REDISTRIBUTION, PENSION SYSTEMS AND CAPITAL ACCUMULATION

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Abstract

In this paper we study the macroeconomic impact of a policy which changes the redistributive properties of an unfunded pension system. Using an overlapping generations model with a closed economy and heterogeneous agents, we show that a weaker link between contributions and benefits has an impact on the level of capital per capita if and only if there are inequalities in the length of life. Furthermore, this policy has positive implications for every economic agent if the system has a defined-benefit structure. The tax rate and inequalities decrease, whereas the wealth of each agent increases. However, with a defined-contribution pension system, this policy has a negative impact on every macroeconomic variable except on the wealth of the poorest agents.

Keywords: inequality, pension systems, redistribution, capital

1 Introduction

Pension systems can be classified according to three dimensions (Feldstein and Liebman (2002)). Firstly, they can adopt either a Pay-As-You-Go (PAYG) or a fully-funded structure[1]. The size of unfunded pension systems is large in most industrialized co-

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[1] In this paper we only consider PAYG pension systems. In a portfolio approach, Dutta *et al.* (2000) show that a mixed pension system is desirable because of the differences in the risks between funded and unfunded pension systems.

untries. For example, the payroll tax rate used to finance them ranges from 12.4% for the United-States to 29.6% for Italy (Nyce and Schieber (2005: 236)).

Secondly, pension systems can have either a defined-benefit or a defined-contribution structure. A pension system has a defined-benefit structure if it is the tax rate that adjusts itself to changes in the economic and demographic environment. Conversely, it has a defined-contribution organization if it is the replacement rate that adjusts itself. Most countries have chosen a defined-benefit pension system (Nyce and Schieber (2005)). However, because of the increase in life expectancy, the fiscal burden of this structure has increased strongly. Consequently, some countries, such as Italy, have adopted a defined-contribution pension system[2].

Thirdly, pension systems can be more Beveridgeian or more Bismarckian. A pension system is purely Beveridgeian if every agent receives the same pension. Conversely, a pension system is purely Bismarckian if pensions depend completely on the wages of agents. A pension system is mixed if it has a Beveridgeian and a Bismarckian component. The more Beveridgeian a pension system, the higher the intra-generational transfers. Countries differ sharply by this intra-generational component. France, Germany and Italy have a Bismarckian structure. The systems in Canada, the Netherlands and New-Zeland are essentially Beveridgeian. Finally, Japan, the United-Kingdom and the United States have mixed pension systems (Sommacal (2006), Casamatta et al. (2000)).

Theoretical literature has explored the impact on the economic activity of the size of PAYG pension systems[3] that has either the Beveridgeian or Bismarckian structure[4]. The usual result is that the Bismarckian systems provide more incentives to accumulate human and physical capital and thus induce a higher growth rate than the Beveridgeian pension systems. But in fact, pension systems are usually a combination of these two elements. Only a few authors have studied the impacts of a change along the third axis mentioned above[5]. However, it is a central issue given the wide dispersion of countries along this third axis.

The main idea of this paper is that PAYG pension systems can adopt a structure which combines the Bismarckian and Beveridgeian components, and we study the impacts of a policy which increases the Beveridgeian component of pension systems. We show that these effects are different, depending on whether the pension system has a defined-benefit or a defined-contribution structure. Last but not least, the inequalities of the length of life play an important role in the qualitative and quantitative results of this paper.

There is a growing empirical literature which analyzes these inequalities. Mesrine (1999) studies the inequalities in the length of life according to socio-professional gro-

[2] In this paper we do not explain the switch from a defined-benefit to a defined-contribution pension system. A model with a representative agent and an increasing life expectancy would be more appropriate for this study.

[3] See Belan and Pestieau (1998), Breyer and Straub (1993) or Homburg (1990) among others for the analysis of the transition from unfunded pension systems towards fully-funded pension systems.

[4] See Docquier and Paddison (2003), or Casarico and Devillanova (2007). These results are questioned notably by Groezen et al. (2007), Lambrecht et al. (2005) or Le Garrec (2005).

[5] Except Sommacal (2006) using an endogenous labor supply model with a defined-contribution pension system.

ups in France[6]. The most striking feature of his paper is that a worker has an almost twice higher probability to die at the age between 35 and 65 years than an executive manager. Furthermore, their life expectancies at the age of 35 are 38 and 44 respectively. The same qualitative results are recorded in the United States (Panis and Lillard (1995), Deaton and Paxson (2000)).

Finally, Robert-Bobbée and Cadot (2007) show that this inequality is also observed for elderly people. For agents who are 86, those with the highest education level can expect to live 20% longer than the ones with the lowest education level.

Only a few papers have explored the economic impacts of these health inequalities. Mitchell and Zeldes (1996: 365) emphasized that these health inequalities have implications on the redistributive properties of pension systems, but they did not provide any empirical or analytical analysis. Drouhin (2001) showed by a small open economy that a Bismarckian PAYG pension system induces transfers from agents with a short life expectancy to agents with a long life expectancy. His model is the first step in studying the impacts of the inequalities in the length of life but he only uses the Bismarckian structure and there are no general equilibrium effects in his model. The political economy literature has recently become interested in the implications of the link between life expectancy and wages[7].

In this paper we study the macroeconomic impact of a policy which modifies the redistributive properties of an unfunded pension system[8]. In order to obtain clear qualitative results for every macroeconomic variable, we first present an analytical resolution of our model. Then, because the impact is ambiguous for some variables, we calibrate our model on French data and we numerically solve our model. We work with French data because the French pension system is highly Bismarckian (Casamatta et al. (2000)) and because the efficiency of such a system is widely questioned. However, we also show that our numerical results do not depend on this specific case. Using an overlapping generations model with a closed economy and heterogeneous agents, we show that a weaker link between contributions and benefits has an impact on the level of capital per capita if and only if there are inequalities in the length of life. We also show that this redistributive policy has positive implications for every economic agent if the system has a defined-benefit structure. The tax rate and inequalities decrease, whereas the wealth of each agent increases. However, with a defined-contribution pension system, this policy has a negative impact on every macroeconomic variable except on the wealth of the poorest agents.

Gorski et al. (2007) also emphasize the role of the mortality differential in analyzing the impact on educational choices of a change towards a more Beveridgeian pension system. They find that this impact is positive. In this paper, we analyze the impact of this policy on physical capital accumulation.

[6] These inequalities also depend on other factors like sex or geographical localization. For example, in France the life expectancy of women is 84.1, whereas that of men is only 77.2 (INSEE, 2006). Moreover, Rican and Salem (1999) show that there are strong disparities according to the localization of people in France.

[7] Borck (2007) shows that the size of a pension system can be determined by a coalition of elderly, very poor and very rich agents. Poor agents benefit from the Beveridgeian part of the pension system, whereas rich agents benefit from the pension system for the longest time.

[8] In this paper the term "redistributivity" means that we change the Bismarckian structure of pension systems. A decrease in the redistributivity means that there is a stronger link between wages and pensions per unit of time.

This paper is organized as follows: Section 2 presents the main elements of our model. In Section 3, we detail the dynamics of the economy and its properties. The implications in terms of utility and inequalities are studied in Section 4. In Section 5 we calibrate and solve our model. Finally, Section 6 includes some closing remarks.

2 The Model

It is assumed that two generations, the young and the old, overlap in each period t. Their respective sizes are N_t and N_{t-1}. The population grows at a constant rate n, so that $N_t = (1+n)\, N_{t-1}$. At the beginning of his life, each member of a generation receives a productivity endowment a. This productivity takes its values in the interval $\Omega_a = [a_-, a_+]$. The density function and the cumulative distribution function of a are denoted by $f(a)$ and $F(a)$ respectively. These functions are such that: $\bar{a} = \int_{\Omega_a} af(a)da = 1$, $F(a_-) = 0$ i $F(a_+)$. Furthermore, $\bar{a}$ denotes the average productivity of the economy:

$$\bar{a} = \int_{\Omega_a} af(a)da \tag{1}$$

The density function $f(a)$ is assumed to be independent of time and of the level of capital.

Each agent lives completely his first period of life[9] but only a fraction $T(a)$ of his second period of life[10]. We assume that $T'(a) > 0$. The higher the productivity, the larger the length of life. In this exercise we assume that the length of life depends positively on the productivity level of each agent. In our model, the wage level is an increasing function of the productivity level. Consequently, the assumption on $T(a)$ uses the empirical evidence that the wage level is a significant variable to explain the mortality differential between agents (Adams *et al.* (2003))[11]. Borck (2007) uses the same assumption in a political economy framework.

The average length of life is denoted by $\bar{T}$ and is determined by:

$$\bar{T} = \int_{\Omega_a} T(a)f(a)da \tag{2}$$

The link between productivity and the length of life is measured by the covariance:

$$COV_{T(a),a} = \int_{\Omega_a} T(a)af(a)da - \bar{T}\bar{a} \tag{3}$$

[9] The length of each period is normalized to 1.

[10] There is no uncertainty in this economy to simplify our model. However, our model can also be interpreted as a model with uncertain lifetime. In that case, there is a probability that an agent survives at the end of his first period of life, as in Drouhin (2001). It also implies that there exists a perfect annuity market for each group of agents.

[11] See also Mesrine (1999), Pannis and Lillard (1995), or Deaton and Paxson (2000).

This covariance is positive because of our assumption on the sign of $T'(a) > 0$ (See Appendix E). The stronger the link between $T(a)$ and a, the bigger the covariance. Conversely, if $T'(a) = 0$, i.e. if the length of life is the same for every agent, then this covariance is zero.

2.1 Consumers

The utility of consumers depends on their consumption flows in their two periods of life. For an agent born in the period endowed with a productivity level a, $c_t(a)$, and $d_{t+1}(a)/T(a)$ denote the first period and the second period consumption flows respectively. Their utility function is intertemporally separable and has the following form[12]:

$$U_t(a) = u(c_t(a)) + \beta T(a) u\left(\frac{d_{t+1}(a)}{T(a)}\right) \tag{4}$$

where β represents the pure time preference factor for the present, and the $T(a)$ in front of their second period utility implies that the longer the length of life, the more consumers value their utility of this period.[13]

Each agent offers inelastically his work during his first period of life and obtains a wage $w(a)$[14]. This wage is taxed at a rate τ, and the revenues from this tax are used to finance a PAYG pension system. When an agent becomes old, he receives a pension p(a). Budget constraints on an agent born in a period t, are as follows:

$$c_t(a) = w_t(a)\,(1-\tau) - S_t(a) \tag{5}$$

$$d_{t+1}(a) = R_{t+1}S_t(a) + p_{t+1}(a) \tag{6}$$

where R_{t+1} represents the interest factor and $S_t(a)$ the saving function.

We also assume that the utility function has the following form: $u(x) = ln(x)$. It simplifies the analytical expressions[15]. Based on all these assumptions, the saving function is the following:

$$S_t(a) = \frac{\beta T(a) w_t(a)(1-\tau)}{(1+\beta T(a))} - \frac{p_{t+1}(a)}{(1+\beta T(a))R_{t+1}} \tag{7}$$

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[12] It implies that the marginal rate of substitution between c_t and d_{t+1} depends on the length of life.

[13] In this paper we do not represent fertility choices even though life expectancy inequalities and wage inequalities do impact on these choices.

[14] In doing so, we do not model the burden of income taxation on labour supply.

[15] It notably simplifies the conditions that will be obtained and the aggregation of the saving functions. Our qualitative results do not depend on this assumption.

2.2 Firms

We assume a perfect competition on the final goods market and on the inputs market. The production function of firms is as follows:[16]

$$Y_t = AK_t^{\alpha}\left(N_t \int_{\Omega_a} af(a)da\right)^{1-\alpha} \tag{8}$$

where $0 < \alpha < 1$, K_t represents the physical capital level, and $A > 0$ the level of technology. As there is perfect competition on each market, firms take wages and interest factors as given. Profit maximisation implies the following expressions for prices, given that the final good is the *numéraire*:

$$R_t = A\alpha K_t^{\alpha-1}\left(\int_{\Omega_a} af(a)N_t da\right)^{1-\alpha} \equiv A\alpha k_t^{\alpha-1}\overline{a}^{1-\alpha} \tag{9}$$

$$w_t = A(1-\alpha)K_t^{\alpha}\left(\int_{\Omega_a} af(a)N_t da\right)^{-\alpha} \equiv A(1-\alpha)\frac{k_t^{\alpha}}{\overline{a}^{\alpha}} \tag{10}$$

where $k_t \equiv K_t/N$ represents the capital level per young agent. w_t is the wage per efficiency unit of work. The wage for agents with a productivity level a is the following:

$$w_t(a) = w_t a = A(1-\alpha)\frac{k_t^{\alpha}}{\overline{a}^{\alpha}}a \tag{11}$$

It implies that relative wages are independent of the level of capital, whereas absolute differences in wages depend on it.

In the rest of this paper, $\overline{w}_t$ will denote the average wage in the economy at period *t*. It has the following expression:

$$\overline{w}_t = \int_{\Omega_a} w_t(a)f(a)da = A(1-\alpha)k_t^{\alpha}\overline{a}^{1-\alpha} \tag{12}$$

2.3 The Pension System

We assume a PAYG pension system. The revenues of this system come from a proportional tax on wages: τ. It is used to provide a pension for elderly people. Their pension depends on the wages of young agents having the same productivity as theirs, and on the average wage in the economy. Their respective weights are λ and $(1-\lambda)$. The first part of this pension represents the Bismarckian component, whereas the second part re-

[16] Our results do not depend on the form of the production function but it clarifies our analysis with simple analytical results.

presents the Beveridgeian component of this system (Casamatta *et al.* (2000)). λ measures the indexation of pensions to activity wages of agents. The smaller the λ, the more restributive the pension system.[17].

Consumers receive only a fraction ν (with $0 < \nu \leq 1$) of this weighted average, and only during their second period of life $T(a)$. ν denotes the average replacement rate of the pension system. The pension of an agent endowed with a productivity level a is:

$$p_{t+1}(a) = \nu\left(\lambda w_{t+1}(a) + (1-\lambda)\overline{w}_{t+1}\right)T(a) \tag{13}$$

By means of equations (11) and (12) we obtain:

$$p_{t+1}(a) = \nu A(1-\alpha)\frac{k_{t+1}^{\alpha}}{\overline{a}^{\alpha}}(\lambda a + (1-\lambda)\overline{a})T(a) \tag{14}$$

We also assume that the government does not use debt. It implies that for every period we have:

$$\int_{\Omega_a} \tau w_{t+1}(a) f(a) N_{t+1} da = \int_{\Omega_a} p_{t+1}(a) f(a) N_t da \tag{15}$$

As shown in Appendix A, we obtain the following expression:

$$\tau = \frac{\nu}{1+n}\left(\lambda \frac{COV_{T(a),a}}{\overline{a}} + \overline{T}\right) \tag{16}$$

It defines the tax rate in the function of the parameters of the model. We say that it characterizes a defined-benefit organization.

Proposition 1: *In a defined-benefit PAYG pension system, τ is an increasing function of l[18] if and only if $COV_{T(a),a,} > 0$, i.e. if there are inequalities in the length of life.*

This result is very intuitive. Indeed, the richer the agents, the longer their length of life. Therefore, an increase in λ (i.e. a decrease in the redistributivity of the pension system) results in stronger indexation of pensions to their wages. It implies that the pensions of rich agents increase. Moreover, they benefit from these pensions for a longer period of time than other agents. Consequently, in order to finance these additional expenditures the tax rate has to be raised.

We have to note that this result depends only on the budget constraint of the government and not on consumers' preferences.

[17] In this paper the term "redistributivity" only concerns the direct redistribution of pension systems and not the effective redistribution of pension systems. The effective redistribution, which is the difference between tax paid and amount received, can be very different because of life expectancy inequalities.

[18] This proposition can partly explain why Bismarckian pension systems are bigger than Beveridgeian ones.

Let us now assume that we have a defined-contribution PAYG pension system (τ is exogenous). It is the replacement rate ν that adjusts itself in order to maintain the government budget constraint in equilibrium:

$$\nu = \frac{\tau(1+n)}{\left(\lambda \frac{COV_{T(a),a}}{\bar{a}} + \bar{T}\right)} \tag{17}$$

Proposition 1 *(bis): In a defined-contribution PAYG pension system, the replacement rate (ν) is a decreasing function of λ if and only if $COV_{T(a),a} > 0$.*

The intuition is the same as before. A smaller indexation to wages (a smaller λ) is beneficial for poor agents who live for a shorter period of time than the rich ones. Then, for a given replacement rate, expenditures are lower. Finally, government can increase the replacement rate for every agent.

Corollary 1: *In a defined-benefit (defined-contribution) PAYG pension system, the tax rate (replacement rate) is independent of the redistributivity of the pension system if and only if there are no inequalities in the length of life.*

Without inequalities in the length of life a variation in λ does not affect the total amount of pensions paid.

3 The Dynamics and Their Properties

The dynamics of this economy are represented through the equation of capital accumulation. Furthermore, because the marginal return on capital in the production function is decreasing, the economy converges towards a steady state equilibrium, so that the capital level per worker is constant. The dynamics are as follows:

$$K_{t+1} = \int_{\Omega_a} S_t(a) f(a) N_t da \tag{18}$$

It is straightforward to show that we finally obtain:

$$\begin{aligned} & k_{t+1}\left[1+n+\nu\frac{1-\alpha}{\alpha}\int_{\Omega_a}\frac{\lambda\frac{a}{\bar{a}}+(1-\lambda)}{1+\beta T(a)}T(a)f(a)da\right] \\ & = \frac{\beta A(1-\alpha)(1-\tau)k_t^{\alpha}}{\bar{a}^{\alpha}}\int_{\Omega_a}\frac{T(a)a}{1+\beta T(a)}f(a)da \end{aligned} \tag{19}$$

The right-hand-side of this equation is a strictly concave function of k_t. Consequently, there is a unique non-trivial steady state which has the following form:

$$(k^*)^{1-\alpha} = \frac{\dfrac{\beta A(1-\alpha)(1-\tau)}{\overline{a}^{\alpha}} \int_{\Omega_a} \dfrac{T(a)a}{1+\beta T(a)} f(a)da}{1+n+\nu \dfrac{1-\alpha}{\alpha} \int_{\Omega_a} \dfrac{\lambda \frac{a}{\overline{a}} + (1-\lambda)}{1+\beta T(a)} T(a) f(a) da} \tag{20}$$

Proposition 2: *In a defined-benefit PAYG pension system, a decrease in λ has a positive impact on k*.*

Proof: The numerator of equation (20) is a decreasing function of λ because only τ depends positively on λ. Moreover, we know that $T(a)/(1+\beta T(a)$ is an increasing function of a. It implies that $T(a)/(1+\beta T(a)) < T(\overline{a})/(1+\beta T(\overline{a}))$ (>), $\forall a < \overline{a}$ (>). Then $(a - \overline{a})\, T(a)/(1+\beta T(a)) > T(\overline{a}))\,(a - \overline{a})$, $\forall a$. The denominator is an increasing function of λ if the following condition is satisfied: $\int_{\Omega_a} \frac{a-\overline{a}}{1+\beta T(a)} T(a) f(a) da \geq 0$. We know that $\frac{a-\overline{a}}{1+\beta T(a)} T(a) f(a) \geq \frac{a-\overline{a}}{1+\beta T(\overline{a})} T(\overline{a}) f(a)$, $\forall a \in \Omega_a$. Integrating the two sides of this equation on the interval Ω_a, the right-hand-side is equal to zero and the condition mentioned above is satisfied.□

Two kinds of effects play a role when we analyse the effects of a decrease in λ. The former concerns the impact on the tax rate. Indeed, we have shown in proposition 1 that the tax rate is an increasing function of λ. If λ falls, the tax rate decreases for every consumer, which has a positive effect on saving without ambiguity. The latter effect concerns the impact on the pension received by each agent. If λ decreases, consumers with a productivity lower than $\overline{a}$ receive a higher pension, whereas consumers with a productivity higher than $\overline{a}$ receive a lower pension. The first group of agents saves less and the second one saves more. Proposition (2) shows that the net effect on saving is positive. Indeed, the agents whose pensions decreases have a longer length of life than the others. Consequently, the increase in the saving of rich agents overcompensates the decrease in the saving of poor agents.

Proposition 2 (bis): *In a defined-contribution PAYG pension system, a decrease in λ has a positive impact on k* if and only if:*

$$\frac{\int_{\Omega_a} \dfrac{1}{1+\beta T(a)} T(a) f(a) da}{\int_{\Omega_a} T(a) f(a) da} \leq \frac{\int_{\Omega_a} \dfrac{a-\overline{a}}{1+\beta T(a)} T(a) f(a) da}{\int_{\Omega_a} (a-\overline{a}) T(a) f(a) da} \tag{21}$$

Proof: τ is fixed because this is a defined-contribution pension system. It is ν that adjusts itself and only the last term of the denominator depends on λ. The condition ensures that the derivative of this term with respect to λ is positive.□

With a defined-contribution PAYG pension system, we have shown in proposition 1 (bis) that ν is a decreasing function of λ. Then, following an increase in the redistributivity of the pension system (a decrease in λ), the government increases the replacement rate. It has a positive impact on the pension of every consumer *ceteris paribus,* and thus a negative effect on saving. But the decrease in λ has a positive (negative) impact on the saving of agents endowed with a productivity higher (smaller) than $\overline{a}$. The condition of the proposition ensures that the positive effect is higher than the two negative ones.

Proposition 3: *(i) If there are no inequalities in the length of life, then k* does not depend on λ. (ii) This result remains true for every homothetic preference.*

Proof: See Appendix B.□

We have shown in propositions 1 and 1 (bis) that if $T(a) = T$ for all a, then the tax rate (replacement rate) is independent of λ. The only effects concern the increase in the saving of agents endowed with a productivity higher than $\overline{a}$, and the decrease in the saving of agents endowed with a productivity lower than $\overline{a}$. These last two effects exactly compensate.

4 Wealth, Consumption and Redistribution

This section has two main objectives. The first one is to study the evolution of the wealth, consumption and utility of an agent, if the degree of redistribution of the pension system increases (λ decreases). The second one is to study the evolution of inequalities in consumption and welfare if λ decreases.

These analytical results are obtained at a steady state in order to simplify the exposition. Every derivative is thus a comparison between steady states.

4.1 Wealth, Welfare and Redistribution

The wealth of an agent born in a period t endowed with a productivity level a has the following form:

$$W_t(a) = w_t(a)(1-\tau) + \frac{p_{t+1}(a)}{R_{t+1}} \tag{22}$$

We want to know if the wealth of each consumer increases when the redistribution of the pension system is higher (λ decreases).

Proposition 4: *In a defined-benefit pension system, if λ decreases then the wealth of agents endowed with a productivity smaller than $\overline{a}$ increases, whereas the impact on the wealth of other agents is ambiguous. The net effect is positive for every agent if*[19]*:*

$$-\frac{\frac{dk}{d\lambda}}{k} \geq \frac{\frac{a_+}{\overline{a}} - 1}{\lambda \frac{a_+}{\overline{a}} + 1 - \lambda} \tag{23}$$

[19] It is a sufficient condition.

Proof: See Appendix C.□

Proposition 1 has shown that the tax rate is an increasing function of λ. Furthermore, we have shown in proposition 2 that k^* is a decreasing function of λ. Then, the net wage for the first period of life is higher if the redistributivity of the pension system increases. More generally, wages per efficiency unit of work increase.

Moreover, a decrease in λ reduces the indexation of pensions to wages. Consequently, it has a positive impact on the pensions of agents endowed with a productivity smaller than $\overline{a}$ and a negative effect on the pensions of agents endowed with a productivity higher than $\overline{a}$. The condition in the proposition ensures that for rich agents ($a > \overline{a}$) all positive effects overcompensate the decrease in the indexation of pensions to wages.

Proposition 4 (bis): *In a defined-contribution pension system, if λ decreases then:*

• *If the condition of proposition 2 (bis) is true, then the wealth of agents endowed with a productivity smaller than $\overline{a}$ increases, whereas the impact on the wealth of other agents is ambiguous. The net effect is positive for every agent if*[20]*:*

$$-\frac{\nu\frac{dk}{d\lambda}+k\frac{d\nu}{d\lambda}}{k} \geq \frac{\frac{a_+}{\overline{a}}-1}{\lambda\frac{a_+}{\overline{a}}+1-\lambda} \tag{24}$$

• *Otherwise, the net impact is ambiguous for every consumer.*

Proof: See Appendix C.□

If the condition of proposition 2 (bis) is true, then a decrease in λ has a positive impact on k^*. Furthermore, it affects the pensions of agents differently, depending on whether the productivity of consumers is higher or lower than $\overline{a}$. The effects are the same as before, except that τ is fixed exogenously. Every agent benefits from the increase in ν, particularly the agents with a long life expectancy. That is why the condition is less restrictive than that of proposition 4. Nevertheless, if the effect on k^* is negative, then the impact on wealth is ambiguous for every consumer.

The utility of an agent depends on the levels of consumption in the two periods of his life. Using the budget constraints of consumers we obtain:

$$c_t(a) = \frac{W_t(a)}{1+\beta T(a)} \tag{25}$$

and

$$d_{t+1}(a) = \beta T(a) R_{t+1} \frac{W_t(a)}{1+\beta T(a)} \tag{26}$$

[20] It is a sufficient condition.

The consumption level of his first period of life depends on λ only through the wealth level, whereas the consumption level of his second period of life depends on the wealth level and on the interest factor. The utility level is an increasing function of the redistributivity of the pension system if and only if:

$$-(1+\beta T(a))\frac{dW(a)/d\lambda}{W(a)} > -(1-\alpha)\beta T(a)\frac{dk/d\lambda}{k} \tag{27}$$

The left-hand-side represents the evolution of an agent's wealth and the right-hand-side shows the evolution of the interest factor. Indeed, a change in λ affects k^* and thus the interest factor. Let us consider the case of a defined-benefit pension system. A decrease in λ has a positive impact on the wealth of every consumer (($dW(a)/d\lambda < 0$)). But at the same time, it reduces the interest factor ($dk^*/d\lambda < 0$). The net effect on utility is thus ambiguous. More precisely, the net effect can be negative for agents with a long life expectancy because they save a large portion of their wealth and are strongly affected by the decrease in the interest factor.

4.2 Inequalities and Redistribution

To study inequalities, two groups of agents are used: the poorest endowed with a productivity level a_- and the richest endowed with a productivity level a_+[21]. The main objective is to study welfare inequalities, but relative inequalities of wealth have to be studied first.

Proposition 5: *In a defined-benefit pension system, the relative inequality of wealth $W(a_-)/W(a_+)$ is an increasing function of the redistributivity of the pension system (a decrease in λ) if:*[22]

$$\frac{T(a_-)}{a_-} \geq \frac{T(a_+)}{a_+} \tag{28}$$

Proof: See Appendix D.□

The direct impact of a decrease in λ is a reduction in the pensions of rich agents ($a > \overline{a}$) and an increase in those of poor agents ($a < \overline{a}$). It increases the ratio $W(a_-)/W(a_+)$. Moreover, a decrease in λ has a positive effect on net wages because of its positive impact on capital *per capita* and a negative impact on the tax rate. This effect is essentially beneficial for the richest. Finally, a decrease in λ has a positive impact on w_t/R_t. The richest are the ones who essentially benefit from this effect because they live for a longer period of time. The condition of the proposition ensures that the redistributive effect dominates over every other.

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[21] For analytical convenience, we do not use here the Gini coefficient. See section 5 for the estimation of the Gini coefficient in our model.

[22] It is a sufficient condition.

Proposition 5 (bis): *In a defined-contribution pension system, if the condition of proposition 2 (bis) is true, then the relative inequality of wealth* $W(a_-)/W(a_+)$ *is an increasing function of the redistributivity of the pension system (a decrease in* λ*) if*[23]:

$$\frac{T(a_-)}{a_-} > \frac{T(a_+)}{a_+} \times \frac{\lambda \frac{a_+}{\bar{a}} + 1 - \lambda}{\lambda \frac{a_-}{\bar{a}} + 1 - \lambda} \tag{29}$$

Proof: See Appendix D.□

The interpretation is the same as before, except that τ is fixed exogenously and that λ has a negative impact on ν. The increase in the replacement rate is essentially beneficial for agents with a long length of life, i.e. for rich agents. Condition (29) is therefore more restrictive than condition (28) and cannot be true for a λ which tends towards 1.

Now we can study welfare inequalities. These inequalities can be measured as the difference between the utility of the richest ($U(a_+)$) and the utility of the poorest ($U(a_-)$). Analytically, it has the following form:

$$\begin{aligned} U(a_+) - U(a_-) = (1+\beta T(a_+))\ln(W(a_+)) - (1+\beta T(a_-))\ln(W(a_-)) + \\ \beta(T(a_+) - T(a_-))(\alpha - 1)\ln(k) + cste \end{aligned} \tag{30}$$

If the redistributivity of the pension system increases (λ decreases), the previous differential decreases if and only if:

$$\begin{aligned} -(1+\beta T(a_+))\frac{dW(a_+)/d\lambda}{W(a_+)} - \beta(T(a_+) - T(a_-))(\alpha - 1)\frac{dk/d\lambda}{k} < \\ -(1+\beta T(a_-))\frac{dW(a_-)/d\lambda}{W(a_-)} \end{aligned} \tag{31}$$

This equation is useful because it shows in detail the different channels through which redistributivity has an impact on the utility differential. Let us study the case of a defined-benefit pension system. First let us assume that the condition of proposition 5 is true. Then, we show that the wealth ratio ($W(a_-)/W(a_+)$) is an increasing function of the redistributivity of the pension system, i.e.:

$$-\frac{dW(a_+)/d\lambda}{W(a_+)} < -\frac{dW(a_-)/d\lambda}{W(a_-)} \tag{32}$$

[23] It is a sufficient condition.

The condition (31) is more restrictive if, for the moment, we neglect the impact on the interest rate. Indeed, the richest can benefit from their wealth for a longer period of time. Then, a decrease in wealth inequalities does not necessarily imply a decrease in the utility differential. Nevertheless, the left-hand-side also shows that the decrease in interest rate more strongly affects the richest that save more because they live longer. This last effect reduces the utility differential.

5 Calibration and Results

We choose to calibrate our model on French data because the French pension system is purely Bismarckian. As it will be mentioned later, Hairault and Langot (2008) find that in the French pension system λ is 0.885. Then, we can consider the opportunity to switch towards a more Beveridgeian pension system.

The availability of data thanks to the study of Hairault and Langot (2008) is a key factor influencing our choice to consider the French case[24].

First of all we have to define an interval for the set Ω_a. We assume that it is: $\Omega_a = [0.08,1]$. The ratio a_+/a_- is 12.5. It implies that the wage inequality ratio between the poorest and the richest is 12.5. Piketty (2002), studying the distribution of wages in France, finds a ratio of 5 between the wages of the first and of the last decile. The gap between this empirical fact and our calibration can be explained by the fact that we use two extreme values of a *continuum,* which results in greater wage inequalities. We could even say that it underestimates the reality. We choose this interval for Ω_a because once it is combined with the density function of *a,* our model matches the Gini coefficient of wage distribution calculated by Hairault and Langot (2008) on French data.

The density function of productivity levels ($f(a)$) has to respect the essential property: mode<median<mean (Lambert (2001, pp.23)). This property is common for most industrialized countries. It implies that the wage distribution among population is asymmetric. The most common income level is less than the median wage. And, because of strong wage inequalities, the median wage is less than the average wage in the economy.

$f(a) = b - ca$, with $b, c \in R$ is the simplest way to represent it. b and c have to be fixed, so that: $f(a) > 0$, $\forall a$ and $\int_{\Omega_a} f(a)da = 1$. Furthermore, the Gini index has to tend towards 0.32 in order to match the estimation on French data used in Hairault and Langot (2008).

Lambert (2001) shows that the Gini index can be calculated as:

$$G = -1 + 2\int_{a_-}^{a_+} \frac{aF(a)f(a)}{\bar{a}}da \tag{33}$$

The following density function respects these properties:

$$f(a) = 2{,}1129 - 1{,}9a \tag{34}$$

[24] Appendix G sums up our calibration and the main statistics.

Moreover, we can check that the mean is higher than the median because $\int_{\underline{a}}^{\overline{a}} f(a)da > 0{,}5$.

The second important function that we have to specify is $T(a)$. For simplicity reasons and lack of information, we assume that this function has the form: $T(a) = a$. We obtain that $\overline{T} = 0.4167$ and that $COV_{T(a)a} = 0.5533$. It implies that the average length of life of the population is 77 years. It is slightly lower than the average life expectancy observed in France which is 80 years (World Bank)[25].

The initial value of λ is fixed at 0.885. It is the estimate obtained by Hairault and Langot (2008) on French data. It implies that the French pension system is highly Bismarckian. The growth rate of the population is $n = 0.3$. It corresponds to the annual population growth rate of 0.65% calculated by Charpin (1999) on French data. The technology parameter is normalized to 1.

Finally, the last two parameters are presented in most economic literature using calibration to solve the overlapping-generations model. The length of each period is 40 years. The elasticity of the production function with respect to capital is $\alpha = 0.33$. It also represents the share of capital in total output. The pure time preference factor is $\beta = 0.6$ (d'Autume (2003)), i.e. it equals the annual psychological discount rate of 1.3%.

We analyse the effects of a decrease in λ, i.e. an increase in the Beveridgeian part of the pension system[26]. We distinguish between the long term effects and transitional dynamics for defined-benefit and defined-contribution pension systems.

5.1 The Long-Term Effects

In a defined-benefit pension system, it is the tax rate that adjusts itself and the average replacement rate (ν) is fixed at 0.757, i.e. the value obtained by Hairault and Langot (2008) on French data. The annual interest rate obtained is approximately 4.4%.

Table 1: Macroeconomic impact of a more redistributive pension system (2008) on French data. The annual interest rate obtained is approximately 4.4%

$\Delta\lambda$ = -11.3%	Defined-Benefit	Defined-Contribution
$\Delta\tau^{a}$	-2.5%	–
$\Delta\nu$	–	+2.55%
Δk^*	+2.7%	-0.15%
$\Delta W(a_+)$	+1.14%	-0.55%
$\Delta W(a_-)$	+2.68%	+0.053%
ΔRIW^{b}	+1.53%	+1.08%
$\Delta BDPpc^{c}$	+0.88%	-0.05%
$\Delta dUtil^{d}$	-1.18%	-0.85%
IGb^{e}	0.3383	0.3351
IGa	0.3364	0.3338

[25] Appendix F shows that this has no impact on our qualitative results.
[26] Appendix F provides a sensitivity analysis.

[a] *Here we report a change in % and not in % pts.*
[b] *RIW = $W(a_-)/W(a_+)$.*
[c] *GDP_{pc} means GDP per capita*
[d] *dUTIl = $U(a_+) - U(a_-)$.*
[e] *IGb (IGa) denotes the Gini coefficient before (after) the change in λ.*

Note: The annual interest rate is obtained by $R^{1/40}$ - 1, where R is the interest factor obtained by equation (9).
Source: Author

Qualitatively, the results are as expected. Indeed, an increase in the redistributivity of the pension system (a decrease in λ) has a negative impact on the tax rate, and a positive one on the steady state capital per worker, on the GDP per capita and on the wealth level. Welfare inequalities decrease.

Quantitatively, by arbitrarily reducing λ from 0.885 to 0.785, i.e. by 11.3%, we observe a decrease in the tax rate by 2.49%. The steady state level of capital per worker and GDP per capita increase by 2.7% and 0.88% respectively. Welfare inequalities decrease by 1.18. Finally, the Gini coefficient of wealth[27] decreases, which means that wealth inequalities decrease. Table (1) sums up the main results.

We now study the case of a defined-contribution pension system. The tax rate is fixed exogenously at 0.23. It is the value calculated by Hairault and Langot (2008), and it approximates the tax rate reported by Nyce and Schieber (2005). We also study the impact of an arbitrary decrease in the annual interest rate of approximately 3.9%.

Qualitatively the results show an increase in the replacement rate. Furthermore, the net effect on saving is negative since the steady state capital per young decreases. This last effect implies a decrease in the wealth of the richest, whereas the net effect remains positive for the poorest because of the redistributive effect.

Quantitatively, a decrease in λ by 11.3% (from 0.885 to 0.785), results in an increase in the replacement rate by 2.55%. The steady state level of capital per young and GDP per capita decrease by 0.15% and 0.05% respectively. The utility inequalities decrease by 0.85%. As before, we observe a decrease in the Gini coefficient of wealth, i.e. a decrease in wealth inequalities. Table (1) sums up the main results.

Two conclusions can right now be stressed: (i) the net impact is greater for a defined-benefit pension system than for a defined-contribution pension system because in the first case every effect has the same sign. (ii) For a defined-contribution pension system the only positive impact of the redistributivity is to reduce inequalities.

5.2 The Transitory Dynamics

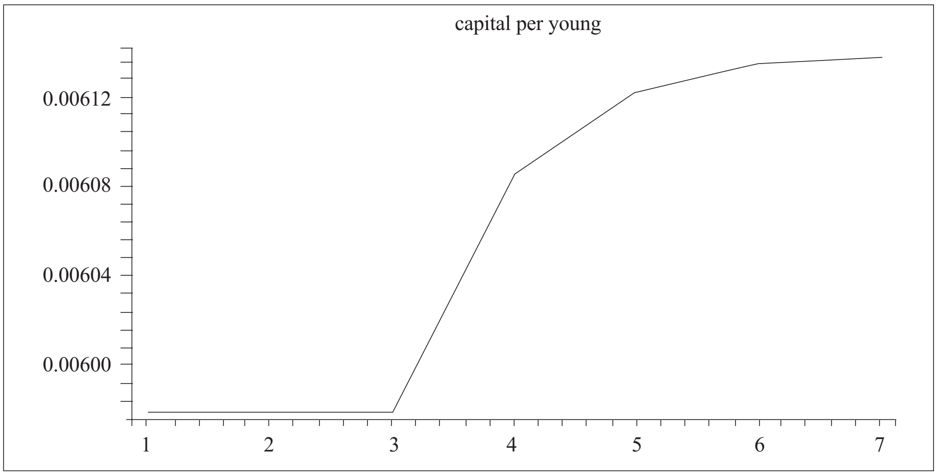
The main objective of this part is to study the short-term effects of an unexpected decrease in λ by 11.3%. We assume that the economy is initially at its steady state. λ is assumed to remain constant during the first two periods and then to decrease to 0.785. Agents born in period 2 do not expect this change and thus do not adjust their saving. But, for every following generation the assumption of perfect foresight implies that they exactly

[27] Using the same methodology as for wage distribution, the Gini coefficient of wealth is obtained by applying the formula of Lambert (2001:33): $G = -1 + 2\int_{a_-}^{a_+} \frac{W(a)F(a)f(a)}{\overline{W}(a)} da$

adjust their saving in order to maximize their utility. Because of the unpredictability of the change in λ, capital per worker remains constant until period 3 and is only adjusted in the following periods.

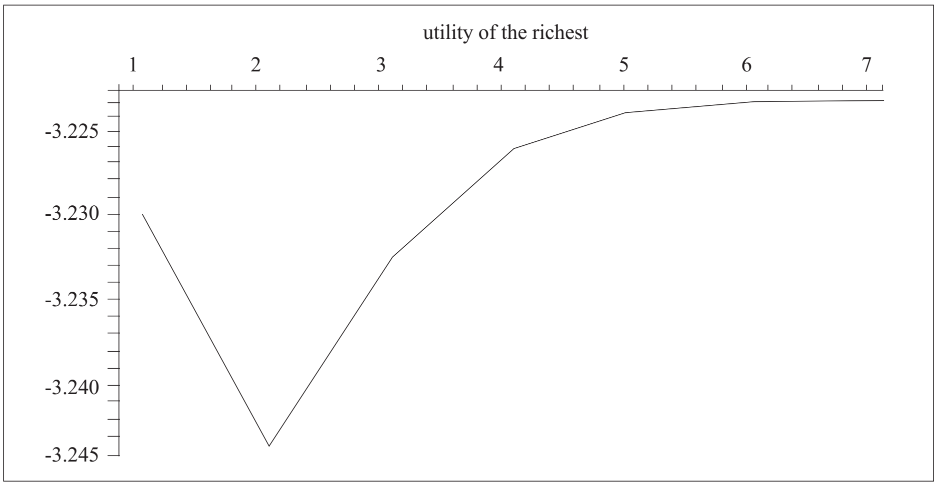
In a defined-benefit pension system the tax rate becomes 0.31 from period 3 (0.3 initially). Agents born in period 1 are not affected by this change and are used as a reference. The capital per young is adjusted progressively to its new steady-state value. The utility

Figure 1 Capital per young (k_t) for defined-benefit pension systems. Periods are reported on the abscissa.



Source: Author's calculation

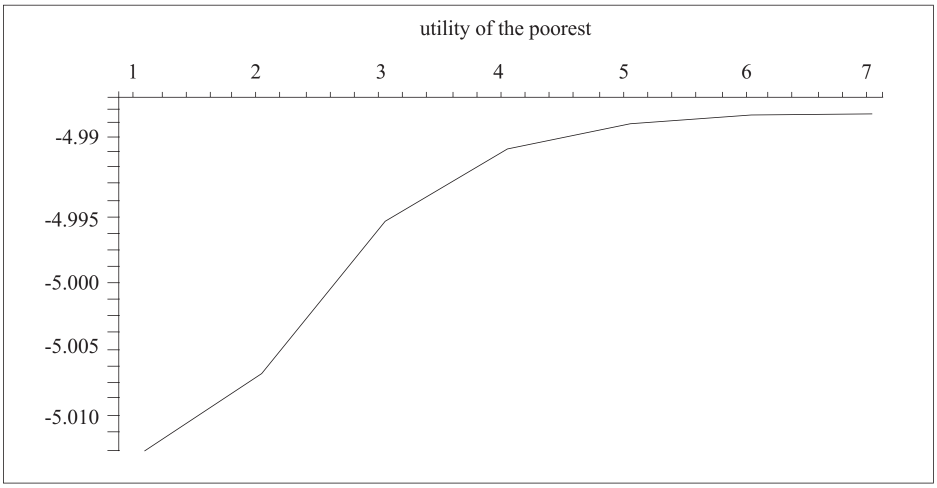
Figure 2 Utility of the richest ($U_t(a_+)$) for defined-benefit pension systems



Note: For example $U_1(a_+)$ is the utility of the richest born in period 1.

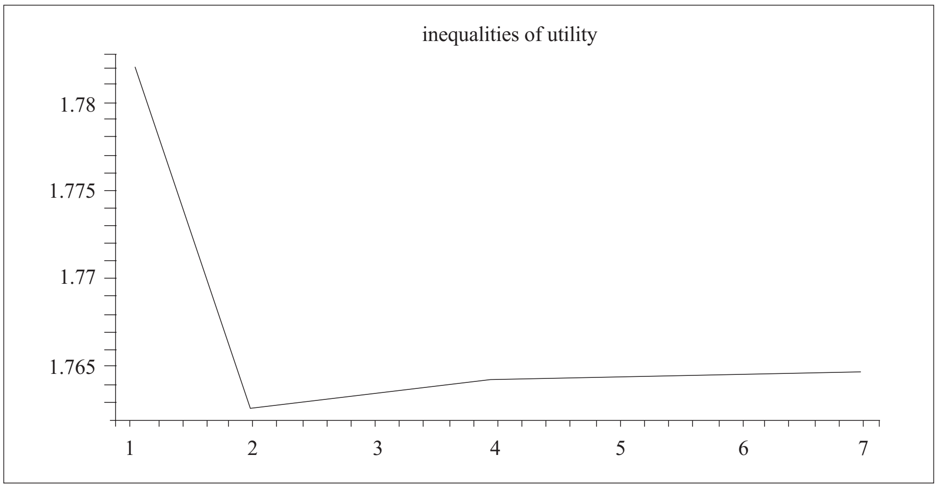
Source: Author's calculation

Figure 3 Utility of the poorest $U_t(a\_)$ for defined-benefit pension systems



Note: For example $U_1(a\_)$ is the utility of the poorest born in period 1.
Source: Author's calculation

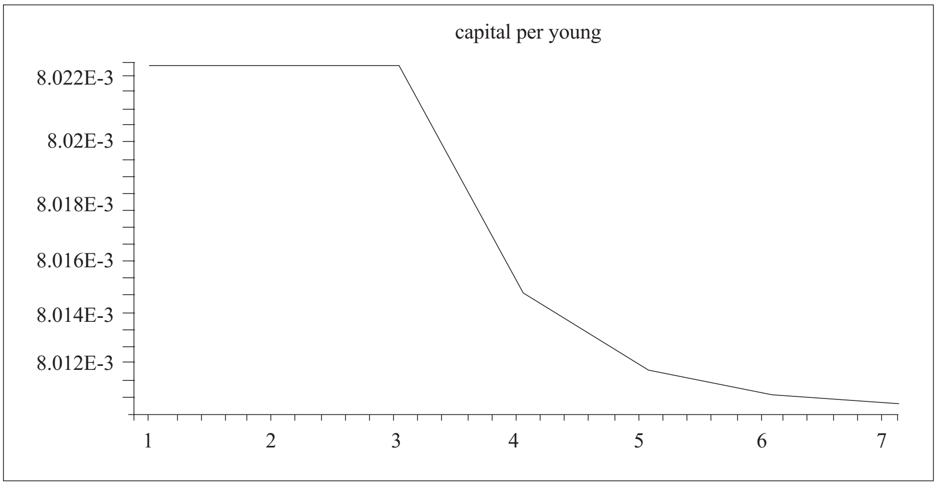
Figure 4 Utility differential for defined - benefit pension systems



Source: Author's calculation

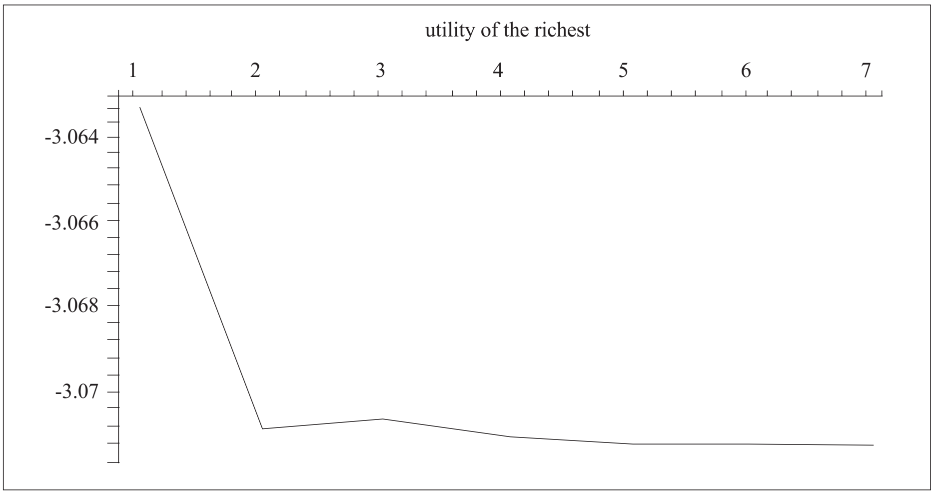
of the richest decreases substantially for agents born in period 2 because they do not sufficiently save for their second period of life. But the utility of the poorest increases until it reaches a new steady state value which is higher.

Figure 5 Capital per young (k_t) for defined-contribution pension systems



Source: Author's calculation

Figure 6 Utility of the richest ($U_t(a_+)$) for defined-contribution pension systems

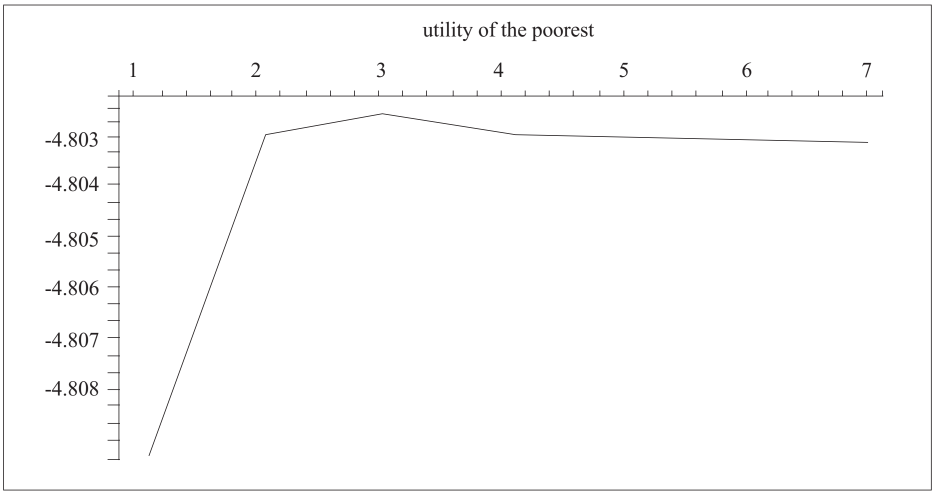


Source: Author's calculation

Utility inequalities decrease strongly, as early as the second generation, and then stabilize after a very small increase because of adjustment in the saving of the richest. Figures 1-4 sum up the main results.

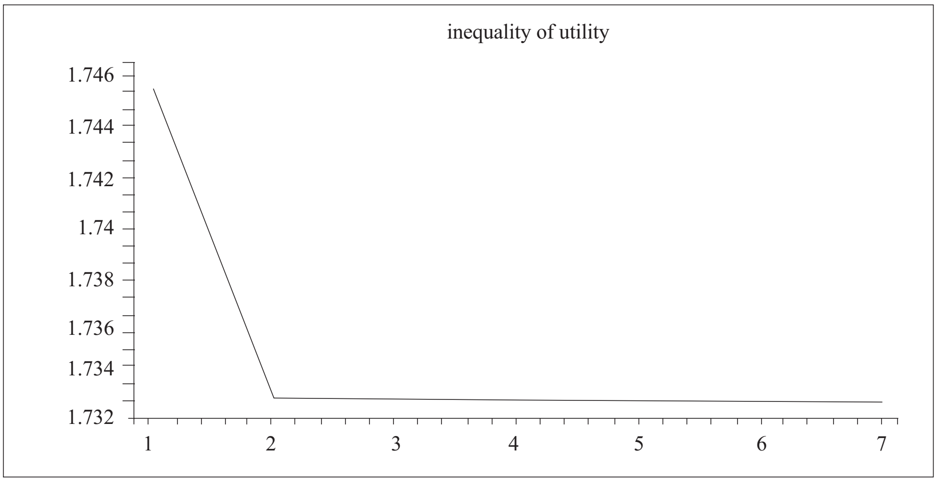
For defined-contribution pension systems the simulation is the same. Qualitative results show a quick adjustment of variables towards their new steady state value. Only the

Figure 8 Utility of the poorest ($U_t(a_-)$) for defined-contribution pension systems



Source: Author's calculation

Figure 9 Utility differential for defined-contribution pension systems



Source: Author's calculation

utility levels of consumers born in period 2 describe a different trajectory. The utility of the richest and that of the poorest decrease and increase respectively. Figures 5-8 sum up the main results.

Remark: Qualitative and quantitative results are very different depending on the nature of the pension system (defined-benefit or defined-contribution). This has to be taken into account in studying the impact of a change in the redistributive properties of a pension system.

6 Conclusion

An increase in the redistributivity of a defined-benefit pension system can: (i) decrease the tax rate of the pension system; (ii) increase the capital per capita; (iii) increase the wealth and welfare of every agent; (iv) reduce inequalities in wealth and welfare. However, if the pension system has a defined-contribution structure, then the only positive effect is that it increases the wealth and utility of poorest agents.

Therefore, both the knowledge of the nature of a pension system (defined-benefit or defined-contribution) and taking into account of life expectancy inequalities are important for the assessment of qualitative and quantitative impacts of a more redistributive pension system.

The first extension of this paper would be to introduce labour supply in order to take into account the distortive impact of our redistributive policy.

Another application of this paper would be to study the impact of redistributive policies on educational choices. In the case of capital-skill complementarity, and given the above described mechanism, a possible implication of a more redistributive pension system is that a larger share of the population decides to educate itself.

Appendix A

A computation of the expression of τ:

$$(1+n)\int_{\Omega_a} \tau w_{t+1}(a) f(a) da = \int_{\Omega_a} p_{t+1}(a) f(a) da. \tag{35}$$

Furthermore, we know that:

$$p_{t+1}(a) = \nu A(1-\alpha)\frac{k_{t+1}^{\alpha}}{\overline{a}^{\alpha}}(\lambda a + (1-\lambda)\overline{a})T(a). \tag{36}$$

By computing the right-hand-side, we obtain the following expression:

$$RHS = \nu A(1-\alpha)\frac{k_{t+1}^{\alpha}}{\overline{a}^{\alpha}}\left(\lambda \int_{\Omega_a} T(a) a f(a) da + (1-\lambda)\overline{a}\int_{\Omega_a} T(a) f(a) da\right) \tag{37}$$

Equation (3) implies: $\int_{\Omega_a} T(a) a f(a) da = COV_{T(a),a} + \overline{a}\overline{T}$. The second part of the expression between brackets is the average length of life. Finally we have:

$$RHS = \nu A(1-\alpha)\frac{k_{t+1}^{\alpha}}{\overline{a}^{\alpha}}(\lambda COV_{T(a),a} + \overline{a}\overline{T}). \tag{38}$$

On the left-hand-side we recognize the average wage: $\int_{\Omega_a} w_{t+1}(a) f(a) da$. Then, by equalizing the left-hand-side to the right-hand-side we obtain equation (16).

Appendix B

a) The study of equation (20) shows that τ and the denominator become independent of λ if, $T(a) = a$, $\forall a$.

b) Let us consider the case of homothetic preferences which have the following form:

$$U_t(a) = U\left(c_t(a), \frac{d_{t+1}(a)}{T(a)}\right). \tag{39}$$

The intertemporal budget constraint of this agent is:

$$c_t(a)+\frac{d_{t+1}(a)}{R_{t+1}}=w_t(a)(1-\tau)+\frac{p_{t+1}(a)}{R_{t+1}}\equiv W_t(a) \tag{40}$$

Given the preferences, the solution for consumers is:

$$c_t(a)=\xi(T(a),R_{t+1})W_t(a) \tag{41}$$

And finally:

$$S_t(a)=w_t(a)(1-\tau)-c_t(a)=\xi_1(T(a),R_{t+1})w_t(a)-\xi_2(T(a),R_{t+1})\frac{p_{t+1}(a)}{R_{t+1}} \tag{42}$$

Therefore, saving is a linear function of wage and of pension. Assuming that the length of life is the same for every agent ($T(a) = \forall a$), then the capital market equilibrium can be written as:

$$(1+n)k_{t+1}=\int_{\Omega_a}S_t(a)f(a)da \tag{43}$$

or:

$$(1+n)k_{t+1}=\xi_1(T,R_{t+1})\overline{w}_t-\xi_2(T,R_{t+1})\frac{\nu T\overline{w}_{t+1}}{R_{t+1}} \tag{44}$$

λ does not appear in this expression.

Appendix C

Proof of Proposition 4:

The derivative of equation (22) with respect to λ gives the following expression:

$$\frac{dW_t(a)}{d\lambda}=aA\frac{1-\alpha}{\overline{a}^{\alpha}}\left(\alpha k^{\alpha-1}\frac{dk}{d\lambda}(1-\tau)-\frac{d\tau}{d\lambda}k^{\alpha}\right)+\nu\frac{1-\alpha}{\alpha}T(a)$$
$$\left[\left(\lambda\frac{a}{\overline{a}}+1-\lambda\right)\frac{dk}{d\lambda}+k(\frac{a}{\overline{a}}-1)\right]$$

We know that $d\tau/d\lambda > 0$ and that $dk^*/d\lambda < 0$. Finally, the previous expression is negative if the second part of the equation is negative, i.e. if:

$$\frac{\frac{dk}{d\lambda}}{k} \leq \frac{1-\frac{a}{\overline{a}}}{\lambda\frac{a}{\overline{a}}+1-\lambda}$$

However, as the right-hand-side is a decreasing function of a, it is sufficient for this inequality to be true for $a = a_+$.

Remark: This inequality is always true for $a < \overline{a}$

Proof of Proposition 4 (bis):

The methodology is the same as before, except that τ is fixed exogenously and that ν is a decreasing function of λ.

Appendix D

Proof of Proposition 5:

Equation (22) can be written as:

$$W_t(a) = A(1-\alpha)\frac{k_t^{\alpha}}{\overline{a}^{\alpha}}a(1-\tau)+\nu\frac{1-\alpha}{\alpha}k_{t+1}\left(\lambda\frac{a}{\overline{a}}+(1-\lambda)\right)T(a) \tag{45}$$

or, in the steady state:

$$W_t(a) = k\left[A(1-\alpha)\frac{k^{\alpha-1}}{\overline{a}^{\alpha}}a(1-\tau)+\nu\frac{1-\alpha}{\alpha}\left(\lambda\frac{a}{\overline{a}}+(1-\lambda)\right)T(a)\right] \tag{46}$$

With equation (20), the left-hand-side between brackets can be written as:

$$a\frac{1+n+\nu\frac{1-\alpha}{\alpha}\int_{\Omega_a}\frac{\lambda\frac{a}{\overline{a}}+(1-\lambda)}{1+\beta T(a)}T(a)f(a)da}{\beta\int_{\Omega_a}\frac{T(a)a}{1+\beta T(a)}f(a)da} \equiv af(\lambda)$$

Equation (46) becomes:

$$W_t(a) = k\left[af(\lambda)+\nu\frac{1-\alpha}{\alpha}\left(\lambda\frac{a}{\overline{a}}+(1-\lambda)\right)T(a)\right] \tag{47}$$

The relative wealth inequalities can be written as:

$$\frac{W_t(a_-)}{W_t(a_+)}=\frac{a_-f(\lambda)+\nu\frac{1-\alpha}{\alpha}\left(\lambda\frac{a_-}{\bar{a}}+(1-\lambda)\right)T(a_-)}{a_+f(\lambda)+\nu\frac{1-\alpha}{\alpha}\left(\lambda\frac{a_+}{\bar{a}}+(1-\lambda)\right)T(a_+)} \tag{48}$$

The result of the proposition is obtained if the derivative of this expression with respect to λ is negative. It is true if and only if:

$$[f(\lambda)-\lambda f'(\lambda)]\left[a_+\left(\frac{a_-}{\bar{a}}-1\right)T(a_-)-a_-\left(\frac{a_+}{\bar{a}}-1\right)T(a_+)\right]<$$

$$<f'(\lambda)[a_+T(a_-)-a_-T(a+)]+\nu\frac{1-\alpha}{\alpha}T(a_+)T(a_-)\left(\frac{a_+-a_-}{\bar{a}}\right)$$

The left-hand-side has two components. The second is obviously negative. It is straight-forward to show that $1>\lambda\frac{f'(\lambda)}{f(\lambda)}$ and then that the left hand side is negative.

It only remains to show that the right-hand-side is positive. It is true under the condition of the proposition.

Proof of Proposition 5 (bis):

The relative wealth inequalities can be written as:

$$\frac{W_t(a_-)}{W_t(a_+)}=\frac{A(1-\alpha)\frac{k^{\alpha-1}}{\bar{a}^{\alpha}}a_-(1-\tau)+\nu\frac{1-\alpha}{\alpha}\left(\lambda\frac{a_-}{\bar{a}}+(1-\lambda)\right)T(a_-)}{A(1-\alpha)\frac{k^{\alpha-1}}{\bar{a}^{\alpha}}a_+(1-\tau)+\nu\frac{1-\alpha}{\alpha}\left(\lambda\frac{a_+}{\bar{a}}+(1-\lambda)\right)T(a_+)} \tag{49}$$

The derivative of this expression with respect to λ is negative, if and only if:

$$\frac{1-\alpha}{\alpha}c\left[(\alpha-1)k^{\alpha-2}\frac{dk}{d\lambda}-\frac{d\nu}{d\lambda}k^{\alpha-1}\right]\left[a_-T(a_+)(\lambda\frac{a_+}{\bar{a}}+1-\lambda)-a_+T(a_-)(\lambda\frac{a_-}{\bar{a}}+1-\lambda)\right]$$

$$+\frac{1-\alpha}{\alpha}ck^{\alpha-1}\left[a_+T(a_-)(\frac{a_-}{\bar{a}}-1)-a_-T(a_+)(\frac{a_+}{\bar{a}}-1)\right]$$

$$+\nu\frac{1-\alpha}{\alpha}T(a_-)T(a_+)\left(\frac{a_--a_+}{\bar{a}}\right)<0$$

where $c=A(1-\alpha)\frac{1-\tau}{\bar{a}^{\alpha}}>0$.

The last two terms are strictly negative. Then, under the condition $dk/d\lambda < 0$, and knowing that $dv/d\lambda < 0$, the sign of the first term depends only on the sign of the condition mentioned in the proposition.

Appendix E

The covariance can also be written as: $\int_{\Omega_a}(a-\bar{a})(T(a)-\bar{T})f(a)da$. But as $\int_{\Omega_a}(a-\bar{a})f(a)da = 0$, we can write that: $\int_{\Omega_a}(a-\bar{a})(T(a)-\bar{T})f(a)da = \int_{\Omega_a}(a-\bar{a})(T(a)-X)f(a)da$ with X a constant, whatever the value of X. So it is particularly true for $X = T(\bar{a})$. Then, we can write that: $\int_{\Omega_a}(a-\bar{a})(T(a)-\bar{T})\,f(a)da = \int_{\Omega_a}(a-\bar{a})(T(a)-T(\bar{a}))f(a)da$. The RHS is positive as it is an integral on a product of terms with the same sign because $T'(a) > 0$.

Appendix F

In this Appendix we try to determine if our qualitative results depend on an initial condition, on the form taken by $T(a)$ or on values taken by our parameters, notably the average replacement rate (ν) or tax rate (τ). In doing so, we extend our results to other countries besides France.

For defined-benefit pension systems

Firstly, let us consider the impact of a decrease in λ in function of its initial value. A simple numeric exercise, using our calibration, shows that our qualitative results remain true whatever the initial value of λ and whatever the percentage of change in λ. It implies that a decrease in λ has always a positive impact on capital per capita and on the wealth of every agent. It also always has a negative impact on wealth inequalities, on the Gini coefficient and on the utility differential (*dUtil*).

Secondly, we do the same exercise but with the new function $T(a) = a^{0,75}$. The form of this function implies that the average life expectancy of agents in our model is 80 years, which matches the observed life expectancy in most industrialized countries. We find the same qualitative results. As previously, our results do not depend on the initial value taken by λ.

Thirdly, we solve our model for different values of ν ($\nu \in \{0.757, 0.6, 0.4\}$)[28]. Whatever the function $T(a)$ which is chosen, our qualitative results are unchanged.

[28] $\nu = 0.4$ seems to be the lowest replacement rate among industrialized countries. See Nyce and Schieber (2005:236).

For defined-contribution pension systems:

We find a monotonous relationship between macroeconomic variables and λ. It implies that the impact of λ on macroeconomic variables has the same sign as this reported in Table 1, whatever its initial value.

As in the defined-benefit case, the use of the functional form $T(a) = a^{0.75}$ has no impact on our qualitative results. λ still has a monotonous impact on macroeconomic variables.

Finally, we check whether our qualitative results remain unchanged for $\tau \in \{0.1;0,23;0,3\}$.

Appendix G

In this Appendix, we sum up our calibrations of the functions and of the parameters of our model. Furthermore, we show some important statistics.

The Basic Calibration

The length of each period is 40 years. Table 2 sums up the basic parameters used for the numerical resolution of our model.

Table 2 Basic Calibration of the Model

Parameter	Meaning	Value	Source(s)
α	$R_t K_t / Y_t$ [a]	0.33	Sommacal (2006) among others
β	Actualization factor	0.6	APDR = 1.3%[b], d'Autume (2003) Heer and Maussner (2005)
	The technology level	1	Normalization
A	Population's growth rate	0.3	AGR = 0.65%[c], Charpin (1999)
n	Average replacement rate[d]	0.757	Hairault and Langot (2008)
ν	Tax rate[e]	0.23	Hairault and Langot (2008)
τ	Initial value of λ[f]	0.885	Hairault and Langot (2008)

[a] *The share of income spent on capital.*
[b] *Annual psychological discount rate.*
[c] *AGR = annual growth rate.*
[d] *For defined-benefit pension systems.*
[e] *For defined-contribution pension systems.*
[f] *We use this value as a reference. We analyse the effects of a decrease in λ knowing that λ is initially λ_I.*

Source: Author

The Calibration of Functions and Their Main Statistics

Firstly, we calibrate the interval Ω_a. We use:

$$\Omega_a = [0.08,1]$$

The ratio a_+/a_- is lower than the one found in Acemoglu (2002), but higher than the one in Piketty (2002). The corresponding density function is:

$$f(a) = 2.1129 - 1.9a \tag{50}$$

These two components respect the two main properties:

• Mode < median < mean; Source: Lambert (2001), and

• $IG_{w} = 0.32$[29] in France; Source: Hairault and Langot (2008), INSEE (1999)

In our model, we have:

$$\begin{gathered} \overline{a} = 0.4167 \\ a_{medium} = 0.378 \\ a_{mode} = a_{-} = 0.08 \\ Var(a) = 0.005533 \end{gathered}$$

Secondly, we calibrate the function $T(a)$:

$$T(a) = a$$

It implies that the distribution of the length of life has the same properties as the distribution of the variable. Furthermore, we have:

$$COV_{T(a),a} = Var(a) = 0.05533$$

Knowing that the length of each period is 40 years, the average length of life[30] is 77 years. It is lower than the figure for France which is around 80 years[31] (Source: INSEE or World Bank). The standard deviation is:

$$\sigma_{T(a)} = 0.24$$

which corresponds to a standard deviation of almost 9.4 years[32].

[29] IG_w denotes the Gini coefficient of wages.

[30] The life expectancy of each individual is $(1+T(a)) * 40$

[31] Appendix F shows that this has no impact on our qualitative results.

[32] The standard deviation for the function $T(a) = a^{0.75}$ is below 9 years.

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