Metodika 17 (2/2008), 433-441
Professional paper
Received: 10.05.2008.

TASK DESIGN AND REDESIGN – OPENING UP, EXTENDING AND AUGMENTING TASKS TO PROMOTE LEARNERS' INITIATIVE AND FULL ENGAGEMENT IN THINKING

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Summary – *The article gives a critical account of the activity undertaken by the author with a small group of secondary school students. Its purpose is to explore ways in which a teacher can redesign an initial task in order to promote students' initiative and engagement in thinking. It describes this group's work on a mathematical task during which the teacher remained merely a participant in the discussion by giving occasional prompts to the learners. It is reported that in the case in view students have shown initiative by engaging actively in the discussion and by directing it in ways they felt appropriate for the task in front of them. This demonstrates that they welcome such an approach to learning.*

Key words: *discussion, student's initiative, task redesign*

INTRODUCTION

The present work reports on, and reflects critically at, a mathematical activity I have undertaken with a small group of secondary school learners. It was the process of working on, and subsequently redesigning of, the initial task as an exploration into the ways students think and use their mathematical powers such as: deduction and generalization, imagining and expressing what is imagined, particularising, conjecturing. This also was an exploration into ways of promoting students' initiative while working on mathematical tasks.

MILIEU

The investigation below was undertaken with a small group of learners in an informal setting (out of school). The learners were of mixed abilities; some were used to getting more instructions in school, others were encouraged to work in a conjecturing atmosphere. Their relationship with the teacher (myself) was a positive one; I did not feel they perceived me as threatening since I am not their assessor (perhaps that is why they freely spoke what they thought). Also, since this was a small group of learners, they had the opportunity to discuss things together, and, after a while questions and answers were exchanged between learners, not necessarily between learners and the teacher (me); thus, I was just another participant. This was all part of the atmosphere.

There are some specific ethical considerations to the task described. The data collected were qualitative – they include excerpts from the discussions with and among the learners, as well as my field notes. As such, parents were told about the process and their permission was obtained to use the data for research. I also explained to the students that these tasks were not for assessment and that their anonymity was assured. Of course, had this task been done in a formal classroom setting, some of the above would not be an issue at all, since a teacher can do tasks for the purpose of probing learning.

WORKING ON THE TASK

Normally I try to get the learners I work with to start each task by asking: What do I know? What do I want? I hope that by the process of scaffolding-fading[1] future initiative on their side will be facilitated. Learning is a gradual process, and consists of a continual building upon and modification of what has gone before. It can be thought of as a See-Experience-Master framework (see Mason, 1999): seeing a concept; experiencing an idea using previously mastered skills; mastering by using newly acquired skills in different contexts. This is the framework I adopted in the case in view in order to initialise the activities.

In the initial tasks learners were expected to master the rules (sine, cosine) they were previously taught by solving simple examples (see Appendix 1, examples 1 & 2), which I intended to redesign with the same group of learners at a later stage. Thus, I got them to start with what they knew. Also, I introduced a memory aid I found on the web. I told learners that the phrase I'm so sorry I ate chocolate might help them remember that "we have One Side: Sine Rule; and One Angle: Cosine Rule," or "1 Side Sine and 1 Angle Cosine," if taken as: I(1)'m S(ide)o S(ine)orry I(1) A(ngle)te C(osine)hocolate. When I mentioned this to my learners I got a positive reaction. That session had begun with an element of fun, which somehow relaxed them and they started talking more openly. Some of them even to-day keep writing first letters in their notebooks as a reminder: 1SS 1AC.

My questions for the learners while working on the first exercises (Appendix 1.1 & 1.2) were mostly closed: "What kind of a triangle is here? Is it relevant at

[1] Scaffolding– instruction which guides the learner to independent and self-regulated competence of skills, and as the learner's knowledge and learning competency increases, the teacher gradually reduces the supports provided (fading), *derived from Wood, Bruner & Ross (1976)*

all? And what about cosine in general, (definition), and the relation to a right angle triangle?" They were also the rehearsing type of questions, which repeat facts already known by students.

From this type of questions and my asking role (as opposed to listening or discussing), further development followed (see Appendix 1.6). This activity (Appendix 1.6) relates to level two (the analysis level) of van Hiele's levels of thinking (Mason & Johnston-Wilder 2004, p.59).[2] As van Hiele explains, "…at this level the concepts can exist for the learners separate from the situations in which they were developed. These concepts exist in a network of related concepts... Arguments can be resolved by referring to the definition…" (Ibid.).

In a further step, students proceeded with the sine formula. This can be recognized as a procept.[3] Each trigonometric formula involves both the process of dividing the length of two sides and the product, and the number which is the ratio of these two lengths. Learners in this case had the ability to see these relationships in a new triangle, and "…the flexibility to perceive that as the angle increases… the sine increases… and to give meaning to the singular cases ... the flexibility to extend to the case when the angle increases beyond 90° or becomes negative" (Gray & Tall 1992, p.7).

In order to open-up the task and get more involvement from the learners, different variations of the same task were presented. Encouragement to talk more and discuss was achieved by asking enquiry type questions, such as: What is the same? What is different? What do you know about triangles and the sum of angles? This shows how the learners connect previous knowledge with the current topic. The questions reflect a style of explanatory enquiry. The purpose here was to get them to talk, to develop a pattern of communication. Since this was not done in a formal setting, with learners that normally work together, an atmosphere of trust and openness had to be established first. I wanted to draw different aspects of student activity together (practising/mastering formulas, conjecturing/discussing, generalising/applying) while working on the task. In order to get the discussion going, learners had to be given the opportunity to choose how they were going to solve a task (which method they would use), or how they were going to pose a problem in the first place (choose the representation, which can be symbolic or picture), which meant adjusting the task for that purpose. Some of the possible variations I presented are: word problem, real-life situations, a puzzle (see samples in Appendix 1.7).

What I found is that using a word problem forces the learners to think how to translate it into 'formal' mathematics, be it a formula, an expression, or a graph. For instance, when I gave such a task (see Appendix 1.3), the learners first wrote

[2] The other levels van Hiele defines are: visualisation, abstraction, informal deduction and formal deduction.

[3] Procept – looking at a symbol both as a process and a concept, *defined by Gray and Tall (1994)*

neatly in two columns the information given: (a) What do we know? (b) What are we after? Then they sketched the situation and marked the unknown with an x. I would like to think that for some of them these were the consequence of a scaffolding-fading process (see fn. 1), as this is the usual way we approach a task. When asked: What shall we do with these two next? How do we connect (a) and (b), all learners drew a picture. Although the pictures were slightly different, there was a clear sense of what the situation was (at this point they compared each other's sketches). I noticed one learner making a comment on the other one's picture: That is not precise, because your angle looks greater than 21°. It is worth noting that the other student replied: It does not matter, it is only a picture. Perhaps this could have been discussed further –what could be the greatest possible angle in a similar situation and why, and subsequently, what effect would that have on the results. I was thinking to myself whether I should get involved in their discussion. However, I decided to let them justify their choices to each other, although in doing this it is probable that the possibility for discussion provided by the task was not transformed into a learning opportunity to be exploited (on these, see Mason & Johnson-Wilder 2004, p.45).

The learners recognized that the situation presented gives a prominent triangle, as one's prompt comment shows: We have to do something with a triangle. It is possible that this reveals a move from the enactive (merely manipulating sine and cosine rules), to iconic (seeing a connection for this particular situation), and later to symbolic (when expressing a problem in terms of formal definitions with a meaning). I got them to express in their words what the unknown distance is in relation to the initial distance, and what they needed in order to solve it. In the initial tasks (Appendix 1.1 & 1.2) they just dived into the formula; now they had to pause and think about the situation.

I felt I had to prompt them to think about the appropriate method for solving the problem posed. However, the question now moved from rehearsing to enquiry: "How can you solve a triangle if various information is given – sides, angles, height, bisectors …, give a few examples with the sketches." As such, my intended role was that of initiator, while trying to involve the learners in the process of making choices. Getting them to do this took some time; however, it gave them the opportunity to work at their own pace, to choose the elements and methods they were giving as an answer, and to eventually come to an understanding of mathematics as "a constructive enterprise" (Mason & Johnston-Wilder 2004, p.48). This also contributed to building a sense of authorship. It is worth noting here that in a formal setting, due to the lack of time and to curriculum constraints, such an approach is not always possible. I suppose, however, that it could be used in homework assignments, even though it would also have to be subsequently discussed in class.

Looking at the possible methods of dealing with the problem posed to them, learners chose the one which applied to this case. I asked: "What do we usually do when a choice is given?" "Justify our choice" was their answer. The fading element is recognizable in the answer. In order for the discussion to take place, I found useful to use enquiry questions such as: "What can we change here so that the distance x remains the same?" This was to give them the opportunity to explore the range-of-permissible-change for the task in view. Thus, I got them more involved by asking, discussing (see conversation excerpt in Appendix 2.1).

Another way of getting more involvement is to try to get learners recognize the particular in the general. This relates to what Tahta (1981) calls the inner task, as they use their mathematical power of specialising. We noted that the cosine rule is a generalisation of Pythagoras' theorem and discussed how this is found. In order for the learners to fully engage themselves in thinking, the task was opened-up even further, by engaging in authentic activity (see Appendix 1.4). As Papy (2004, p.109.) put it: "All of the concepts begin in everyday situations familiar to all students." In Tahta's (1981) terminology, this would be a meta-task.

What is different in the new formulation of the task is that the question asked is an exploratory enquiry. Here learners needed some guidance, so I had to prompt them by explaining the situation clearly: "If you had a distance between points A and B, what else would you need to know if you wanted to calculate the distance AC in the following situation: say we are standing on this side of the river at point A and we can only operate on this side, do any measurements, etc…" It is possible they needed my help in this instance since the question was too ambiguous for them. Thus, first we speculated on what needed to be done/known in order to solve this problem and then a task was devised (see Appendix 1.5) to solve the problem. They continued working independently. I see this as a sampling of Vygotsky's idea of zone of proximal development.[4] Once guidance was given, further individual explorations took place. Furthermore, here a new framework was adopted: Manipulating-getting a sense of-articulating. At the start, learners manipulated the object (here a triangle), then a sense of the underlying structure or the relationships between elements of different triangles developed, and finally, based on what they have understood, these rules were applied to solve new, more complex problems. In this regard, Goos (2004, p.104) has observed that "Mathematical thinking is an act of sense-making, and rests on the processes of specialising and generalising, conjecturing and justifying."

Sometimes, in doing such tasks and trying to boost learners' initiative, students see/do things we hardly anticipate. As one of the conversations we had shows (see Appendix 2.2), the discussion may digress from the initial topic. However, I appreciated the initiative the student took in order to understand. It showed that the learner is genuinely interested in mathematics and wants to understand.

Furthermore, learners initiative is best encouraged by "…having learners make up examples or questions" (Mason & Johnston-Wilder, 2004, p.43). The ex-

[4] ZPD – the difference between what a learner can do without help and what he/she can do with help, *defined by Vygotsky (1978)*

ample in Appendix 2.3 is such a case – a self-generated example (as Mason, 2004, calls it), coming from one of the learners, which shows clear evidence of interest, understanding, and initiative.

In terms of my own reactions, I observed that at times I wanted to tell them the solutions, to give tips before they figured out what to do by themselves. No doubt, the learners were influenced to think in a specific way, since the lesson taught was about sine and cosine rules. I wondered whether they would solve the task in Appendix 1.3 as easily if it was presented some time later, in a different context, one which did not include a prior presentation of the sine and cosine rules. I also had questions about the motivation students received – I noticed that one learner, after the discussion and after finishing calculations, started drawing a ship with all its details (while waiting for the others to catch up). I had to consider whether I should wait until all reach the final result (and eventually give those who finished another, more complex, task meanwhile), or rather interrupt after the conclusion was reached, and leave the calculations for later or for homework. I also realised that if the atmosphere/milieu I create within the group of learners is encouraging, friendly, communicative, then students feel comfortable in expressing their opinions. Penner (1984) has rightly observed that "learning is about establishing effective social relationships with students." Also, Murray (1991) has emphasized that "enthusiasm, clarity and expressiveness help students learn – you have to get them to pay attention. Teacher enthusiasm has consistent correlation with good teaching. Teacher enthusiasm motivates students to explore the subject further outside of class."

CONCLUSION

Having in mind the purpose of promoting students' initiative, the process of redesigning of the initial task has been attempted as an exploration into the ways students think and use their powers such as: deduction and generalization, imagining and expressing what is imagined, particularising, conjecturing and convincing oneself and others. It has shown that students respond positively to this type of activity, as evident from their participation in, and learning outcomes of, their working on the tasks given. Their engagement in discussion improved since the answers to their questions and ideas were not delayed, but on the contrary, explored. Students' initiative was manifested in the dialogues they led where the teacher was not much more than an observer, since her participation was limited to prompts and guiding questions.

It must be mentioned here, though, that the situation in view was advantageous, since working in an informal environment did not impose a pressure to stick to the lesson unit. This is a problem encountered frequently; because of the lack of time not everything gets to be explored in the class. One way of dealing with it, when the possibility to digress from the topic occurs, would be to simply ignore the rest of the lesson and to go in the direction where the discussion is going. Another way would be anticipating moments and places for a fruitful mathematical dialogue and giving those as an introduction to the particular unit, or, furthermore, letting students do it for themselves.

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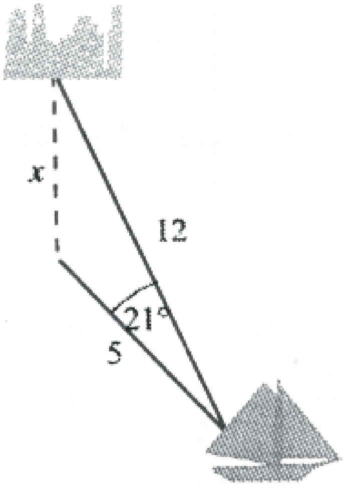
APPENDIX 1:

1. Solve triangle if $a = 40$ cm, $b = 37$ cm, and $\gamma = 18°$
2. Solve triangle if $a = 17$ cm, $b = 10$ cm, $c = 9$ cm

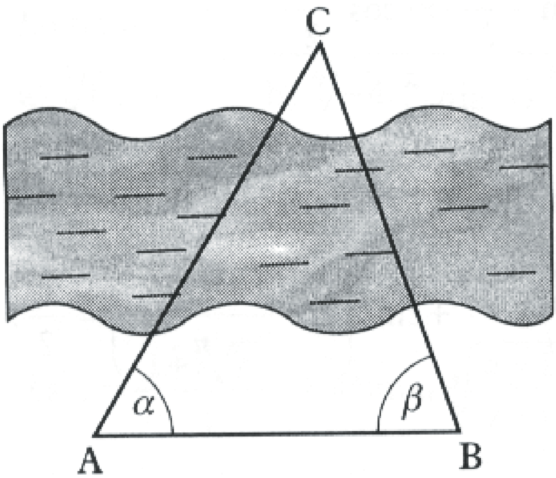
3. The ship is sailing towards the harbour which is 12km away. After 5 km the captain realised that he deflected 21° from the course. How far from the harbour were they then? (See graphics 1)
4. How would you calculate the distance to an inaccessible object?
5. Find distance between A and C on the other side of the river if we know the distance between points on the same side of the river.

   |AB| = 300 m. The angles seen from the A and B are 52°18', 103°40'

   (See graphics 2)



Graphics 1.



Graphics 2.

6. Find sides *a* and *b* if *c* = 10 cm, v_c = 5 cm, α = 62°10'.

   Student: Now we can calculate sine.

   Teacher: How?

   Student: We have a right angle triangle.
7. A puzzle: http://www.geocities.com/mathematicsplus/resources.html

APPENDIX 2:

1. Student: We can change the angle of deflection, but then…also something else?

   Teacher: What?

   S: If the angle is different, then the ship must travel longer or shorter than 5km.

   T: Say the angle changes to 30°. What would then happen?

   S: 5km changes.

   T: In what way? Do you think the ship would sail longer or shorter? Can you explore this? Can you tell me without calculations?

2. Side lengths of a triangle 'are in ratio' 2: 4: 8. Find the smallest angle of this triangle.

   Student: We don't have any actual lengths; can we get a precise result?

   Teacher: What makes you ask that?

   S: Everything is unknown.

   T: What is the unit measure for an angle?

   S: Degree.

   T: Consider this: sine of an angle is 0.5. 0.5 of which units?

   S: Nothing, just a number.

   Other student: Maybe if we have some units on the left and on the right side of equation, than they can be crossed and we remain without units.

   Here a learner is trying to apply something that must have happened before, perhaps she recollects a task where this occurred and tries to make sense of?

   T: Can you give an example?

   S: 2sinγ=2bsinγ.

   T: What would you call the 'units' here?

   S: Sinγ.

   T: Aha, so you cross them on both sides and you get b=1?

   S: Yes.

   T: Good thinking, except can you explain this bit about sinγ being a unit?

   S: Well, they are not units, but maybe expressions… or the unknowns?

   She had a good idea; however the language was not appropriate.

   Student (after group comparing two and two sides): but I thought we are not allowed to do that.

   T: To do what?

   S: To break up the ratio in twos.

   S: And why do you think so? Does it change the ratio for these lengths?

   S: Maybe.

   T: Do you think you can check that? Choose numbers (arbitrary) and try it. Does it work? Can you conclude from this example that it always works? What is important for a ratio?

   Student concluded: For ratio we don't need measure units (as long as we are comparing 'same' things.

3. Investigate distances traversed by players/balls or optimum angles for certain plays.