borts (1960) is of the opinion that railroads must be separated by size and region while evaluating their cost structure. R.K. Koshal (1970) examines the question of economies of scale in railway transport in India by estimating the long run cost curves through multiple regression analysis and concludes that Indian Railways are operating under constant returns to scale. Keeler (1974) makes use of the cost function of rail transportation and separates the passenger and output flows. Parameters of the short-run cost function are estimated by Keeler based on the assumption that the services of track and structures (T) are optimally distributed between the flows of freight and passenger traffic. This means that the costs of common physical plant are optimally distributed between the exogenously determined freight and passenger transportation services in the context of the common carrier character of the railways. Nevertheless, this is not an acceptable assumption. The long-run cost curve is estimated as an envelope of the joint short-run curves, achieved by minimizing the short-run total cost function with respect to track size and physical structures.

Inherent diversities of the costs and output flows of railways accentuate their character as a combined sum of many different groups. Their aggregation in one single figure of cost or output expected hides some significant details. Heterogeneity of the product flow and the consequent operations of railways, thus, necessitate some disaggregation of the cost-output relationship. This is tried in different studies by categorizing the total costs into numerous homogeneous or less heterogeneous elements. The classification may be operation-based as in Borts (1952), or it may be functional as in Meyer et al. (1964), Koshal (1970, 1971, 1972), Stenason and Bandeen (1965) or it may be output-based as in Borts (1960), Keeler (1971), Meyer (1958) and Harris (1977).

Varma (1983, 1988) estimates the cost function both at the aggregate and sub-aggregate cost levels over the period 1951-52 through 1978-79. He attempts temporal analysis of incremental and unit cost of passenger and freight output. His study shows that unit cost of passenger transportation indicates a significant tendency to increase over time. On the contrary, the unit cost of freight transportation shows a significant downward movement.

The criticism against the above study is that costs are not based on input classification in terms of labour, capital and intermediates. Because of this, no inference can be made with respect to the relative contribution of these factors to productivity.

In order to assess the relative contribution of the factors of production, it becomes essential to make use of an appropriate methodology for estimating the cost function. It is in this context that the econometric approach of the cost functions is discussed in the following section.

Econometric Approach

As far as the econometric approach is concerned, functional forms for the cost functions are specified a priori and estimates of parameters are established by making use of observations on outputs, inputs and input prices.

While designing a cost function to represent the nature of cost for any industry, the function must be valid. For the cost function to be valid, it must theoretically have the accurate characteristics of cost. In other words, it should be non-decreasing in factor prices, it should be homogeneous of degree one in factor prices, and it must be concave in factor prices and continuous in nature. If the cost function is not valid, the nature of cost for the industry is not well represented.

The most extensively used production functions to measure technical change are Cobb-Douglas (CD) and Constant Elasticity of Substitution (CES) forms. The major shortcoming of these forms is that they impose severe restrictions on the production structure with respect to elasticity of substitution, (i.e., CD assumes unitary elasticity and CES assumes constant elasticity), neutral technical change and homotheticity. Apart from this, both CD and CES forms impose separability restrictions and generally deal with two-input case. In order to overcome the restrictions associated with the above functions, flexible functional forms such as the Generalized Leontief (GL), Translog (TL), Quadratic (QF), Generalized BOX-COX (GBC), Generalized Cobb-Douglas (GCD), Generalized Square Root Quadratic (GSRQ) have been formulated and used in quantitative analysis of economic problems.

For the present study a brief discussion only of the three functional forms, namely, GL, TL and QF is made.

Given a multi-output production function with m outputs and n inputs at time T as in the following equation:

$$H(Y_{1T}, Y_{2T}, \ldots Y_{mT}; X_{iT}, X_{2T}, \ldots, X_{nT}; T) = 0 \tag{1}$$

There exists a unique dual cost function with a strictly convex input structure as in equation 2

$$C = g(Y_{1T}, Y_{2T}, \ldots Y_{mT}; P_{1T}, P_{2T}, \ldots, P_{nT}, T) \tag{2}$$

The flexible functional forms for the cost function specified in equation (2) are:
(i) The Generalized Leontief (GL) specified by Diewert (1971) as:

$$\begin{gathered} C = \sum_{j}^{m} \sum_{j'}^{m} \sum_{i}^{n} \sum_{i'}^{n} a_{jj'ii'T} (Y_j Y_{j'}, P_i P_{i'}, T)^{1/2} \\ j, j' = 1, \ldots, m \\ i, i' = 1, \ldots, n \end{gathered} \tag{3}$$

(ii) The Translog (TL) cost function formulated by Christensen, Jorgensen and Lau (1971), used by Burgess (1974) for a multi-product industry is:

$$\begin{aligned} \ln C = {} & \alpha_0 + \sum_{i}^{n} \alpha_i \ln P_i + \tfrac{1}{2} \sum_{i}^{n} \sum_{i}^{n} \alpha_{ii'} \ln P_{i'} \\ & + \sum_{j}^{m} \beta_j \ln Y_j + \tfrac{1}{2} \sum_{j}^{m} \sum_{j'}^{m} \beta_{jj'} \ln Y_j \ln Y_{j'} + \sum_{i}^{n} \sum_{j}^{m} \gamma_{ij} \ln P_i \ln Y_j \\ & + \delta_T T + \tfrac{1}{2} \delta_{TT} T^2 + \sum_{i}^{n} \delta_{Ti} T \ln P_i \\ & + \sum_{j}^{m} \delta_{Tj} \ln Y_j \end{aligned} \tag{4}$$

(iii) Quadratic cost function specified by Lau (1974) is:

$$\begin{aligned} C = {} & \alpha_0 + \sum_{i}^{n} \alpha_i P_i + \tfrac{1}{2} \sum_{i}^{n} \sum_{i'}^{n} a_{ii'} P_i P_{i'} \\ & + \sum_{j}^{m} \beta_j Y_j + \tfrac{1}{2} \sum_{j}^{m} \sum_{j'}^{m} \beta_{jj'} Y_j Y_{j'} \\ & + \sum_{i}^{n} \sum_{j}^{m} \gamma_{ij} P_i Y_j + \delta_T T + \tfrac{1}{2} \delta_{TT} T^2 \\ & + \sum_{i}^{n} \delta_{Ti} T P_i + \sum_{j}^{m} \delta_{Tj} T Y_j \end{aligned} \tag{5}$$

Lau (1986) projected that any functional form needs to be evaluated based on the following criteria:

(i) Theoretical consistency, (ii) Domain of applicability, (iii) Flexibility, (iv) Computational facility and factual conformity. It is observed that since any functional form does not meet all the above conditions, there is a necessity to make trade-offs. An attempt is made to discuss broadly whether the three functional forms given above meet these criteria (Lau, 1986). The chief prerequisite for the use of duality theory is that linear homogeneity in input prices for all feasible prices and output levels are to be satisfied. Both GL and TL forms satisfy this condition on imposition of suitable restrictions. On the other hand, QF does not satisfy this restriction. If homogeneity is imposed, QF does not satisfy the flexibility condition also. Hence, it is argued that QF is a restrictive form for a multi-product cost function. Among the GL and TL functions, Lau (1986) has shown that TL function possesses a larger domain of theoretical consistency than that of GL function. However, both functions may not be well behaved internationally, except under inflexible conditions.

Given a flexible functional form, it is advantageous that the number of parameters to be estimated is small. With the imposition of linear homogeneity restrictions, the TL function with m outputs and n inputs has $[m(m+1)]/2+[n(n+1)]/2+mn$ parameters. The QF has m+n+1 parameters more than TL. GL has $[m(m+1)n(n+1)]/4$ parameters when constant returns to scale is imposed. Except for the two outputs, two input case, the parameters to be estimated in GL is greater than in TL. If the assumption of constant returns to scale is relaxed, then parameters to be estimated in GL will further increase.

Therefore, the translog functional form is theoretically consistent, flexible, economical in parameters and computationally tractable. It is because of all the above characteristics that TL function is considered superior over QF and GL forms. The significant feature of this function is that it does not make a priori assumptions with respect to separability, substitution and transformation, returns to scale, homogeneity and homotheticity of input structure and neutrality of technical change.

Having decided on for the translog functional form, the choice between production and cost function is a subject of analytical objective and statistical expediency. The major concerns are to see whether:

- The level of output should be considered endogenous (production function) or exogenous cost function)
- Factor inputs are exogenous (production function) or factor prices are exogenous (cost function)

In case, cost function is adopted, duality theory can be exploited so that no restrictions are imposed on returns to scale. The derivative of cost function with respect to factor price yields the derived demand for input, which implies a set of cost share equations (Sheppard's Lemma). Joint estimation of cost share equations coupled with the cost function results in the increase in degrees of freedom without any addition to the number of parameters to be estimated. In addition to this, cost shares add up to unity, which does not impose any restriction on the returns to scale. On the other hand, if production functions along with value shares are estimated jointly with value shares summing up to unity, then the implicit assumption of constant returns to scale is made.

The Allen-Uzawa partial elasticities of substitution can be easily obtained with respect to TL cost function estimates. With respect to the production function, calculation of elasticities of substitution involves the inversion of certain matrices after the estimation of production function coefficients. This increases the estimation errors and decreases the statistical precision of the computed elasticity of substitution (Binswanger, 1974a).

The Translog Model

The purpose of this section is to present the methodology for the analysis of cost structure of Indian Railways. The nature and rapidity of technical change and elasticities of substitution are examined by estimating a flexible form of the cost function, namely, the translog (TL) form with minimum restrictions discussed in the earlier section.

The TL cost function for the present study is estimated as follows:

$$C = g(Y_1, Y_2, P_L, P_F, P_K, T) \tag{6}$$

Where C is total cost, Y_1 is freight output, Y_2 is passenger output, P_L is the price of labour, P_F is price of fuel, and P_K is price of capital. Most commonly used indicator of technology T is the time trend. In the present study, an explicit technology index is considered instead of a general indicator in terms of a trend variable. This has a unique application in the case of Indian Railways, which has been characterized by technological advances largely on broad gauge track. Both advanced rolling stock and modern equipment have been installed on broad gauge network. It is likely that this has a positive impact on freight output. Technology index is taken to be proportion of freight-ton kilometres carried by diesel and electric engines on broad gauge track to total freight-ton kilometres carried. This facilitates to capture the explicit characteristics of technical change in Indian Railways. The cost function is then modified as:

$$C = G(Y_1 h(A), Y_2, P_L, P_F, P_K) \quad (7)$$

Where '*h*' is augmentation function such that for any given freight output, an increase in technology index *A* will result in decline in total costs. $Y_1^* = Y_1 h(A)$ represents augmented output. The exponential function *h (A)* is specified as follows:

$$h(A) = e^{\lambda A}$$

Thus, the augmented freight output is represented by '$e^{\lambda A}$' and an increase in '*A*' will result in the decrease in costs. This kind of representation is called as model 1. This approach to model technology index clearly in the cost function was followed by Denny, Fuss and Waverman (1981).

Then the cost function in (7) can be written as

$$C = G(Y_1 e^{\lambda A}, Y_2, P_L, P_F, P_K) \quad (8)$$

The translog form of model 1 as

$$\begin{aligned}
\ln C &= \alpha_0 + \alpha_L \ln P_L + \alpha_K \ln P_K + \alpha_F \ln P_F + \tfrac{1}{2}\alpha LL(\ln P_L)^2 \\
&+ \alpha_{LK} \ln P_L \ln P_K + \alpha_{LF} \ln P_L \ln P_F + \tfrac{1}{2}\alpha_{KK}(\ln P_K)^2 \\
&+ \alpha_{KF} \ln P_K \ln P_F + \tfrac{1}{2}\alpha_{FF}(\ln P_F)^2 + \beta_1 \ln Y_1 \\
&+ \beta_2 \ln Y_2 + \tfrac{1}{2}\beta_{11}(\ln Y_1)^2 + \beta_{12} \ln Y_1 \ln Y_2 \\
&+ \tfrac{1}{2}\beta_{22}(\ln Y_2)^2 + \gamma_{L1} \ln Y_1 \ln P_L + \gamma_{K1} \ln Y_1 \ln P_K \\
&+ \gamma_{F1} \ln Y_1 \ln P_F + (\beta_2\lambda)A + \tfrac{1}{2}(\beta_{22}\lambda^2)A^2 + (\beta_{12}\lambda)A \ln Y_1 \\
&+ (\beta_{22}\lambda)\mathrm{A} \ln Y_2 + (\gamma_{L2}\lambda)\mathrm{A} \ln P_L + (\gamma_{K2}\lambda)A \ln P_K \\
&+ (\gamma_{I2}\lambda)A \ln P_F
\end{aligned} \quad (9)$$

Symmetry and linear homogeneity in price impose the following restrictions:

$$\begin{aligned}
&\alpha_L + \alpha_K + \alpha_F = 1 \\
&\alpha_{LL} + \alpha_{LK} + \alpha_{LF} = 0 \\
&\alpha_{KL} + \alpha_{KK} + \alpha_{KF} = 0 \\
&\alpha_{FL} + \alpha_{FK} + \alpha_{FF} = 0
\end{aligned} \quad (10)$$

$$\gamma_{LI} + \gamma_{K1} + \gamma_{F1} = 0$$
$$\gamma_{L2} + \gamma_{K2} + \gamma_{F2} = 0$$

Since the number of parameters estimated is large, we do not attempt a direct estimation of (9). Instead, Shephard's Lemma is used to obtain the cost share equations. Since the cost shares sum to unity at every observation, one of the share equations can be deleted. For the present study, the share equation for fuel input is deleted. Thus, we rewrite the labour and capital share equations as:

$$\begin{aligned} W_L &= \alpha_1 + \alpha_{LL} \ln(P_L/P_F) + \alpha_{LK} \ln(P_K/P_F) + \gamma_{LI} \ln Y_1 + \gamma_{L2} \ln Y_2 + (\gamma_{L2}\lambda)A \\ W_K &= \alpha_K + \alpha_{KL} \ln(P_L/P_F)\alpha_{KK} \ln(P_K/P_F) + \gamma_{KI} \ln Y_1 + \gamma_{K2} \ln Y_2 + (\gamma_{K2}\lambda)A \end{aligned} \quad (11)$$

The translog cost function in (9) and the two share equations in (11) are treated as a multivariate Seemingly Unrelated Regression (SUR) System and estimated by Zellner's iterative method for SUR models, with restrictions in (10) imposed. The computation is carried out using the micro version of the Stata software package.

Technical change is measured by the rate of cost diminution over time, $-\partial \ln C/\partial T = \varepsilon_{CT}$. This model is referred to as Model 2. For model 2, the translog form with a trend variable T is given by

$$\begin{aligned} \ln C &= \alpha_0 + \alpha_L \ln P_L + \alpha_K \ln P_K + \alpha_I \ln P_F + \tfrac{1}{2}\alpha_{LL} (\ln P_L)^2 \\ &+\alpha_{LK} \ln P_L \ln P_K + \alpha_{LF} \ln P_L \ln P_F + \tfrac{1}{2}\alpha_{KK} (\ln P_K)^2 \\ &+\alpha_{KF} \ln P_K \ln P_F + \tfrac{1}{2}\alpha_{FF} (\ln P_F)^2 + \beta_1 \ln Y_1 \\ &+\beta_2 \ln Y_2 + \tfrac{1}{2}\beta_{11} (\ln Y_1)^2 + \beta_{12} \ln Y_1 \ln Y_2 \\ &+\tfrac{1}{2}\beta_{22} (\ln Y_2)^2 + \gamma_{L1} \ln Y_1 \ln P_L + \gamma_{K1} \ln Y_1 \ln P_K \\ &+\gamma_{F1} \ln Y_1 \ln P_F + \gamma_{I2} \ln Y_2 \ln P_L + \gamma_{K2} \ln Y_2 \ln P_K \\ &+\gamma_{F2} \ln Y_2 \ln P_F + \delta_T T + \tfrac{1}{2}\delta_{TT} T^2 \\ &+\delta_{TL} T \ln P_L + \delta_{TK} T \ln P_K + \delta_{TF} T \ln P_F \\ &+\delta_{TI} T \ln Y_1 + \delta_{T2} T \ln Y_2 \end{aligned} \quad (12)$$

Symmetry and linear homogeneity in price restrictions are imposed as in (10).

The share equations for model 2 are given by:

$$\begin{aligned} W_L &= \alpha_L + \alpha_{LL} \ln(P_L/P_F) + \alpha_{LK} \ln(P_K/P_F) + \gamma_{L1} \ln Y_1 + \gamma_{L2} \ln Y_2 + \delta_{TL} T \\ W_K &= \alpha_K + \alpha_{KL} \ln(P_L/P_F) + \alpha_{KK} \ln(P_K/P_F) + \gamma_{K1} \ln Y_1 + \gamma_{K2} \ln Y_2 + \delta_{TK} T \end{aligned} \quad (13)$$

The tests carried out on this general model are given in table 1. The cost function is homothetic[1] if it can be written as a separable function of factor prices and outputs. This implies that scale economies can be defined independently of factor proportions. Homogeneity[2] is tested as a hypothesis conditional on the acceptance of homotheticity. If homotheticity is rejected, then homogeneity is automatically rejected. If homogeneity is rejected, it can be inferred that scale economies are important in cost and production structure. Under constant returns to scale, the cost flexibility is equal to unity. That is, economies of scale are independent of the firm's scale of operation, relative changes in input prices, etc. The hypothesis that technical progress does not have a significant impact costs implies that $\lambda = 0$.

Table 1: Hypotheses Tests on the TL Cost Function

Null Hypothesis	Test Restrictions
1. Homotheticity	$\gamma_{L1} = \gamma_{K1} = \gamma_{F1} = \gamma_{L2} = \gamma_{K2} = \gamma_{F2} = 0$
2. Homogeneity	$\gamma_{L1} = \gamma_{K1} = \gamma_{F1} = \gamma_{L2} = \gamma_{K2} = \gamma_{F2} = \beta_{11} = \beta_{12} = \beta_{22} = 0$
3. Constant returns to Scale	Restrictions in (2) and $\beta_1 + \beta_2 = 1$
4. Absence of technical change	$\lambda = 0$
5. Strong separability of all inputs	$\alpha_{LL} = \alpha_{LK} = \alpha_{LF} = \alpha_{KK} = \alpha_{KF} = \alpha_{FF} = 0$
6. Strong separability of all outputs	$\beta_{11} = \beta_{12} = \beta_{22} = 0$
7. Global separability	$\alpha_{LL} = \alpha_{LK} = \alpha_{LF} = \alpha_{KK} = \alpha_{KF} = \alpha_{FF} = \beta_{11} = \beta_{12} = \beta_{22} = 0$

The possible levels of input and output aggregation can be examined by testing the separability conditions. For example, if the hypothesis of strong separability of outputs is accepted, it implies that the industry output can be taken as consistent aggregate output even though its actual components may be varied in nature. On the other hand, if the hypothesis is rejected, then it necessitates a model specification, keeping in view, the multiple-output nature of the industry. In case, both homogeneity and strong separability restrictions hold, then the cost structure is better represented by Cobb-Douglas technology. The Likelihood Ratio (LR) statistic, $-2\log_e$ (LR), is calculated. Under the null hypothesis, it is asymptotically distributed as χ^2 where q is the number of independent restrictions implied by the null hypothesis.

In terms of the cost function (9), the j^{th} output cost elasticity is

$$\begin{aligned} \varepsilon_{CYj} &= \delta \ln C / \delta \ln Y_j \\ &= \beta_j + \sum_{j'} \beta_{jj'} \ln Y_j + \sum_i \gamma_{ij} \ln P_i \end{aligned} \tag{14}$$

Having estimated the parameters of the cost function, the rate of technical progress for each year is estimated. In terms of the cost function (9), the rate of technological progress is given as follows:

$$-\dot{B} = \partial \ln C / \partial \ln A \dot{A} = \varepsilon_{CA} \dot{A} \quad (15)$$

Where

$$\varepsilon_{CA} = \partial \ln C / \partial \ln Y_1^* X \partial \ln Y_1^* / \partial \ln A$$
$$= (\lambda \varepsilon_{CY1*}) A$$

One of the major advantages of the *TL* cost function is that it allows the elasticity of substitution between any pair of factors of production to be variable. The Allen-Uzawa partial elasticity of substitution ($\sigma_{ii'}$) between two factors *i* and *i'* (keeping price of the third factor and output levels constant) are given in terms of factor cost shares and estimated parameters of the *TL* cost function. Pairs of inputs are classified as substitutes or complements based on the signs of corresponding cross elasticities. Positive values signify that the pairs are substitutes and negative values indicate that the pair of inputs is complements. These elasticities are not constrained to be maximum but differ with the values of cost shares.

Data used for estimation of models 1 and 2 are time series on outputs, input prices, and total cost and cost shares running from 1982-83 through 2002-03. Indicator of technical change and the cost shares are expressed in terms of proportions. The remaining variables are expressed in terms of index numbers with base 1981-82 as 1.0. The most important benefit of this scaling procedure is that it will make the estimates of the translog model independent of units of measurement.

To obtain the total cost figures, the data on actual expenditure on labour and intermediate inputs is utilized and the imputed capital cost is worked as $K(r+\gamma)$ where K is the total capital stock, γ is the rate of dividend payable by the railways to the government, r is the replacement rate. As a consequence of regulation in this sector, the government makes available capital-at-charge with very low rate of dividend, which is significantly lower than the long-term interest rates in the economy. Because of this, it is likely that capital investments in Indian Railways are higher than otherwise, owing to cheaper capital obtainable to them.

Estimates of Translog Cost Function for Indian Railways

The present study takes into consideration the cross-section data for nine zones on Indian Railways, namely, Central Railway (CR), Eastern Railway (ER), North

Eastern Railway (NER) Northeast Frontier Railway (NFR), Northern Railway (NR), South Central Railway (SCR), South Eastern Railway (SER), Southern Railway (SR), and Western Railway (WR).

Data Sources and List of Variables

The methodology discussed in the previous sections represents a multi-product production or cost function with:

- With two outputs, namely, passenger and freight outputs
- Three inputs, namely, labour, fuel and capital
- Three input prices
- Total costs

The implementation of methodology necessitates time series data on all the above variables for the period 1981-82 – 2002-03.

Data Sources

Data is collected from various publications of the Railway Board, Ministry of Railways, Government of India. Majority of the data is taken from Annual Statistical Statements published by the Railway Board. Data pertaining to outputs, costs, labour and fuel along with their prices is taken from these statements. The data on capital input is taken from Appropriation Accounts (Annexure G), Works, Machinery and Rolling Stock Program published by the Railway Board, Report of the Working Group on Depreciation, Report on Capital Restructuring on Indian Railways. The data on capital price is collected from the Index Numbers of Wholesale Price Indices-Monthly Bulletins, Ministry of Industry, Chandhok (1990) and Year Books, Indian Railways. In addition to the above publications, various reports like General Manager's Annual reports, Rail Convention Committee Reports, Lok Sabha Reports, Report on Currency and Finance published by the Reserve Bank of India are also used.

List of Variables

The service provided by railways is expressed in terms of transportation of a unit of freight or a passenger over some specified distance. Hence, the output measurement has to take into consideration both weight and distance components. In order to measure passenger output and freight output, passenger kilometres and freight ton-kilometres per unit of time (for a period of one year), are taken as basic units of measurement. Passenger kilometres are defined as the total number of passengers multiplied by the average distance over which they travel. Likewise, freight ton-kilometres represents the number of tons of freight carried multiplied by the average distance over which it is transported.

Nevertheless, these two services are not strictly homogeneous. For example, with respect to the freight output, the kind of engines and wagons required for a bulk commodity, such as, cement/food grains might not be the same for commodities like iron ore/coal. As far as the passenger output is concerned, different cost and quality features characterize each of the passenger service. For example, the service provided by an air-conditioned coach is not the same as that provided in a second-class or a general class service. As a consequence of this, there is a variation in the cost characteristics of these individual services

Three inputs, labour, fuel and capital are taken into consideration. Labour is categorized into managerial group called as Group A and B, skilled labour represented by Group C and semi-skilled labour represented by Group D. The second component, fuel consists of coal, diesel, petrol, kerosene and electricity. Capital as an input consists of structural engineering works (track, stations, bridges, land etc.), rolling stock comprising of locomotives, coaches and wagons. The third category of capital input, equipment, consists of electrical and machinery equipment. Capital is taken to be fixed capital stock arrived at by making use of Perpetual Inventory Method. Analogous to the input data, price data for all the above input categories is collected.

The main feature of this study is to exploit all the available data sources in order to construct time series on output, input and input price variables for the period 1981-82 through 2002-03. The consistent data series for passenger and freight outputs, labour, capital and fuel, wage rate, rental and fuel input price are taken into consideration.

The two outputs are Y_1 (Freight) and Y_2 (passenger). The inputs are labour (X_1), fuel (X_2), capital (X_3). The price of labour is given as P_L, the price of fuel as P_F and the price of capital as P_k.

The exercise of construction of variables has been undertaken only for inputs. This exercise is not carried for the outputs as only the physical quantities of outputs are considered. In order to estimate the cost function total expenses are divided into three broad categories namely, labour (L), fuel (F) and capital (K). The price index of labour is derived form by dividing the total expenditure on labour by the total number of labour. The category fuel includes all items like diesel, electricity, and insignificant quantities like coal, wood and kerosene. A weighted price index for fuel is obtained by dividing the expenditure on fuel by total fuel consumed. All the elements of fuel are converted into coal equivalents. (With respect to petrol, diesel oil, kerosene the converted coal equivalent figures are already available in the Annual Statistical statements. With regard to electricity, in order to convert them into coal equivalents, the procedure as given by Nachane, Manohar Rao et. al. (1981, 1982) was followed. The sum of these two broad categories was utilized for arriving at the total fuel consumed in terms of physical quantities). Using the respective wholesale price indices weighted price index is derived for all these items. The Perpetual Inventory Method is used to create the real capital stock (Christensen and Jorgenson 1969)

The methodology for the construction of real capital asset is given as follows:

Let G_{n60} be the benchmark capital of n^{th} asset in year 1960-61, (where n = 1, 2, 3) in 1981-82 prices. B_{nT} represents the book value figures for n^{th} asset in T^{th} year. This value is in gross terms. Gross additions to assets in T^{th} year from n^{th} category of capital asset is

$$A_{nT} = B_{n(T)} - B_{n(T-1)} \qquad n = 1,2,3$$

The gross investment at 1981-82 prices is obtained as follows:

$$I_{nT} = A_{nT} / q_{nT}$$

Where, q_{nT} is the asset price of n^{th} asset in T^{th} year with base 1981-82. Gross capital stock at constant prices is then,

$$K_{nT} = G_{n60} + \sum_{c=1960-61}^{T} I_{nc}$$

Where, I_{nc} is the gross investment in year c.

Aggregate capital stock is then written as:

$$K_T = \sum_{n=1}^{3} K_{nT} \qquad n = 1,2,3$$

The Tornquist index of aggregate capital stock K_{T*} is

$$\dot{K}_{T*} = \ln K(T) - \ln K(T-1)$$

The service price of capital given by Christensen and Jorgenson (1969) is used to arrive at the rental values of the capital assets.

By making use of the above stated outputs and inputs, we estimate the translog cost function for Indian Railways which is represented in table 2. The translog cost function is estimated for two models. In model 1, an explicit technology indicator is introduced, while in model 2, the conventional trend variable is used.

Table 2: Translog Cost Function for Indian Railways

Variable	Coefficient	Model 1 Estimate	Model 2 Estimate
Constant	α_0	7.37 (4.12)	10.62 (4.77)
Price of Labour (LPL)	α_L	3.59** (1.07)	3.61 (1.07)
Price of Fuel (LPF)	α_F	-2.79 (1.16)	-2.81 (1.40)
Price of capital (LPK)	α_K	0.204 (1.15)	0.19 (1.65)
½ (LPL.LPL)	α_{LL}	0.27 (0.043)	-0.32** (0.11)
LPL.LPK	α_{LK}	0.05 (0.12)	0.71 (0.18)
LPL.LPF	α_{LF}	-0.31** (0.12)	-0.40** (0.19)
½(LPK.LPK)	α_{KK}	-0.061 (0.09)	-0.74 (0.11)
LPK.LPF	α_{KF}	0.01 (0.16)	0.06 (0.18)
½(LPF.LPF)	α_{FF}	0.30 (0.10)	0.36 (0.11)
Freight output (LY1)	β_1	0.009** (0.01)	0.009** (0.001)
Pass. Output (LY2)	β_2	0.011** (0.01)	0.013** (0.001)
½(LY1.LY1)	β_{11}	0.008 (0.026)	-0.04** (0.03)
LY1.LY2	β_{12}	0.06 (0.036)	0.11 (0.03)
½(LY2.LY2)	β_{22}	-0.06** (0.02)	0.05* (0.02)
LY1.LPL	γ_{L1}	-0.01 (0.07)	-0.08 (0.05)
LY1.LPK	γ_{K1}	-0.06 (0.04)	-0.04 (0.06)
LY1.LPF	γ_{F1}	0.07 (0.08)	0.12 (0.07)
LY2.LPL	γ_{L2}	0.19** (0.06)	0.08 (0.05)
LY2.LPK	γ_{K2}	-0.04 (0.02)	0.01 (0.05)
LY2.LPF	γ_{F2}	-0.15 (0.06)	-0.09 (0.06)
Technical change indicator	λ	-0.04 (0.032)	-0.12 (0.12)

A LY1	λ_{Y1}	-0.003* (0.001)	0.006 (0.002)
A LY2	λ_{Y2}	0.002* (0.001)	-0.005 (0.001)
½(LA.LA)	$\lambda\lambda$	0.00009** (0.00003)	0.002 (0.002)
ALPL	λ_L	-0.009** (0.002)	-0.08** (0.02)
ALPF	λ_F	-0.0003 (0.003)	0.03 (0.03)
ALPK	λ_K	0.004 (0.005)	0.008 (0.02)

Note: Figures in parenthesis indicate standard errors, Model 1 takes into consideration augmented technical change, Model 2 considers 't' as the trend variable, * Significant at 5% level of significance, ** Significant at 1% level of significance.

(i) Model 1

Table 2 presents the estimates of the translog cost function. Adjusted R^2 is 0.9851 for the cost function, 0.77 for the labour share equation and 0.81 for the capital share equation. In terms of input prices (*L*, *F* and *K*) the estimated equation reveals that the prices of labour and capital have a positive association with cost as expected. The monotonicity condition of the well-behaved cost function is satisfied with respect to the labour and capital. As far as the price of the fuel is concerned the coefficient has negative sign. This may be, perhaps, due to the presence of x-inefficiencies. However, the fitted regression coefficients of the first order factor prices added up to one. This implies that a one percent increase in prices resulted in one percent increase in costs, thus, satisfying the first property of cost. The estimated cost function is non-decreasing in output.

The coefficient of technology has a negative sign implying that the technology has resulted in cost efficiency as per the theoretical expectations. The outputs have significant 't' ratios, while technology coefficient seems to be less significant.

As far as the second order coefficients are concerned, passenger output – passenger output, passenger output – labour, technology – freight output, technology – passenger output, technology – technology are statistically significant.

(ii) Model 2

The last column of table 2 presents the estimates of the translog cost function for model 2. Adjusted R^2 is 0.9862 for the cost function, 0.71 for the labour share equation and 0.84 for the capital share equation. In terms of input prices (*L*, *F* and *K*) the estimated equation reveals that the price of labour and price of cost have a positive association. The monotonicity condition of the well-behaved cost function is satisfied with respect to labour and capital. As in model 1, fuel has a negative association with cost. The fitted cost functions are found to be increasing in output.

The coefficient of technology indicated by the trend variable has a negative sign implying that the technology has resulted in the cost efficiency as per the theoretical expectations. The outputs have significant ‘t’ ratios, while technology coefficient seems to be less significant.

In the case of second order coefficients, labour – labour, labour – fuel, freight output-freight output, passenger output-passenger output, technology – labour are statistically significant.

Next, we test the model for homotheticity, separability and absence of technical change. Various hypotheses are given in table 3. Under the null hypothesis, likelihood ratio statistic *[-2ln (LR)]* is distributed asymptotically as chi-square with degrees of freedom equal to the number of independent restrictions imposed by null hypothesis. This statistic is worked out to test each of the null hypothesis.

Table 3: Results of LR test

Null Hypothesis	LR Statistic	95% significance level	Hypothesis accepted/rejected
1. Homotheticity $\gamma_{L1} = \gamma_{K1} = \gamma_{I1} = \gamma_{L2} = \gamma_{K2} = \gamma_{I2} = 0$	10.3	Significant	Hypothesis Rejected
2. Homogeneity $\gamma_{L1} = \gamma_{K1} = \gamma_{I1} = \gamma_{L2} = \gamma_{K2} = \gamma_{I2} = \beta_{11} = \beta_{12} = \beta_{22} = 0$	20.13	Significant	Hypothesis Rejected
3. Constant returns to Scale Restrictions in (2) and $\beta_1 + \beta_2 = 1$	25.73	Significant	Hypothesis Rejected
4. Absence of technical change $\lambda = 0$	13.89	Significant	Hypothesis Rejected
5. Strong separability of all inputs $\alpha_{LL} = \alpha_{LK} = \alpha_{LI} = \alpha_{KK} = \alpha_{KI} = \alpha_{II} = 0$	40.27	Significant	Hypothesis Rejected
6. Strong separability of all outputs $\beta_{11} = \beta_{12} = \beta_{22} = 0$	40.32	Significant	Hypothesis Rejected
7. Global separability $\alpha_{LL} = \alpha_{LK} = \alpha_{LI} = \alpha_{KK} = \alpha_{KI} = \alpha_{II} = \beta_{11} = \beta_{12} = \beta_{22} = 0$	40.12	Significant	Hypothesis Rejected
8. Homogeneity and separability restrictions in (2) and (6)	57.13	Significant	Hypothesis Rejected

The cost function for Indian Railways illustrates that scale economies cannot be defined independent of factor proportions. That is hypothesis 1 in table 1, that the cost structure is homothetic has been rejected. Since homogeneity is tested conditional on the acceptance of homotheticity, the former is automatically rejected. The implication of this is that scale economies are important in the cost and production structure of railways. Moreover, economies of scale are dependent on the railways scale of operation, relative changes in input prices etc. In other words, hypothesis 3 of constant returns to scale is not tenable.

Both input separability and output separability assumptions are restrictive as is evident from hypothesis 4 and 5, suggesting the restrictive functional forms like Cobb-Douglas or CES functions may not correctly represent the rail technology for Indian Railways. Since the hypothesis of separability of inputs and outputs are not accepted, global separability of inputs and outputs are not accepted, global separability (hypothesis 6) is automatically rejected. Hypothesis 7 that imposes both separability and homogeneity restrictions is also rejected. The absence of output augmenting technical change (hypothesis 8) is also rejected.

Thus results given in table 3 show that the hypotheses of homotheticity, separability, absence of technical change and Cobb-Douglas production/cost function, as an explanation of the structure of costs in Indian Railways are all rejected.

Technical Change – An Analysis

The analysis of technical change and of the factors contributing to it is discussed as follows: Elasticity of cost with respect to the technology index is given as follows:

$$\begin{aligned}\varepsilon_{CA} &= (\lambda\beta_2)A + (\lambda\beta_{12})A\ln Y_1 + (\lambda\beta_{22})A\ln Y_2 + (\lambda^2\beta_{22})A^2 \\ &+ (\lambda\gamma_{L2})A\ln P_1 + (\lambda\gamma_{K2})A\ln P_K + (\lambda\gamma_{I2})A\ln P_I\end{aligned} \tag{16}$$

In this section, the elasticity of cost with respect to technology index is estimated, annual rate of cost reduction and the contribution of the three inputs. As a next step, we attempt to compute partial elasticities of substitution between labour, fuel and capital, elasticities of substitution and also the own and cross price elasticities of demand for both the models.

Table 4 presents the elasticities with respect to technology index for the entire period. Decomposition in terms of contributions of labour, capital and fuel i.e. the last three terms on the RHS of (16) is also indicated in the table. The cost saving effects of technical change is clearly brought out by the foregoing results. This is corroborated by results relating to model 2, which is the substitute formulation of the translog cost function incorporating a trend variable. Next, the contributions of the three inputs, namely labour, capital, fuel to technical change are computed. That is for each of the i^{th} input; this effect can be gauged from 5th, 6th and 7th terms on the RHS of equation (17).

Table 4: Elasticity of Cost w.r.t. Technology Index and the annual rate of cost reduction and the contributions of the three inputs. 1981-82 – 2002-03

Elasticities	Model 1	Model 2
Labour	-0.9057	-0.2293
Fuel	-1.3282	-0.1371
Capital	2.2174	0.5295
$\varepsilon_{CA} = \partial \ln C / \partial \ln A$	-2.1175	-0.4864
$-B = \partial \ln C / \partial T$	-0.012	-0.609

For the period as a whole, one percent increase in technology is associated with 1.32 percent decline in fuel, decrease in labour by 0.90 percent and an increase in capital by 2.22 percent in model 1. In other words, the nature of technical change is such that it reduced fuel and labour services and increased capital services. This seems reasonable as most of the technical changes are capital intensive and skilled labour is to be used to adopt the technology. As far as the trend variable is concerned, a one percent increase in technology resulted in decline in labour by 0.23 percent, in fuel by 0.14 percent and increase in capital by 0.53 percent. The most distinctive feature of the above table is that the rate of cost diminution as a consequence of technical change increased over the years.

Further, we evaluate the partial elasticities of substitution between labour, fuel and capital for model 1 and model 2.

Table 5: Partial elasticities of substitution between Labour, Fuel and Capital

	Labour		Fuel		Capital	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
Labor	-0.82	-0.24	-0.05	-0.07	0.004	0.03
Fuel			-0.027	-0.04		
Capital					-0.06	-0.75

The results on partial elasticities of substitution between labour, fuel and capital are represented in table 5. The economic theory proposes that all the own price elasticities should have a negative sign. In the present study this has been accurate. The own price elasticity for the labour is –0.82 in model 1 and –0.24 in model 2. The own price elasticity for the fuel stood at –0.27 in model 1 and –0.04 in model 2. Similarly, the own price elasticity of capital is –0.06 in model 1 and –0.74 in model 2.

Next, we turn to the evaluation of the substitution elasticities for both the models. The results are presented in table 6

Table 6: Elasticities of Substitution – Model Wise

		Model 1	Model 2
σ_{LL}	Labour-Labour	-0.8226	-0.2366
σ_{FF}	Fuel-Fuel	-0.2706	-0.0431
σ_{KK}	Capital-Capital	-0.0597	-0.7458
σ_{LK}	Labour-Capital	0.0040	0.0325
σ_{LF}	Labour-Fuel	-0.0503	-0.0706
σ_{FK}	Fuel-Capital	0.0179	0.0097

If the value of the elasticity of substitution σ_{ij} is positive then the inputs *i* and *j* are substitutes and if they are negative then the two inputs are complements to each other. Accordingly it is observed that over the years on IR, the elasticity of substitution between labour and fuel showed a value of –0.05 in model 1 and –0.07 in model 2, indicating complementarity between the two inputs in both the models. With respect to the elasticity of substitution between labour and capital it was 0.004 in model 1 and 0.0325 in model 2. This implies that the elasticity between labour and capital are substitutes in both the models. Like-wise, the elasticity of substitution between fuel and capital is 0.0179 and 0.0097 in models 1and 2 respectively, indicating that both the inputs are substitutes. With respect to the elasticity of substitution between fuel and capital the trend has indicated a different result in both the models that is there is not much stability.

Results of the own price elasticity of demand and cross price elasticites of demand are represented in table 7.

Table 7: Own and Cross Price Elasticities of Demand

		Model 1	Model 2
P_{LL}	Labour	-0.6124	-0.1762
P_{FF}	Fuel	-0.0602	-0.0096
P_{KK}	Capital	-0.0020	-0.0247
P_{LK}	Labour-Capital	0.0001	0.0008
P_{LF}	Labour-Fuel	-0.0083	-0.0117
P_{FK}	Fuel-Capital	0.0001	0.0001

The estimates indicate that practically all elasticities are less than one representing inelastic demand for inputs along with rigidity in their combination. As expected all the own price elasticities of demand have the correct sign, in other words they are all negative. The cross price elasticities of demand are positive between labour and capital and also between the capital and fuel. This indicates that an increase in the price of labour will result in substitution towards capital; nevertheless the shift will be comparatively small.

One percent increase in the labour price has resulted in decrease in the demand for the labour by 0.61 percent in model 1 and 0.18 in model 2. In opposition to this, one percent increase in labour price resulted in marginal increase in the demand for capital by 0.0001 percent in model 1 and 0.0008 in model 2. As far as the demand for fuel is concerned, one percent increase in the price of labour resulted in the decrease in demand for fuel by 0.008 percent in model 1 and 0.012 percent in model 2. Further, one percent increase in the price of fuel will decrease the demand for fuel by 0.06 percent and 0.009 percent in models 1 and 2 respectively. Likewise one percent increase in the price of capital will decrease the demand for capital by 0.002 percent and 0.02 percent in models 1 and 2 respectively.

Summary and Conclusions

The main contribution of the study is the estimation of a well-specified translog system in which the error terms in the cost and cost-share equations are internally consistent. In addition to this, the most important aspect of the analysis is the explicit introduction of technological characteristics in the cost function in order to assess the influence of technological change on costs. The results clearly indicate the cost saving effects of technical change. Another important finding relates to the successively higher rate of cost reductions due to increase in technical change over a period of time.

A significant result from a practical point of view is that the analysis of the cost structure of Indian Railways using translog cost function yields reasonable estimates of technical change, scale and substitution analysis. Results show that the hypothesis of constant returns to scale can be rejected. The hypothesis of homotheticity is also rejected. Another restrictive assumption, which has been broadly used in productivity studies on Indian Railways, is that of separability. The hypothesis of both weak separability of outputs and inputs have been rejected. The estimates show that the value of ë reflecting the effect of technical change on cost decline is negative. This implies that as the proportion of freight traffic carried by the electric and diesel locomotives on Broad Gauge track to total freight traffic increases the total cost of operations decline. Similar is the case in model 2 where trend is taken as technology indicator. In addition to this the rate of cost diminution due to technical change resulted in a cost reduction in both the models. The nature of technical change is such that it reduced fuel and labour services and increased capital services. The estimate on price elasticity of demand has indicated that the own price elasticities of all the three inputs have a negative sign. The cross price elasticities of demand are positive between labour and capital and so is the case with the capital and fuel. As far as the elasticity of substitution is concerned, the elasticity of substitution between the labour and fuel indicated complementarity and the elasticity between the labour and capital showed substitutability. The results also show that fuel and capital are substitutes.

The own price elasticities of demand for labour, fuel and capital had negative signs indicating that when prices of the respective inputs increase, the demand for them decreases.

NOTES

[1] Let *u(x)* be a function homogeneous of degree one in *x*. Let *g(y)* be a function of one argument that is monotonically increasing in *y*. Then *u(g())* is a homothetic function of *y*. So a function is homothetic in *y* if it can be decomposed into an inner function that is monotonically increasing in *y* and an outer function that is homogeneous of degree one in its argument.

[2] If $y = f(x)$ is a homogeneous function of degree r, then the first derivatives, $f_i(x)$, are homogeneous functions of degree *r-1*. Thus, for the constant returns to scale case, as $y = f(x)$ is homogeneous of degree 1, then $f_i(x)$ is homogeneous of degree zero, i.e. a proportional increase in inputs will *not* change marginal products at all.

LITERATURE

Allen. R. C., (1983), Recent Developments in Production, Cost and Index Number Theory with an Application to International Differences in the Cost and Efficiency of Steelmaking in 1907/9 in Fremdling and O'Brien (eds.), Ibid, pp. 90-98.

Binswanger, H. P., (1974a), A Cost Function Approach to the Measurement of Elasticities of Factor Demand and Elasticities of Substitution, *American Journal of Agricultural Economics*, Vol. 56, May, pp. 377-86.

Borts, G. H., (1960), The Estimation of Rail Cost Functions, *Econometrica*, Vol. 28, No.1, Jan., pp. 108-31.

Borts, G. H., (1952), Production Relations in the Railway Industry, *Econometrica*, pp. 71-79.

Braeutigam, R. R., Daughety, A. F., Turnquist, M. A., (1982), The Estimation of a Hybrid Cost Function for a Railroad Firm, *The Review of Economics and Statistics*, Vol. 64, No.3, Aug., pp. 394-404.

Brown, R. S, Caves, D. W., Christensen, L. R, (1979), Modeling the Structure of Cost and Production of Multiproduct Firms, *Southern Economic Journal* ,Vol. 46, Part 1, pp. 256-73.

Burgess, D. F., (1974), A Cost Minimization Approach to Import Demand Equations, *Review of Economics and Statistics*, Vol. 56, pp. 225-34.

Caves, D. W., Christensen, L. R., (1980a), Global Properties of Flexible Functional Forms, *American Economic Review*, Vol.70, N0.3, June, pp. 422-32.

Christensen, L. R., Jorgenson, D. W., Lau, L. J., (1973), Transcedental Logarithmic Production Function, *The Review of Economics and Statistics*, Vol. 55, Feb. pp. 28-45.

Christensen, L. R., Jorgenson, D. W., Lau, L. J., (1971), Conjugate Duality and the Transcedental Logarithmic Function, *Econometrica*, Vol. 39, July, pp. 255-6.

Diewert, W. E., (1971), An Application of the Shephard's Duality Theorem: A Generalized Leontief Production Function, *Journal of Political Economy*, Vol. 79, pp. 481-507.

Griliches, Z., (1972), Cost Application in Railroad Regulation, *Bell Journal of Economics*, Vol. 3, No.1, Spring, pp.92-116.

Harris, R. G., (1977), Economies of Traffic Density in the Rail Freight Industry, *Bell Journal of Economics*, Vol. 8, pp.556-64.

Keeler, T. B., (1974), Railroad Costs, Returns to Scale and Excess Capacity, *The Review of Economics and Statistics*, Vol.56, pp.201-8.

Keeler, T. B., (1971), The Economics of Passenger Trains, *Journal of Business*, pp. 148-74.

Koshal, R. K., (1972), Marginal Cost and Resource Allocation in Transport Industry in India, *Sankhya*, Series B, pp. 421-32.

Koshal, R. K., (1971), Economics of Broad Gauge Railways: Indian Experience, *Indian Economic Journal*, Vol. XVIII, No. 3, pp.347-67.

Koshal, R. K., (1970), Economies of Scale in Rail Transport, *Indian Economic Review*, Vol. 5, No.2, Oct., pp. 177-91.

Lau, L. J., (1986), Functional Forms in Econometric Model Building, in Griliches, Z, and Intriligator, M.D, (eds.) Handbook of Econometrics, Vol. III, North Holland, pp. 1515-66.

Lau, L. J., (1974), Comments, in Intriligator and Kendrick (eds.), Ibid, pp. 176-99.

Manohar Rao, M. J., Nachane, Ajit, V. Karnik, D. M, Subba Rao V.V., (1982), A Framework for Optimizing Energy Usage in Industries – A Case Study of IR, *Indian Economic Journal,* Vol. 30, No.2 pp. 39-54.

McGeehan, H., (1993), Railway Costs and Productivity Growth, The Case of the Republic of Ireland, *Journal of Transport Economics and Policy*, January, Vol. 27, No. 1, Page 19.

Meyer, J. R., (1958), Some Methodological Aspects of Statistical Costing as Illustrated by the Determination of Rail Passenger Costs, *American Economic Review*, pp. 209-222.

Meyer, J. R., Peck, M., J., Stenason, J., Zwick, C., (1964), The Economics of Competition in the Transportation Industries, Harvard University Press.

Shepard, R., (1953), Cost and Production Functions, Princeton University Press, N.J.

Shephard, W., (1970), The Theory of Cost and Production Functions, Princeton University Press, Princeton, N.J.

Stenason, W. J., Bandeen, R. A., (1965), Transportation Costs and their Implications: An Empirical Study of Railway Costs in Canada, in NBERC, pp. 121-138.

Varma, H. O., (1988), Econometric Cost Functions, Anmol Publications, New Delhi.

Varma, H. O., (1983), The Study of Cost Function of Indian Railways, Ph.D Thesis, Department of Economics, Delhi School of Economics, University of Delhi, Delhi.