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DEFLECTION OF OVER-REINFORCED CONCRETE BEAMS: COMPARISON OF ANALYTICAL, NUMERICAL AND EXPERIMENTAL RESULTS

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Subject review

Prediction and forecasting of deflection is a complex assignment because of the factors which are varying along the axis and with the time. Thus, the approximate procedures are used, in which the obtained results are mutually different and different in a comparison with values measured on real structure. In this paper an analytical calculation of deflection of five over-reinforced concrete beams according to Eurocode 2 is presented. The beams had different dimensions in order to investigate the impact of beam size on load-bearing capacity and deflection of over-reinforced concrete beams. Comparison of deflection obtained analytically, according to Eurocode 2, and the results obtained experimentally and numerically by other authors, is also presented. The major objective of this research is to determine the differences between the analytically obtained results, and experimentally and numerically obtained results.

Keywords: *over-reinforced concrete beam, reinforcement ratio, deflection, analytical calculation, analytical results, experimental results, numerical results*

Progib prearmiranih betonskih greda: usporedba analitičkih, numeričkih i eksperimentalnih rezultata

Pregledni članak

Predviđanje i prognoziranje progiba kompleksan je zadatak zbog čimbenika koji se mijenjaju uzduž osi elementa i vremenski. Stoga se upotrebljavaju približni postupci kojima se rezultati razlikuju međusobno kao i u odnosu na izmjerene vrijednosti na konstrukciji. U radu je prikazan analitički proračun progiba pet prearmiranih betonskih greda prema Eurokodu 2. Grede su različitih dimenzija kako bismo mogli ispitati utjecaj dimenzije prearmirane betonske grede na njezinu nosivost i pojavu progiba. Također je prikazana usporedba progiba prearmiranih betonskih greda dobivenih analitičkim proračunom prema Eurokodu 2 i rezultata autora koji su izvršili eksperimentalna mjerenja i numeričku analizu za iste grede. Osnovni cilj ovog istraživanja je određivanje odstupanja analitički dobivenih rezultata progiba, i eksperimentalno i numerički dobivenih rezultata.

Ključne riječi: *prearmirane grede, koeficijent armiranja, progib, analitički proračun, analitički rezultati, eksperimentalni rezultati, numerički rezultati*

1 Introduction
Uvod

In most national standards for the design of concrete structures there are limits for maximal reinforcement ratio in a tensile zone, in order to avoid the brittle failure of concrete. It is known that the compressive failure of reinforced concrete member is a brittle failure, even if the concrete of normal strength, which is more ductile than the high strength concrete, is used. In the compression failure of reinforced concrete beams, concrete crushes before steel yields. Such a beam is said to be over-reinforced. Concrete of over-reinforced beam reaches ultimate stress, but steel does not reach the yield strain. Fracturing of concrete is a process that may take different forms in structures of different size and shape. Therefore, it is important to predict correctly the compressive failure of the structural elements of different sizes. In order to investigate whether the existing numerical models are suitable for predicting the experimental response of the over-reinforced concrete beams, RILEM Technical Committee has organized a research program to study the response of over-reinforced concrete beams subjected to four-point loading conditions. Authors of this paper have applied the Eurocode 2 procedures and compared the results with numerical and experimental results obtained by research program of RILEM Technical Committee, i.e. they have not performed either numerical or experimental investigations. Presented is also an analytical calculation of deflection according to Eurocode 2 for the five over-reinforced concrete beams of different geometry. Results of the analytical calculation are compared with the experimental and numerical results of other authors for the same over-reinforced concrete beams. The major objective of this research is to determine the disagreement between the analytical results, and experimental and numerical results.

2 Calculation of curvature and deflection according to Eurocode 2
Proračun zakrivljenosti i progiba prema Eurokodu 2

Analysis of the deflections should consider two possibilities: uncracked conditions, in which steel and concrete participate in load-bearing capacity, and fully cracked conditions where the contribution of the tensile zone of concrete is neglected. On a load-deflection diagram for a reinforced concrete beam, shown in Figure 1, three significant zones may be distinguished.

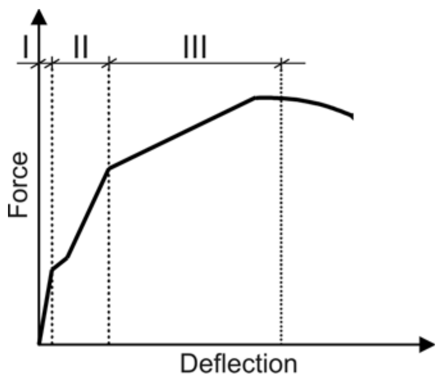

Figure 1 *Load-deflection diagram*
Slika 1 *Dijagram opterećenje-progib*

Zone I is the one before the appearance of the first crack, i.e. the zone in which a reinforced concrete member behaves elastic. Reinforced concrete member is in zone II after appearance of the first crack on the reinforced concrete member, and before the steel yields. Hence, zone II is a region after the cracking of the member that may not be fully cracked. Zone III appears after the steel yields and in that region the curve on load-deflection diagram has a lower slope. Furthermore, with the small increasing of the load values there is an abrupt increase of the deflection. The neutral axis in this zone translates to the compressive edge of cross section, until the full cracking of the concrete in the critical cross section compression zone appears.

For members subjected to a flexure mainly, an adequate prediction of behavior in zone I and zone II is given by Expression (1) [1, 2]:

$$\alpha = (1-\zeta)\cdot\alpha_{\mathrm{I}} + \zeta\cdot\alpha_{\mathrm{II}} \tag{1}$$

where:
α - is the deformation parameter, in this case it is a curvature
α_{I}, α_{II} - are the values of the parameter calculated for the uncracked and fully cracked conditions respectively
ζ - is a distribution coefficient given by Expression (2) [1, 2]:

$$\zeta = 1-\beta\cdot\left(\frac{\sigma_{\mathrm{sr}}}{\sigma_{\mathrm{s}}}\right)^2 \tag{2}$$

$\zeta=0$ for uncracked sections
β - is a coefficient taking into account the influence of the duration of the loading, or repeated loading, on the average strain
σ_{s} - is the stress in the tension reinforcement calculated on the basis of a cracked section
σ_{sr} - is the stress in the tension reinforcement calculated on the basis of a cracked section under the loading conditions causing first cracking.

Arithmetic mean value of the curvature due to load and shrinkage ($1/r_{\mathrm{m}}$) is given by Expression (3):

$$\frac{1}{r_{\mathrm{m}}} = (1-\zeta)\cdot\frac{1}{r_{\mathrm{I}}} + \zeta\cdot\frac{1}{r_{\mathrm{II}}} \tag{3}$$

where:
$1/r_{\mathrm{I}}$ – is a curvature for the uncracked condition I according to Expression (4):

$$\frac{1}{r_{\mathrm{I}}} = \frac{M_{\mathrm{Sd}}}{E_{\mathrm{c,eff}}\cdot I_{\mathrm{id}}} \tag{4}$$

I_{id} – is a filled rectangular area moment of inertia for the uncracked condition I
M_{sd} – is a design value of the applied internal bending moment.
α_{e} - is the effective modular ratio according to Expression (5) [1, 2]:

$$\alpha_{\mathrm{e}} = \frac{E_{\mathrm{s}}}{E_{\mathrm{c,eff}}} \tag{5}$$

where:
E_{s} – is the design value of modulus of elasticity for reinforcing steel
$E_{\mathrm{c,eff}}$ – is an effective modulus of elasticity for concrete.

For the long duration loads which may cause the creep, the total deformation including the creep may be calculated by using an effective modulus of elasticity for concrete according to Expression (6) [1]:

$$E_{\mathrm{c,eff}} = \frac{E_{\mathrm{cm}}}{1+\varphi(t_\infty, t_0)} \tag{6}$$

where:
$\varphi(t_\infty, t_0)$ - is the creep coefficient relevant for the load and time interval.
$1/r_{\mathrm{II}}$ – is a curvature for the fully cracked condition II according to Expression (7):

$$\frac{1}{r_{\mathrm{II}}} = \frac{\varepsilon_{\mathrm{s}}}{d-x} \tag{7}$$

where:
ε_{s} – is the strain of reinforcement given by Expression (8) [1, 2]:

$$\varepsilon_{\mathrm{s}} = \frac{\sigma_{\mathrm{s}}}{E_{\mathrm{s}}} \tag{8}$$

σ_{s} - is the stress in the tension reinforcement calculated on the basis of a cracked section.

Shrinkage curvature ($1/r_{\mathrm{cs}}$) may be assessed using Expression (9) [1]:

$$\frac{1}{r_{\mathrm{cs}}} = \frac{\varepsilon_{\mathrm{cs}}\cdot\alpha_{\mathrm{e}}\cdot S}{I} \tag{9}$$

where:
$\varepsilon_{\mathrm{cs}}$ – is the free shrinkage strain
S – is the first moment of area of the reinforcement about the centroid of the section
I – is the second moment of area of the section
α_{e} - is the effective modular ratio according to (5).

S and I should be calculated for the uncracked and the fully cracked condition; the final curvature ($1/r_{\mathrm{cs,m}}$) may be assessed using Expression (10):

$$\frac{1}{r_{\mathrm{cs,m}}} = \zeta\cdot\frac{1}{r_{\mathrm{cs,II}}} + (1-\zeta)\cdot\frac{1}{r_{\mathrm{cs,I}}} \tag{10}$$

where:
$1/r_{\mathrm{cs,I}}$ – is the shrinkage curvature for the uncracked condition I
$1/r_{\mathrm{cs,II}}$ – is the shrinkage curvature for the fully cracked condition II.

Total curvature ($1/r_{\mathrm{tot}}$) may be assessed by Expression (11):

$$\frac{1}{r_{\mathrm{tot}}} = \frac{1}{r_{\mathrm{m}}} + \frac{1}{r_{\mathrm{cs,m}}} \tag{11}$$

For the element with a constant height, a simplified method is often used, where the deflection (v_{tot}) may be assessed according to Expression (12):

$$v_{\text{tot}} = k \cdot L^2 \cdot \frac{1}{r_{\text{tot}}} \tag{12}$$

where:
k – is a coefficient which depends on different structural systems and applied load
L – is the span of element.

3 Analytical calculation of curvature and deflection for over-reinforced concrete beams

Analitički proračun zakrivljenosti i progiba prearmiranih betonskih greda

An analytical calculation of deflection of five over-reinforcement concrete beams was conducted according to the Eurocode 2, under the load conditions shown in Figure 2. The beams are made with the normal strength concrete (NSC). The cross section and the span of the beams are shown in Table 1 and Figure 3. The properties of used materials, as well as geometry characteristics and load condition of two beams – NSC/B1 and NSC/B3 – have been chosen to be identical to the parameters of beams investigated in research of RILEM Technical Committee [3-6], in order to compare the results obtained analytically with numerical and experimental results of RILEM's research. The properties of used materials, as well as geometry characteristics and load condition of other three beams – NSC/B2, NSC/B4 and NSC/B5 – have been chosen in order to obtain the trend of load impact on beam deflection.

The beams are designed for compressive failure of the concrete. For that purpose, the beams of normal strength concrete have the reinforcement ratio ρ=7,3 %.

The properties of the concrete and steel are shown in Table 2. For the over-reinforced concrete beams NSC/B1 to NSC/B5 that are exposed to the load, as it is shown in Figure 2, the analytical calculation of deflection according to Eurocode 2 was carried out. The results of the analytical calculation, as well as the trend of load impact on beam deflection, are shown in Table 3 and Figure 4.

4 Comparison of analytical with the experimental and numerical results

Usporedba analitičkih s eksperimentalnim i numeričkim rezultatima

The RILEM Technical Committee "Strain Softening of Concrete" has organized interinstitutional research program with the aim to predict the load-deformation behavior of over-reinforced concrete beams. The major objective of the program was to research whether existing numerical models are suitable to predict the experimental response of over-reinforced concrete beams. Thereby, the numerical analysis and experimental research were carried out for the beams that were made with the three different materials, [3-6]. The different concretes were the Normal Strength Concrete (NSC), the High Strength Concrete (HSC) and the Fibre Reinforced High Strength Concrete (FRHSC). To investigate the size effect on failure of over-reinforced concrete beams, the experimental research and numerical analysis were performed on two different sizes of over-reinforced beams.

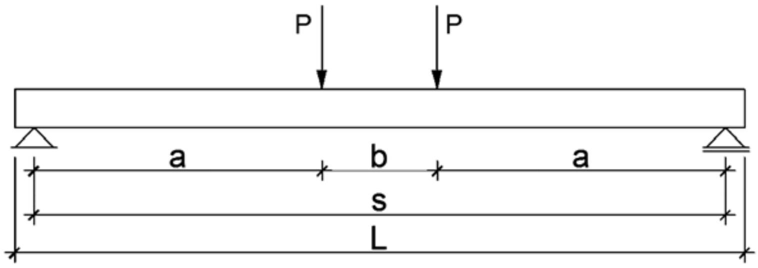

Figure 2 *Load condition for beams*
Slika 2 *Prikaz opterećenja greda*

Table 1 *Geometry characteristics of the beams*
Tablica 1 *Geometrijske karakteristike greda*

Beam	Cross section (mm)	Distance *a* (mm)	Distance b (mm)	Span *s* (mm)	Total length *L* (mm)
NSC/B1	100 × 200	1500	600	3600	3800
NSC/B2	150 × 300	2250	900	5400	5650
NSC/B3	200 × 400	3000	1200	7200	7500
NSC/B4	250 × 500	3750	1500	9000	9350
NSC/B5	300 × 600	4500	1800	10800	11200

Table 2 *Properties of used materials* [3]
Tablica 2 *Svojstva upotrijebljenog materijala* [3]

Normal strength concrete		Steel	NSC/B1	NSC/B2	NSC/B3	NSC/B4	NSC/B5
E_c (MPa)	22000	∅ (mm)	8 ∅15,2	9 ∅19	9 ∅20	9 ∅32	9 ∅40
ν	0,18	E_s (MPa)	195000	222000			
f_t (MPa)	1,89	f_y (MPa)	1650	650			
f_{ck} (MPa)	22,75	α_e	31,02	35,32			

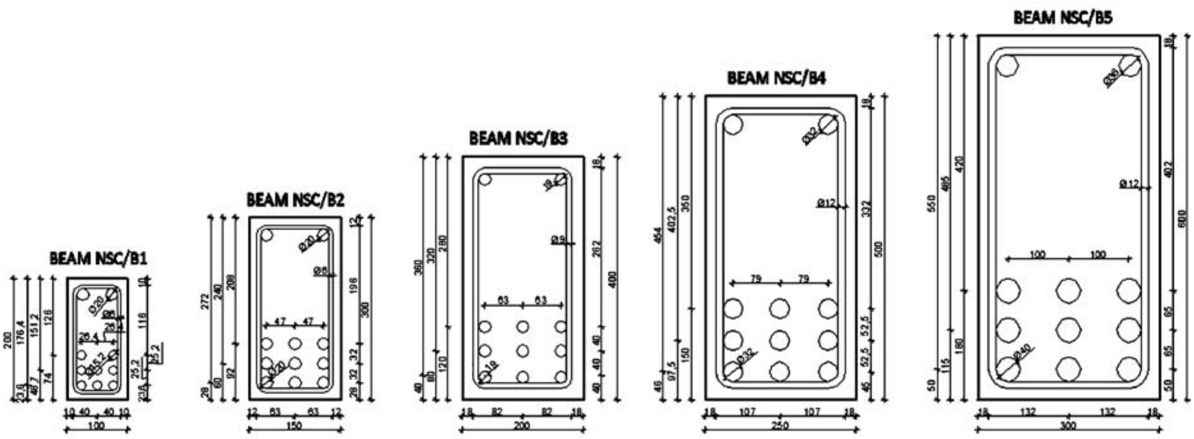

Figure 3 *Schematic drawing of the beams*
Slika 3 *Shematski prikaz greda*

Table 3 *Results of analytical calculation in accordance to Eurocode 2*
Tablica 3 *Rezultati analitičkog proračuna prema Eurokodu 2*

Beam	P (kN)	M_{Sd} (kNm)	$1/r_I$ (mm^{-1})	$1/r_{II}$ (mm^{-1})	$1/r_{cs,I}$ (mm^{-1})	$1/r_{cs,II}$ (mm^{-1})	$1/r_{tot}$ (mm^{-1})	δ_{tot} (mm)
NSC/B1	16,4	24,6	$7{,}2\times10^{-5}$	$9{,}2\times10^{-5}$	$2{,}3\times10^{-5}$	$6{,}0\times10^{-6}$	$9{,}8\times10^{-5}$	12,24
NSC/B2	36,9	83,0	$5{,}9\times10^{-5}$	$8{,}7\times10^{-5}$	$2{,}7\times10^{-5}$	$7{,}7\times10^{-6}$	$9{,}5\times10^{-5}$	26,51
NSC/B3	64,1	192,3	$4{,}1\times10^{-5}$	$7{,}2\times10^{-5}$	$2{,}2\times10^{-5}$	$7{,}4\times10^{-6}$	$8{,}0\times10^{-5}$	39,97
NSC/B4	102,5	384,2	$3{,}2\times10^{-5}$	$5{,}0\times10^{-5}$	$1{,}6\times10^{-5}$	$4{,}6\times10^{-6}$	$5{,}4\times10^{-5}$	42,28
NSC/B5	147,5	663,9	$2{,}5\times10^{-5}$	$4{,}2\times10^{-5}$	$1{,}3\times10^{-5}$	$3{,}9\times10^{-6}$	$4{,}6\times10^{-5}$	51,28

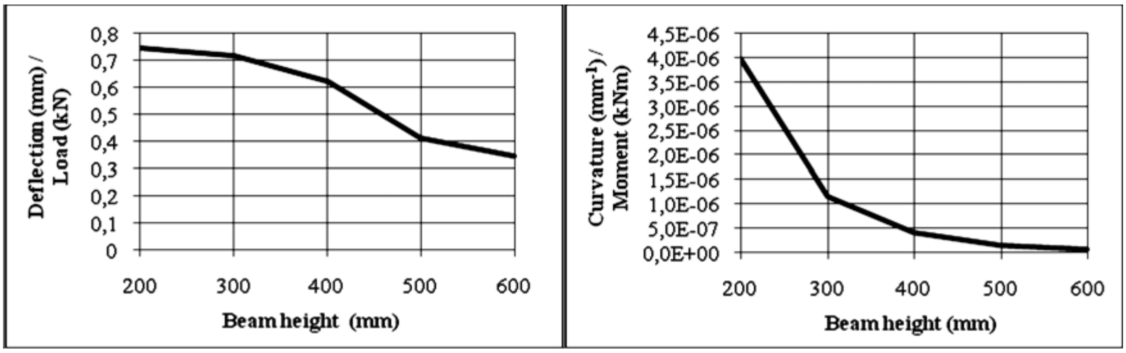

Figure 4 *Graphical representation of the analytical results*
Slika 4 *Grafički prikaz analitičkih rezultata*

Table 4 *Comparison of numerically, experimentally and analytically obtained deflection*
Tablica 4 *Usporedba progiba dobivenih numerički, eksperimentalno i analitički*

Type of the beam	Experimental [3]		Numerical [3]		Analytical	
	P_u (kN)	δ_u (mm)	P_u (kN)	δ_u (mm)	P_u (kN)	δ_u (mm)
NSC/B1	16,4	21,0	14,7	13,5	16,4	12,24
NSC/B3	64,1	47,8	56,2	39,00	64,1	39,97

The cross section areas of the small (NSC/B1) and large (NSC/B3) beams are presented in Figure 3. Material properties for the NSC beams are given in Table 2. Some authors [3] use a complete three-dimensional finite element code. Numerical analysis was conducted with the program MASA3 for three-dimensional finite element modeling. Steel bars are modeled as truss elements. Results of experimental research are obtained as the arithmetic mean value of three values obtained experimentally. In this paper, the results obtained analytically, as it is presented in Sections 2 and 3, are compared with numerical and experimental results for the normal strength concrete beams (NSC) only. Results obtained analytically, as well as results of numerical analysis and experimental research are shown in Table 4.

Numerically obtained ultimate load capacity and related deflection are undervaluated. However, qualitative behavior of construction is predicted in a satisfying manner [3]. The comparison of the maximal load capacity implies that the size effect on the load-bearing capacity is not significant. Geometrical reduction of beam size in all three directions with the factor 2 causes a decreasing of nominal load-bearing capacity for about 5 % [3]. Experimental results for the failure of the beam confirm clearly the existence of a significant size effect on the increasing of the failure brittleness.

Figure 5 presents load-deflection diagram and indicates the position of analytically obtained deflection, according to Eurocode 2, in a comparison with Figure 9 in reference

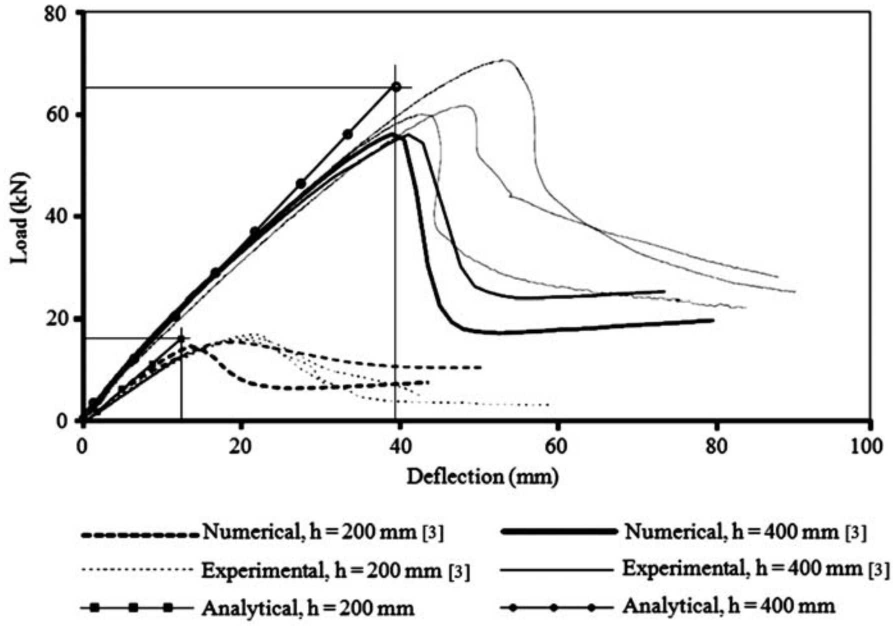

Figure 5 *Load-deflection diagram for NSC/B1 and NSC/B3 beams*
Slika 5 *Dijagram opterećenje-progib za NSC/B1 i NSC/B3 grede*

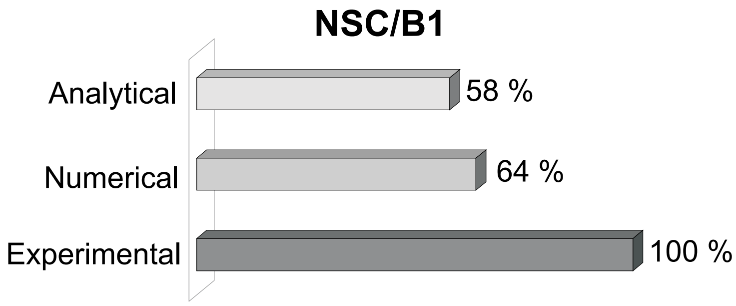

Figure 6 *Normalized value of deflection of NSC/B1 beam at the peak load*
Slika 6 *Normalizirana vrijednost progiba NSC/B1 grede pri vršnom opterećenju*

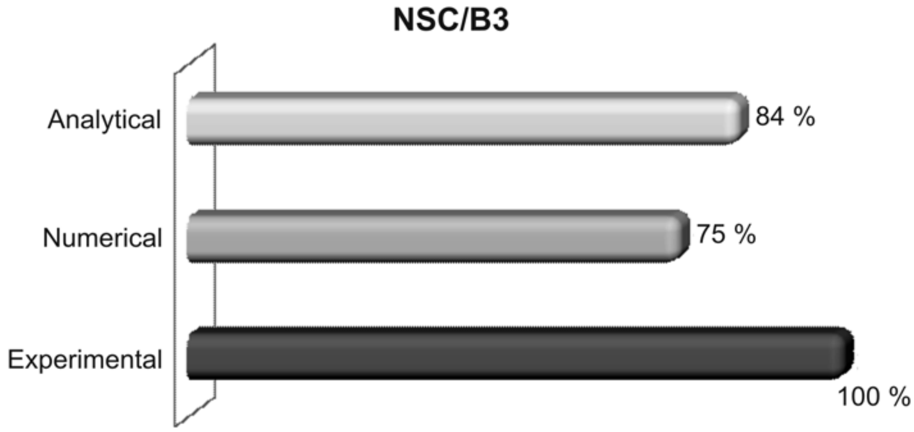

Figure 7 *Normalized value of deflection of NSC/B3 beam at the peak load*
Slika 7 *Normalizirana vrijednost progiba NSC/B3 grede pri vršnom opterećenju*

[3]. Contrary to the results of experiment and numerical analysis, the results obtained analytically cover the zone up to the peak load only.

Figures 6 and 7 show that, at the peak load, the disagreement between the numerical and analytical results is not significant for the two analyzed beams. Thereby, analytically obtained deflection for NSC/B1 is about 36 % smaller than the deflection obtained by the experiment. For NSC/B3 difference between the deflection obtained by experiment and analytically is smaller (about 16 %).

5 Conclusion
Zaključak

The major objective of this paper is to find out how the analytically obtained results according to Eurocode 2 deviate from experimentally and numerically obtained results. In this paper an analytical calculation of total curvature and the deflection of five over-reinforced concrete beams according to Eurocode 2 are presented.

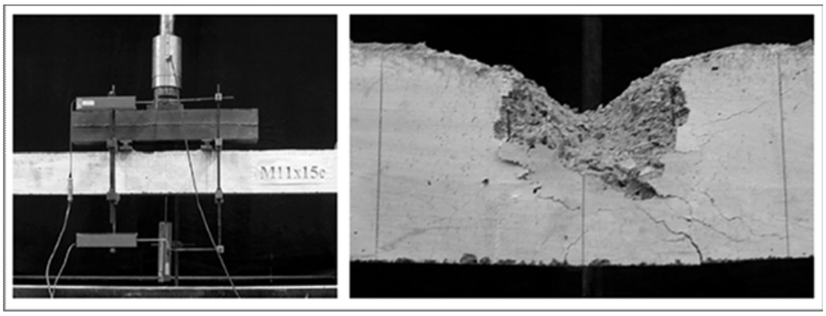

Figure 8 *Failure of an over-reinforced concrete beam* [7]
Slika 8 *Slom prearmirane betonske grede* [7]

Deflection obtained analytically, according to Eurocode 2, is compared with the results obtained experimentally and numerically by RILEM Technical Committee research program. Based on the results of this research the following conclusions are drawn for the analyzed beams:

The beams under the constant bending moment fail by crushing in the compression zone around the peak load. Some local cracks were seen prior to crushing. The typical mode of failure of an over-reinforced beam is shown in Figure 8 [7].

Analytical calculation of the deflection according to Eurocode 2 is carried out for five beams. For two of them, the results obtained analytically are compared with numerical and experimental results obtained by RILEM's research. At the peak load, the disagreement between the numerical and analytical results is not significant for the two beams. Thereby, analytically obtained deflection for smaller beam (NSC/B1) is about 36 % smaller than the deflection obtained by the experiment. For larger beam (NSC/B3) difference between the deflections obtained by experiment and analytically is smaller (about 16 %).

From the comparison of the numerically and experimentally [3, 4, 5] obtained maximal load capacity it can be seen that there is no significant size effect on the load-bearing capacity. Experimental results of the failure of the beam confirm clearly the existence of a significant size effect on the increasing of the failure brittleness [3, 5, 7]. Predicting the ultimate load-bearing capacity with numerical methods is possible with the accuracy of 10 - 15 % [5].

6
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