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NUMERICAL SOLVING OF BALLISTIC FLIGHT EQUATIONS FOR BIG BORE AIR RIFLE

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Subject review

This paper is about solving ballistic equations by means of numerical mathematics. Ballistic flight equations are applied for modern big bore air rifles, operated with high pressure carbon-dioxide (CO_2) gas. Ballistic equations use air drag function for obtaining results. Today there are many complex commercial ballistic programs on market, based on modified mass point model – "MPMM" or "6DOF" model. Big bore air rifle commonly uses ball projectiles, and velocities of the projectiles are lower than the speed of sound. Therefore simplified models for quick calculations of ballistic trajectories and projectile velocities can be used.

Key words: *ballistic equations, big bore air rifle, air drag*

Numeričko rješavanje balističkih jednadžbi za zračnu pušku velikog kalibra

Pregledni članak

Tema ovog rada je rješavanje balističkih jednadžbi uz pomoć numeričke matematike. Balističke jednadžbe su primijenjene za moderne zračne puške velikog kalibra, punjene s ugljik-dioksid (CO_2) plinom. Balističke jednadžbe koriste otpor zrakom ispunjenog prostora pri izračunu rezultata. Danas postoje raznovrsni komplicirani komercijalni balistički programi na tržištu, koji su bazirani na "modified point mass – MPMM" modelu, ili pak na "6 Degrees of Freedom" modelu. Zračna puška velikog kalibra koristi kuglu kao projektil, i brzine projektila su uglavnom manje od brzine zvuka. Stoga se mogu koristiti pojednostavnjeni modeli za brzo izračunavanje balističkih putanja i brzina projektila.

Ključne riječi: *balističke jednadžbe, zračna puška velikog kalibra, otpor zraka*

1 Introduction
Uvod

Today on the world market one can find various commercial ballistic software, which can calculate ballistic trajectories, and projectile speed loss during a flight. All of these programs use database with significant projectile characteristics, needed for ballistic calculations. Main characteristics of the projectiles are given by manufacturers, and this data is obtained by measurements or numeric simulations. These programs are complex because they also take into account a change of drag coefficient during flight of projectile as a variable. The drag coefficient C_d is a function of projectile velocity among other variables. It is also measured or obtained by simulations. If velocity of projectile is smaller than the speed of sound in the air, the drag coefficient is assumed to be constant. Therefore for velocities air rifle can produce, C_d is constant. By raising projectile velocity over speed of sound, drag coefficient changes. Generally Mach number is ratio between projectile velocity (v) at some moment during flight, and the speed of sound in the air (v_s), [1]:

$$M_a = \frac{v}{v_s} \tag{1}$$

If M_a <1, it is subsonic region
if M_a >1, it is supersonic region
if M_a= 1, it is transitional region.

Speed of sound in the air can be calculated by [2]:

$$v_s = \sqrt{\kappa \cdot R \cdot T} \ \text{ m/s}, \tag{2}$$

Where:
κ - ratio of specific heat capacities c_p/c_v, for air κ=1,4
R_{air} - individual gas constant, J/(kg·K)
T – thermodynamic temperature, K

$$R_{air} = \frac{R_m}{M} = \frac{8314}{28,97} = 286,987 \ \text{ J/(kg·K)} \tag{3}$$

R_m – gas constant → R_m = 8314 J/(kmol·K)
M – molecular mass of air → M = 28,97 kg/kmol
For the given data, and temperature ϑ =20 °C,
T = 293,15 K

$$v_s = \sqrt{\kappa \cdot R_{air} \cdot T} = \sqrt{1,4 \cdot 286,987 \cdot 293,15} = 343,2 \ \text{ m/s}$$

For big bore air rifle model, velocity of a projectile is highest when projectile is leaving bore, and it is getting smaller with every shot because CO_2 tank pressure reduces. Even with the first shot, maximal velocity of projectile is smaller than the speed of sound.

Drag function can be calculated by using:

$$F_d = \frac{1}{2} \cdot \rho \cdot A \cdot v^2 \cdot C_d, \ \text{ N} \tag{4}$$

where :
ρ – air density, kg/m^3
A – cross section of projectile, m^2
v – projectile velocity, m/s
C_d – drag coefficient.

C_d is the value that is determined for every shape of the projectile. If C_d is small, then the drag force on body/projectile is small and vice-versa. C_d depends on dimension of projectile, shape of projectile, surface roughness and Reynold's number. For this model approximate value for small sphere C_d = 0,45 will be used [3].

Also for calculating trajectories commercial software uses other factors like projectile/bullet spin, which affects dynamical stabilization. So if projectile has a high frequency of rotation it is over stabilized and vice-versa.

Figure 1 illustrates: a) perfectly stabilized projectile, b) under stabilized projectile, c) over stabilized projectile. The variable which describes this phenomenon is called yaw of angle and it represents the angle between projectile principal axis and tangent on ballistic trajectory.

In the case of air rifle, a ball is used as a projectile, and for its symmetry this phenomenon of stabilized projectile can be ignored. In order to determine the yaw of angle in every moment of time, complex mathematical solutions like MPMM (modified mass point model), or 6DOF (6 degrees of freedom) are to be used.

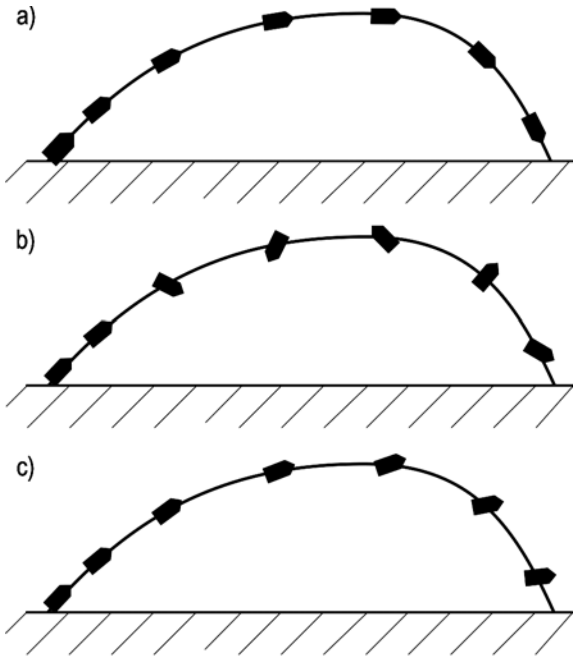

Figure 1 *Various cases of projectile stabilization*
Slika 1 *Različiti slučajevi stabilizacije projektila*

For comparison of cases, in literature [8, 9], one can find expressions for ballistic equations in vacuum ("in vacuo") model.

2 Ballistic equations including wind drag
Balističke jednadžbe s obzirom na otpor zrakom ispunjenog prostora

As already mentioned(4), drag function can be expressed as:

$$F_\mathrm{d} = \frac{1}{2} \cdot \rho \cdot A \cdot v^2 \cdot C_\mathrm{d}\,, \mathrm{N}.$$

It is assumed that projectile is flying in x-y coordinate system, with x coordinate representing the range, and y coordinate representing the height of flight. The drag force can be projected on axes – force divided to components. In the case of side wind, the projectile drifts sideways, and this is projected on z axis. There can also be range wind (blowing in positive or negative direction of x axis), and vertical wind, but it is very rare. Vertical wind has almost no effect on the projectile height, so it is neglected in equations. While calculating with side and range wind, components of velocity change, and velocity vector composed of those components also changes. This motion is complex. Coordinate system for this problem is called – carried local coordinate system, and velocity is marked as $\tilde{v}$.

2.1 Euler model without wind
Eulerov model putanje zrna - bez bočnog vjetra

According to Figure 2, one can see forces acting upon projectile. Force of air drag is trying to slow down the projectile, and the force of gravity is pulling the projectile down. Vector equation can be written as [4]:

$$m \cdot \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = m \cdot \vec{g} - \frac{\rho \cdot v^2}{2} \cdot A \cdot C_\mathrm{d} \cdot \frac{\vec{v}}{v}\;. \tag{5}$$

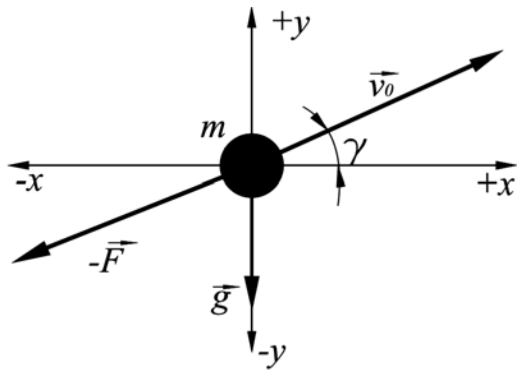

Figure 2 *Drag force, and Earth gravity on the projectile*
Slika 2 *Sila otpora zrakom ispunjenog prostora i ubrzanje Zemljine sile teže koje djeluju na projektil*

The term

$$\frac{\vec{v}}{v} = \frac{v_x}{v}\vec{i} + \frac{v_y}{v}\vec{j} + \frac{v_z}{v}\vec{k} = (\cos\alpha)\vec{i} + (\cos\beta)\vec{j} + (\cos\gamma)\vec{k}$$

represents direction cosines of trajectory tangent.

Vector of velocity is by definition, derivation of radii-vector in time. Radii-vector describes position of projectile from the origin of the coordinate system.

$$\frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \vec{v} \tag{6}$$

First vector equation (5) is projected on tangent and normal of trajectory. Equation (6) is projected on axes of local coordinate system [4].

$$\left.\begin{aligned} \frac{\mathrm{d}v}{\mathrm{d}t} &= -g \cdot \sin\gamma - \frac{\rho \cdot v^2}{2m} \cdot A \cdot C_\mathrm{d}, \\ \frac{\mathrm{d}\gamma}{\mathrm{d}t} &= \frac{-g \cdot \cos\gamma}{v}, \\ \frac{\mathrm{d}x}{\mathrm{d}t} &= v \cdot \cos\gamma, \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= v \cdot \sin\gamma, \end{aligned}\right| \tag{7}$$

where γ is angle between velocity vector $\vec{v}$, and positive x axis.

Solving these equations in classical way would be very difficult, but they can be approximated by means of numerical mathematics.

With the initial conditions set as: v_0 = 183 m/s, mass of bullet m = 14,6 grams, angle of departure measured from x axis γ=3°, bullet diameter d =12,7 mm (0,50 inch), ambient pressure p_0 = 101325 Pa, thermodynamic temperature T = 293,15 K, density of air can be calculated as follows:

$$\rho = \frac{p}{R_{\text{air}} \cdot T} = \frac{101325}{286{,}987 \cdot 293{,}15} = 1{,}2044 \ \text{kg/m}^3 \qquad (8)$$

With dt = 0,02 s, and applying Runge-Kutta method on equations (7), discrete values for the variables are calculated. Fourth order Runge-Kutta method is applied on equations (7) in such a way that first the velocity v is calculated. Next step is calculation of dv, dγ, dx and dy. These values are added to values of v, γ, x, y, respectively. Values v, γ, x, y are set at the beginning of program from the initial conditions. If condition is met, program is instructed to stop, and display results. If condition is not met, by the iterative procedure program calculates new values of dv, dγ, dx, dy, and adds them to the values v, γ, x, y, until condition is met. In this way for each value of time t, values v, γ, x, y numerically approximate analytical solution. Programming was done in Mathematica 5.2 for students. Programming can be done in C++, Fortran or other software. Following diagrams were constructed from results.

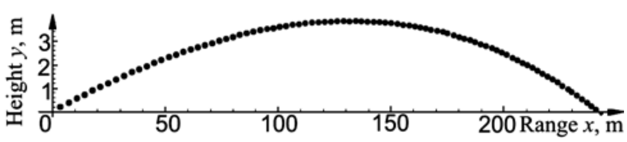

Figure 3 *Ballistic trajectory solved by Euler's equations*
Slika 3 *Balistička putanja riješena Eulerovim jednadžbama*

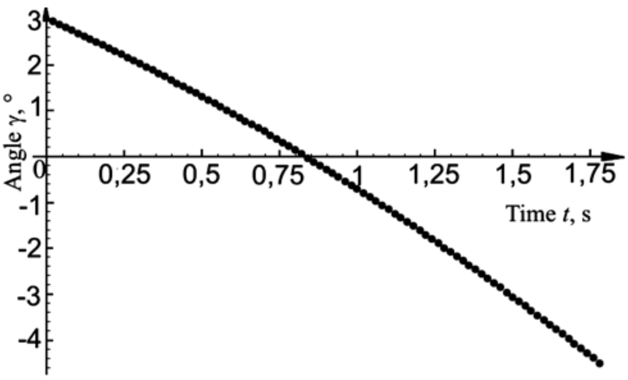

Figure 4 *Angle γ as a function of time t*
Slika 4 *Kut γ izražen kao funkcija vremena t*

From Figure 3, one can see that trajectory is more curved at the end, and this is clearly visible effect of the air drag. Figure 4 represents the angle between the velocity vector and x axis. The angle of departure at the beginning was set as 3°, and in the moment when projectile hit the ground, calculated angle was γ=−4,4°. In vacuum model (without air drag) there is no difference between the angle of departure and the angle γ when projectile hits the ground (flat).

Figure 5 shows scalar value of velocity vector decreasing through time because of air drag.

Initial velocity was set as v_0 = 183 m/s. Calculated velocity in the moment when bullet hits the ground is v = 105,3 m/s.

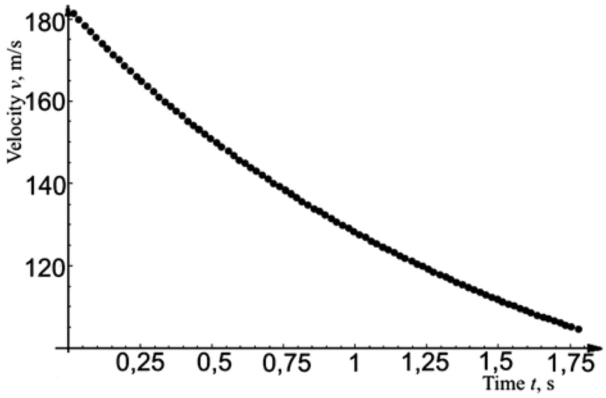

Figure 5 *Velocity of projectile as a function of time t*
Slika 5 *Brzina projektila kao funkcija vremena t*

2.2 Euler model with side wind
Eulerov model putanje zrna - s bočnim vjetrom

This model is actually just a minor modification of a classical Euler model. It uses wind velocity components to change the projectile velocity components, and thus the projectile velocity vector. According to [4] modified Euler's equations are as follows:

$$m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = m \cdot \vec{g} - \frac{\rho \cdot v^2}{2} \cdot A \cdot C_{\mathrm{d}} \cdot \frac{\vec{v}}{v} \qquad (9)$$

Equation of radii vector:

$$\left.\begin{array}{l} \dfrac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \vec{v}, \ \ \vec{v} = \vec{v_{\mathrm{k}}} + \vec{v_{\mathrm{w}}}, \\ \vec{v_{\mathrm{k}}} = u_{\mathrm{k}}\vec{i} + h_{\mathrm{k}}\vec{j} + w_{\mathrm{k}}\vec{k}, \\ \vec{w_{\mathrm{w}}} = u_{\mathrm{w}}\vec{i} + 0 \cdot \vec{j} + w_{\mathrm{w}}\vec{k}, \end{array}\right| \qquad (10)$$

where $\vec{v}_{\mathrm{w}}$ is wind velocity, $\vec{v}_{\mathrm{k}}$ projectile velocity according to classical Euler's model.

From equations (9), (10) components can be written as:

$$\left.\begin{array}{l} \dfrac{\mathrm{d}u}{\mathrm{d}t} = -E \cdot C_{\mathrm{d}} \cdot \dfrac{(u_{\mathrm{k}} - u_{\mathrm{w}})}{v}, \ \dfrac{\mathrm{d}x}{\mathrm{d}t} = u, \\ \dfrac{\mathrm{d}h}{\mathrm{d}t} = -E \cdot C_{\mathrm{d}} \cdot \dfrac{h_{\mathrm{k}}}{v} - g, \ \dfrac{\mathrm{d}y}{\mathrm{d}t} = h, \\ \dfrac{\mathrm{d}w}{\mathrm{d}t} = -E \cdot C_{\mathrm{d}} \cdot \dfrac{(w_{\mathrm{k}} - w_{\mathrm{w}})}{v}, \ \dfrac{\mathrm{d}z}{\mathrm{d}t} = w, \end{array}\right| \qquad (11)$$

where:

$$E = \frac{\rho \cdot v^2 \cdot A}{2 \cdot m}$$

$$v = \sqrt{(u_{\mathrm{k}} - u_{\mathrm{w}})^2 + h_{\mathrm{k}}^2 + (w_{\mathrm{k}} - w_{\mathrm{w}})^2} \quad \text{(see Figure 6)}$$

Assuming initial conditions as: v_0 = 183 m/s , mass of bullet m = 14,6 grams, the angle of departure measured from x axis γ = 3°, bullet diameter d = 12,7 mm (0,50 inch), density of air ρ = 1,2044 kg/m^3, with ambient pressure p_0 = 101325 Pa, thermodynamic temperature T = 293,15 K, components of wind velocity vector u_{w} = 5 m/s, w_{w} = 15 m/s, dt = 0,02 s, and applying Runge-Kutta method on equations

(11), discrete values for variables are calculated, and following diagrams constructed (Figures 7, 8, 9 and 10).

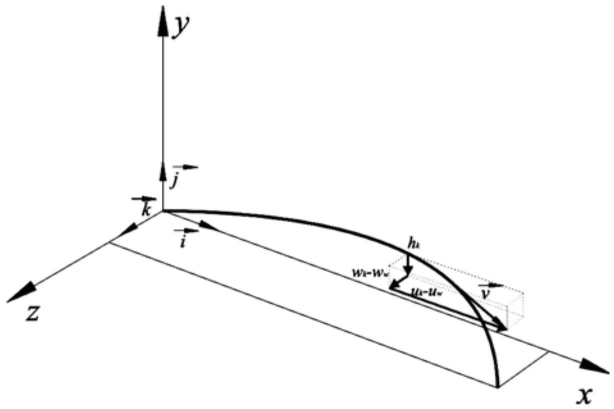

Figure 6 Ballistic trajectory in space, with components of velocity vector shown
Slika 6 Balistička putanja u prostoru, s prikazanim komponentama vektora brzine

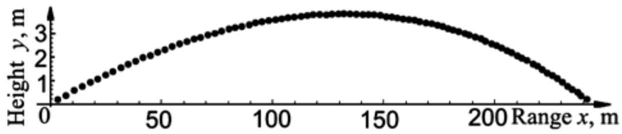

Figure 7 Ballistic trajectory in x-y coordinate system, solved by Euler's equations including wind
Slika 7 Balistička putanja u x-y koordinatnom sustavu, riješen pomoću Eulerovih jednadžbi uzimajući u obzir i bočni vjetar

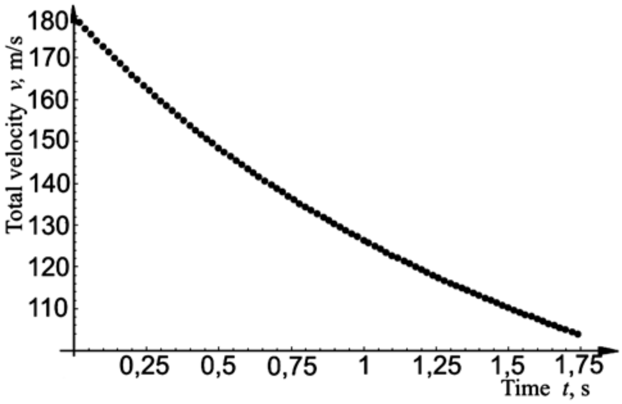

Figure 8 Total velocity of projectile as a function of time
Slika 8 Ukupna brzina projektila kao funkcija vremena

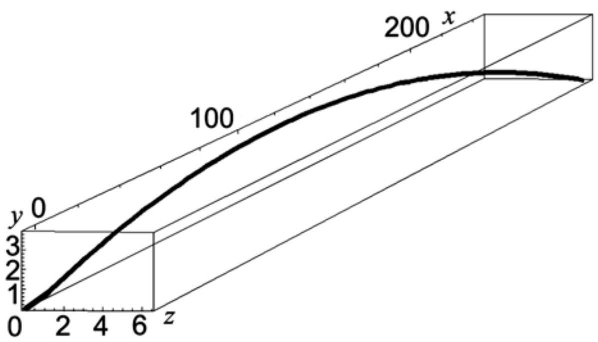

Figure 9 Ballistic trajectory in 3D, solved by Euler's equations including wind
Slika 9 Prostorni prikaz balističke krivulje riješene prema Eulerovim jednadžbama uključujući i bočni vjetar

From initial velocity 183 m/s, because of wind drag, velocity drops to v = 104 m/s, when projectile hits the ground.

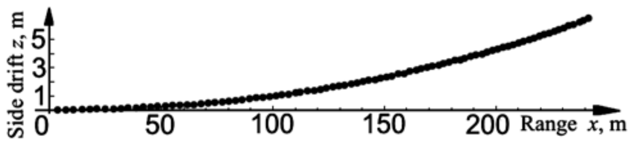

Figure 10 Projectile side drift as a function of range
Slika 10 Bočno zanošenje projektila kao funkcija dometa

Difference from previous results (Euler's equations without wind), can be seen in Figure 10.

As the initial speed was set rather small, the effect of air drag is not quite as visible in Figure 3 and Figure 7. Results obtained by numerically solving equations (7), (11) would have been different, if the initial speed was set higher, like v_0 = 300 m/s. Limit is that the initial speed must not be set higher than the speed of sound.

Air rifle with smaller caliber projectile than 12,7 mm, has higher initial velocity than 183 m/s, and the results calculated for that case show higher expressed appearance of the air drag, in the means of ballistic trajectories.

2.3 Flat fire model

Flat fire model

This is a bit different way of determining ballistic trajectories. It uses the same equations from the classic Euler's model. Main difference in this way of solving is that velocity vector $\vec{v}$ is divided on v_x and v_y components. Because of this separation, variables v_x and v_y have certain limitations. This model was developed during World War One, and in that time rifles were made with smooth bores. Projectiles had small exit velocities at bore end, and small range. The army still uses this model for calculating ballistics of high caliber guns, with small bullet velocities. These conditions are similar to the problem of big bore air rifle, therefore this model can be used. Assumptions are following: there are no Coriolis acceleration, Magnus force and force of aerodynamic lift.

Equations are [5]:

$$\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \frac{v_x\vec{i} + v_y\vec{j} + v_z\vec{k}}{\mathrm{d}t} = -\frac{A\cdot\rho\cdot C_\mathrm{d}}{2m}\cdot v\cdot\vec{v} - g\cdot\vec{j} \tag{12}$$

Components can be written:

$$\left.\begin{aligned}
\dot{v}_x &= \frac{\mathrm{d}v_x}{\mathrm{d}t} = -\overline{C}^*_\mathrm{d}\cdot v\cdot v_x,\\
\dot{v}_y &= \frac{\mathrm{d}v_y}{\mathrm{d}t} = -\overline{C}^*_\mathrm{d}\cdot v\cdot v_y - g,\\
\dot{v}_z &= \frac{\mathrm{d}v_z}{\mathrm{d}t} = -\overline{C}^*_\mathrm{d}\cdot v\cdot v_z,\\
v &= \sqrt{v_x^2 + v_y^2 + v_z^2},
\end{aligned}\right| \tag{13}$$

where:

$$\overline{C}^*_\mathrm{d} = \frac{A\cdot\rho\cdot C_\mathrm{d}}{2m}.$$

Assumptions:
$v_z = 0$ (there is no side wind)
$|v_y/v_x| = \tan\gamma < 0{,}1 \rightarrow \gamma < 5{,}7°$
$v_x/v < 0{,}5$ %.

According to [5] equations (13) can be transformed to use variable of range x, instead of variable of time t.

$$\frac{\mathrm{d}v_x}{\mathrm{d}x}\cdot\frac{\mathrm{d}x}{\mathrm{d}t} = -\bar{C}_{\mathrm{d}}^{*}\cdot v_x\cdot v_x \Rightarrow v_x^{'} = -\bar{C}_{\mathrm{d}}^{*}\cdot v_x\,, \tag{14a}$$

$$\frac{\mathrm{d}v_y}{\mathrm{d}x}\cdot\frac{\mathrm{d}x}{\mathrm{d}t} = -\bar{C}_{\mathrm{d}}^{*}\cdot v_x\cdot v_y - g \Rightarrow \frac{\mathrm{d}v_y}{\mathrm{d}x} = -\bar{C}_{\mathrm{d}}^{*}\cdot v_y - \frac{g}{v_x}. \tag{14b}$$

If the air drag coefficient C_{d} is constant (velocity of the projectile smaller than speed of sound), than equation (14a) can be solved as follows:

$$v_x = v_{x0}\cdot \mathrm{e}^{-\bar{C}_{\mathrm{d}}^{*}\int_0^x \mathrm{d}x_1}$$

and applying

$$k_1 = \bar{C}_{\mathrm{d}}^{*} \Rightarrow v_x = v_{x0}\cdot\mathrm{e}^{-k_1\cdot x}$$

Expression for time t is calculated as:

$$\left.\begin{aligned}
&\mathrm{d}t = \frac{\mathrm{d}x}{v_x},\quad v_x = v_{x0}\cdot\mathrm{e}^{-\int_0^x \bar{C}_{\mathrm{d}}^{*}\mathrm{d}x_1}\\
&t = \int_0^x \frac{\mathrm{d}x_2}{\left(v_{x0}\cdot\mathrm{e}^{-\int_0^{x_2}\bar{C}_{\mathrm{d}}^{*}\mathrm{d}x_1}\right)} = \int_0^x \frac{\mathrm{e}^{\int_0^{x_2}\bar{C}_{\mathrm{d}}^{*}\mathrm{d}x_1}}{v_{x0}}\,\mathrm{d}x_2\\
&t = \frac{1}{v_{x0}}\int_0^x \mathrm{e}^{k_1 x_2}\,\mathrm{d}x_2\\
&t = \frac{1}{v_{x0}\cdot k_1}\left(\mathrm{e}^{k_1\cdot x} - \mathrm{e}^{0}\right) = \frac{1}{v_{x0}\cdot k_1}\left(\mathrm{e}^{k_1\cdot x} - 1\right)
\end{aligned}\right| \tag{15}$$

Equation (14b):

$$\frac{\mathrm{d}v_y}{\mathrm{d}x} + \bar{C}_{\mathrm{d}}^{*}\cdot v_y = -\frac{g}{v_x}\,,$$

is linear differential equation of first order, and with the initial conditions, $v_y = v_{y0}$, $t = 0$ and $x = 0$, it is solved as:

$$v_y = v_x\left[\tan\gamma_0 - \frac{g\cdot t}{v_{x0}}\left(1 + \frac{v_{x0}\cdot k_1\cdot t}{2}\right)\right],$$

$$\frac{v_y}{v_x} = \tan\gamma,$$

From:

$$v_x = v_{x0}\cdot\mathrm{e}^{-k_1\cdot x} \rightarrow k_1 = \frac{1}{x}\cdot\ln\left(\frac{v_x}{v_{x0}}\right),$$

follows:

$$v_y = v_x\left[\tan\gamma_0 - \frac{g\cdot t}{v_{x0}}\left(1 + \frac{v_{x0}\cdot\frac{1}{x}\cdot\ln\left(\frac{v_{x0}}{v_x}\right)\cdot t}{2}\right)\right] \tag{16}$$

$$\tan\gamma = \tan\gamma_0 - \frac{g\cdot t}{v_{x0}}\left[1 + \frac{v_{x0}\cdot t\cdot\ln\left(\frac{v_{x0}}{v_x}\right)}{2x}\right] \tag{17}$$

Equation for describing projectile height y [5]:

$$y = y_0 + x\cdot\tan\gamma_0 - \frac{g}{2}\left[\frac{x}{v_{x0}}\cdot\frac{1}{\ln\left(\frac{v_{x0}}{v_x}\right)}\right]\left[\frac{1}{2}\left(\frac{v_{x0}}{v_x} - 1\right)^2 + \left(\frac{v_{x0}}{v_x} - 1\right) - \ln\left(\frac{v_{x0}}{v_x}\right)\right] \tag{18}$$

Assuming initial conditions as: $v_0 = 183$ m/s , mass of bullet $m = 14{,}6$ grams, angle of departure measured from x axis $\gamma = 3°$, bullet diameter $d = 12{,}7$ mm (0,50 inch), density of air $\rho = 1{,}2044$ kg/m^3, with ambient pressure $p_0 = 101325$ Pa, thermodynamic temperature $T = 293{,}15$ K, d$t = 0{,}02$ s, and applying Runge-Kutta method on equations (14 - 18), discrete values for variables are calculated, and following diagrams constructed (Figure 11, 12, 13, 14 and 15).

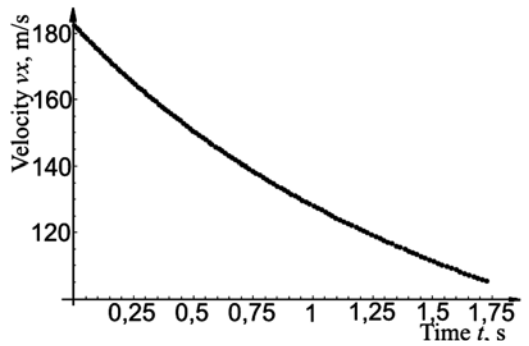

Figure 11 *Component v_x of velocity as a function of time*
Slika 11 *Komponenta v_x brzine kao funkcija vremena*

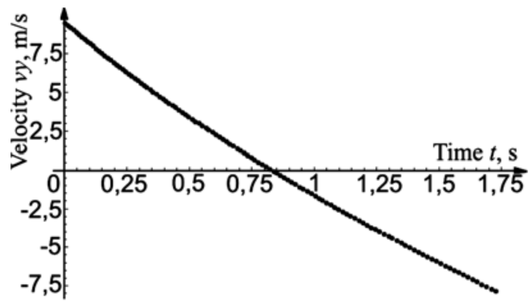

Figure 12 *Component v_y of velocity as a function of time*
Slika 12 *Komponenta v_y brzine kao funkcija vremena*

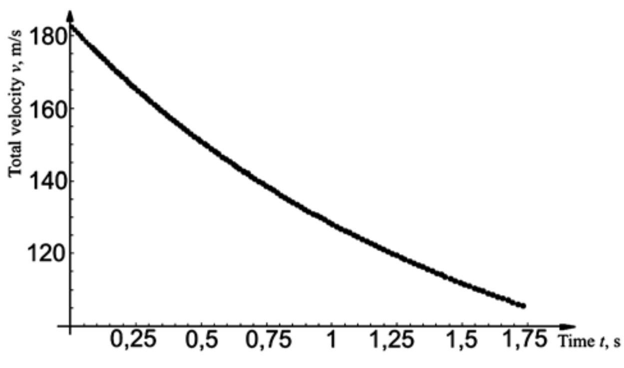

Figure 13 *Total velocity v as a function of time*
Slika 13 *Ukupna brzina v kao funkcija vremena*

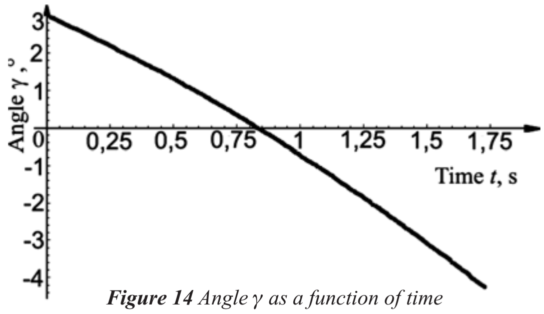

Figure 14 *Angle γ as a function of time*
Slika 14 *Kut γ kao funkcija vremena*

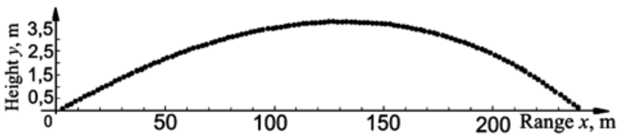

Figure 15 *Ballistic trajectory in x-y coordinate system*
Slika 15 *Balistička putanja u x-y koordinatnom sustavu*

3
Projectile kinetic energy calculations at different ranges
Računanje kinetičke energije hica na različitim udaljenostima

For same initial conditions as before, value of departure angle can be varied, in such way that projectile hits target at defined range. With small ranges angle of departure is small, and trajectory itself is almost flat. With long ranges it is necessary to increase the angle of departure in order for projectile to hit the target at desired range. For this purpose, a classical Euler's model was used (7).

Example 1: For distance x = 10 m, angle of departure calculated is γ = 0,085°. Applying values of variables from the initial conditions, and Runge-Kutta method on equations (7), with range set as x = 10 m, as a result calculated height amounts y = 0,0000368 m. Transformed it is 0,0368 mm deviation from gun's line of sight, and it is satisfactory solution. As bullet diameter is 12,7 mm, deviation from aimed target at range 10 m is 0,0368 mm, which is precise enough. Calculated velocity of projectile according to these initial conditions is 178,7 m/s. Now projectile kinetic energy can be calculated according to expression:

$$m = \frac{14{,}6}{1000} \quad \text{kg}$$

$$E_\mathrm{k} = \frac{m \cdot v^2}{2} = \frac{14{,}6 \cdot 178{,}7^2}{2000} = 232{,}12 \text{ J}$$

Calculated time of flight is t = 0,055 s.

Example 2: For target at range x = 50 m, calculated angle of departure is γ= 0,455°, and value of height amounts y = 0,000278 m. Converted it is 0,278 mm, and this is also small deviation from aimed point at target. Calculated velocity of projectile at impact v =162,4 m/s. Kinetic energy is calculated as follows:

$$E_\mathrm{k} = \frac{m \cdot v^2}{2} = \frac{14{,}6 \cdot 162{,}4^2}{2000} = 192{,}53 \text{ J}$$

Calculated time of flight t = 0,291 s.

Example 3: For target at range 100 m, calculated angle of departure is γ= 0,988°, and value of height y = 0,000332 m. Converted it is 0,332 mm. Calculated velocity of projectile at target impact v = 144,2 m/s. Kinetic energy is calculated as follows:

$$E_\mathrm{k} = \frac{m \cdot v^2}{2} = \frac{14{,}6 \cdot 144{,}2^2}{2000} = 151{,}8 \text{ J}$$

Calculated time of flight t = 0,62 s.

4
Conclusion
Zaključak

This paper is about utilizing numerical mathematic method for numerical approximation of ballistic trajectories for a specific problem. It is shown how simplified ballistic models can be applied for quick ballistic calculations. Models were applied for modern high pressure air rifles. It is shown that new design of air rifles is quite advanced. Today's modern high pressure air rifles can produce kinetic power of projectile leaving bore exit equivalent to some low power fire weapons. For some European countries legal projectile energy limit for air rifles is limited to 17 J.

5
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