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On a non-continuous and stronger form of Levine's semi-continuous functions

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Abstract. The concept of a (δ, β)-irresolute function in topological spaces is introduced and studied. Some of their characteristic properties are considered. We also investigate the relationships between these classes of functions and other classes of non-continuous functions.

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1. Introduction and preliminaries

Levine [10] defined semiopen sets which are weaker than open sets in topological spaces. Since the advent of Levine's semiopen sets, many researchers offered different and interesting new modifications of open sets which showed to be fruitful. In 1968, Veličko [17] introduced δ-open sets, which are stronger than open sets, in order to investigate the characterizations of H-closed spaces. In 1997, Park et al. [16] introduced the notion of δ-semiopen sets which are stronger than semiopen sets but weaker than δ-open sets and investigated the relationships between several types of these open sets. Recently, Caldas et al. [7] and [8] investigated this class of sets further and also studied some of its applications.
The purpose of the present paper is to introduce and investigate a new class of functions, namely (δ, β)-irresolute functions and give several of their characterizations and properties. Relations between this class of functions and other classes of functions are obtained.

In what follows (X, τ) and (Y, σ) (or X and Y) denote topological spaces. Let A be a subset of X. We denote the interior, the closure and the complement of a set A by $Int(A)$, $Cl(A)$ and $X \backslash A$, respectively.

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A subset A of a topological space X is said to be a semiopen [10](resp. preopen [12], α-open [15], β-open [1]) set if $A \subset Cl(Int(A))$ (resp. $A \subset Int(Cl(A))$, $A \subset Int(Cl(Int(A)))$, $A \subset Cl(Int(Cl(A)))$). A point $x \in X$ is called the δ-cluster point of A if $A \cap Int(Cl(U)) \neq \emptyset$ for every open set U of X containing x. The set of all δ-cluster points of A is called the δ-closure of A and denoted by $Cl_\delta(A)$. A subset A of X is called δ-closed if $A = Cl_\delta(A)$. The complement of a δ-closed set is called δ-open. A subset A is said to be a δ-semiopen [16] if there exists a δ-open set U of X such that $U \subset A \subset Cl(U)$. The complement of a δ-semiopen (resp. semiopen, preopen, α-open, β-open) set is called a δ-semiclosed (resp. semiclosed, preclosed, α-closed, β-closed). A point $x \in X$ is called the δ-semicluster point of A if $A \cap U \neq \emptyset$ for every δ-semiopen set U of X containing x. The set of all δ-semicluster points of A is called the δ-semiclosure of A, denoted by $\delta Cl_S(A)$. Recall that the β-closure of A [3] is the intersection of all β-closed sets containing A and will be denoted by $Cl_\beta(A)$. We denote the collection of all δ-semiopen (resp. β-open, δ-open) sets by $\delta\mathrm{SO}(X)$ (resp. $\beta O(X)$, $\delta O(X)$). We set $\delta SO(X,x) = \{U : x \in U \in \delta\mathrm{SO}(X)\}$, $\beta O(X,x) = \{U : x \in U \in \beta O(X)\}$ and $\delta O(X,x) = \{U : x \in U \in \delta O(X)\}$.

Lemma 1 (Park et al. [16]). *The intersection (resp. union) of an arbitrary collection of δ-semiclosed (resp. δ-semiopen) sets in (X,τ) is δ-semiclosed (resp. δ-semiopen). And $A \subset X$ is δ-semiclosed if and only $A = \delta Cl_S(A)$.*

Definition 1. *A function $f : X \to Y$ is said to be:*
(i) β-continuous if $f^{-1}(V)$ is β-open in X for each open set V of Y (see [1]).
(ii) β-irresolute if $f^{-1}(V)$ is β-open in X for each β-open set V of Y (see [11]).
(iii) strongly β-irresolute if $f^{-1}(V)$ is open in X for each β-open set V of Y (see [13]).
(iv) strongly M-precontinuous if $f^{-1}(V)$ is open in X for each preopen set V of Y (see [4]).
(v) almost α-irresolute if $f^{-1}(V)$ is β-open in X for each α-open set V of Y (see [6]).
(vi) semi α-irresolute if $f^{-1}(V)$ is semiopen in X for each α-open set V of Y (see [5]).

2. (δ, β)-irresolute functions

Definition 2. *A function $f : X \to Y$ is said to be (δ,β)-irresolute at $x \in X$ if for each β-open set V of Y containing $f(x)$ there exists a δ-semiopen set U in X containing x such that $f(U) \subset V$. If f is (δ,β)-irresolute at every point of X, then it is called (δ,β)-irresolute.*

Theorem 1. *For a function $f : X \to Y$, the following are equivalent:*
(1) f is (δ,β)-irresolute;
(2) $f^{-1}(V)$ is δ-semiopen in X for each β-open set V of Y;
(3) $f^{-1}(V) \subset Cl(Int_\delta(f^{-1}(V)))$ for every β-open set V of Y;
(4) $f^{-1}(F)$ is δ-semiclosed in X for every β-closed set F of Y;
(5) $Int(Cl_\delta(f^{-1}(B))) \subset f^{-1}(Cl_\beta(B))$ for every subset B of Y;
(6) $f(Int(Cl_\delta(A))) \subset Cl_\beta(f(A))$ for every subset A of X.

Proof. (1) ⇒ (2): Let V be an arbitrary β-open set in Y. We are going to prove that $f^{-1}(V)$ is δ-semiopen in X. For this purpose, let x be any point in $f^{-1}(V)$. Then $f(x) \in V$. Since f is (δ, β)-irresolute, there exists a δ-semiopen set U of X containing x such that $f(U) \subset V$. This implies $x \in U \subset f^{-1}(V)$. By Lemma 1, it follows that $f^{-1}(V)$ is a δ-semiopen set in X.
(2) ⇒ (1): Let $x \in X$ and V be any β-open set of Y containing $f(x)$. By (2), $f^{-1}(V)$ is δ-semiopen in X and $x \in f^{-1}(V)$. Set $U = f^{-1}(V)$; then, U is a δ-semiopen set of X containing x such that $f(U) \subset V$.
(1) ⇒ (3): Let V be any β-open set of Y and $x \in f^{-1}(V)$. By (1), there exists a δ-semiopen set U of X containing x such that $f(U) \subset V$. Thus we have $x \in U \subset Cl(Int_\delta(U)) \subset Cl(Int_\delta(f^{-1}(V)))$ and hence $f^{-1}(V) \subset Cl(Int_\delta(f^{-1}(V)))$.
(3) ⇒ (4): Let F be any β-closed subset of Y. Set $V = Y\backslash F$, then V is β-open in Y. By (3), we obtain $f^{-1}(V) \subset Cl(Int_\delta(f^{-1}(V)))$ and hence $f^{-1}(F) = X\backslash f^{-1}(Y\backslash F) = X\backslash f^{-1}(Y\backslash F) = X\backslash f^{-1}(V)$ is δ-semiclosed in X.
(4) ⇒ (5): Let B be any subset of Y. Since $Cl_\beta(B)$ is a β-closed subset of Y, $f^{-1}(Cl_\beta(B))$ is δ-semiclosed in X and hence $Int(Cl_\delta(f^{-1}(Cl_\beta(B)))) \subset f^{-1}(Cl_\beta(B))$. Therefore, we obtain $Int(Cl_\delta(f^{-1}(B))) \subset f^{-1}(Cl_\beta(B))$.
(5) ⇒ (6): Let A be any subset of X. By (5), we have

$$Int(Cl_\delta(A)) \subset Int(Cl_\delta(f^{-1}(f(A)))) \subset f^{-1}(Cl_\beta(f(A)))$$

and hence $f(Int(Cl_\delta(A))) \subset Cl_\beta(f(A))$.
(6) ⇒ (2): Let V be any β-open subset of Y. Since $f^{-1}(Y\backslash V) = X\backslash f^{-1}(V)$ is a subset of X and by (6), we obtain

$$f(Int(Cl_\delta(f^{-1}(Y\backslash V)))) \subset Cl_\beta(f(f^{-1}(Y\backslash V))) \subset Cl_\beta(Y\backslash V) = Y\backslash Int_\beta(V) = Y\backslash V$$

and hence

$$\begin{aligned} X\backslash Cl(Int_\delta(f^{-1}(V))) &= Int(Cl_\delta(X\backslash f^{-1}(V))) = Int(Cl_\delta(f^{-1}(Y\backslash V))) \\ &\subset f^{-1}(f(Int(Cl_\delta(f^{-1}(Y\backslash V))))) \subset f^{-1}(Y\backslash V) = X\backslash f^{-1}(V). \end{aligned}$$

Therefore, we have $f^{-1}(V) \subset Cl(Int_\delta(f^{-1}(V)))$ and hence $f^{-1}(V)$ is δ-semiopen in X. □

Remark 1. *From the above definitions, we have the following diagram:*

DIAGRAM

(δ, β)-irresoluteness ⇒	*semi α-irresoluteness* ⇒	*semi continuity*
⇓	⇓	⇓
β-irresoluteness ⇒	*almost α-irresoluteness* ⇒	*β-continuity*

By the following examples, remarks and ([5], [6], [13], [14]), the converse implications in the above diagram are not true in general:

1) Semi continuity does not imply (δ,β)-irresoluteness: It follows from the fact that every strongly M-precontinuous function is semi continuous, every (δ,β)-irresolute function is β-irresolute (see [6], Example 3.2).
2) β-irresoluteness does not imply (δ,β)-irresoluteness, since β-irresoluteness does not imply semi continuity (see [6], Example 3.1).
3) continuity and (δ,β)-irresoluteness are independent concepts.

Example 1. *By [6, Example 3.3], we have a continuous function which is not almost α-irresolute hence it is not (δ,β)-irresolute.*

Example 2. *Let $X=\{a,b,c\}$ and $Y=\{a,c\}$ with topologies*

$$\tau=\{X,\emptyset,\{a\},\{b\},\{a,b\}\} \quad \text{and} \quad \sigma=\{X,\emptyset,\{a\}\}.$$

Let a function $f:(X,\tau)\to(Y,\sigma)$ be defined by $f(b)=f(c)=a$ and $f(a)=c$. Then f is not continuous but (δ,β)-irresolute.

Lemma 2 ([2] and [9]). *Let $\{X_i : i\in\Omega\}$ be any family of nonempty topological spaces and A_{i_j} a nonempty subset of X_{i_j} for each $j=1,2,...,n$. Then $A=\prod_{i\neq i_j}X_i\times\prod_{j=1}^{n}A_{i_j}$ is a nonempty β-open [2] (resp. δ-semiopen [9]) subset of $\prod X_i$ if and only if A_{i_j} is β-open (resp. δ-semiopen) in X_{i_j} for each $j=1,2,...,n$.*

Theorem 2. *Let $f:X\to Y$ be a function and $g:X\to X\times Y$ the graph function, given by $g(x)=(x,f(x))$ for every $x\in X$. Then f is (δ,β)-irresolute if g is (δ,β)-irresolute.*

Proof. Let $x\in X$ and V be any β-open set of Y containing $f(x)$. Then, by Lemma 2 $X\times V$ is a β-open set of $X\times Y$ containing $g(x)$. Since g is (δ,β)-irresolute, there exists a δ-semiopen set U of X containing x such that $g(U)\subset X\times V$ and hence $f(U)\subset V$. Thus f is (δ,β)-irresolute. □

Theorem 3. *Let $\{X_i : i\in\Omega\}$ be any family of topological spaces and $Pr_i:\prod X_i\to X_i$. If $f:X\to\prod X_i$ is a (δ,β)-irresolute function, then $Pr_i\circ f:X\to X_i$ is (δ,β)-irresolute for each $i\in\Omega$, where Pr_i is the projection of $\prod X_i$ onto X_i.*

Proof. Let U_i be an arbitrary β-open subset of X_i. Since Pr_i is continuous and open, it is β-irresolute (see [1, Theorem 2.2]), and hence $Pr_i^{-1}(U_i)$ is β-open in $\prod X_i$. Since f is (δ,β)-irresolute, by definition we have $f^{-1}(Pr_i^{-1}(U_i))=(Pr_i\circ f)^{-1}(U_i)$ is δ-semiopen in X. Therefore, $Pr_i\circ f$ is (δ,β)-irresolute for each $i\in\Omega$. □

Theorem 4. *If the product function $f:\prod X_i\to\prod Y_i$ is (δ,β)-irresolute, then $f_i:X_i\to Y_i$ is (δ,β)-irresolute for each $i\in\Omega$.*

Proof. Let $i_0\in\Omega$ be an arbitrary fixed index and V_{i_0} any β-open in Y_{i_0}. Then $\prod Y_j\times V_{i_0}$ is β-open in $\prod Y_i$ by Lemma 2, where $i_0\neq j\in\Omega$. Since f is (δ,β)-irresolute, $f^{-1}(\prod Y_j\times V_{i_0})=\prod X_j\times f_{i_0}^{-1}(V_{i_0})$ is δ-semiopen in $\prod X_i$ and hence, by Lemma 2, $f_{i_0}^{-1}(V_{i_0})$ is δ-semiopen in X_{i_0}. This implies that f_{i_0} is (δ,β)-irresolute. □

Lemma 3. *Let A and Y be subsets of (X,τ). If $A\in\delta SO(X)$ and $Y\in\delta O(X)$, then $A\cap Y\in\delta SO(Y)$.*

Theorem 5. *If $f:(X,\tau)\to(Y,\sigma)$ is (δ,β)-irresolute and A is a δ-open subset of X, then the restriction $f_A: A\to Y$ is (δ,β)-irresolute.*

Proof. Let V be a β-open set of Y. Since f is (δ,β)-irresolute, $f^{-1}(V)$ is δ-semiopen in X. By Lemma 3, $(f_A)^{-1}(V)=A\cap f^{-1}(V)$ is δ-semiopen in A and hence f_A is (δ,β)-irresolute. □

Lemma 4. *Let A and Y be subsets of (X,τ). If $A\in\delta SO(Y)$ and $Y\in\delta O(X)$, then $A\in\delta SO(X)$.*

Theorem 6. *Let $f:(X,\tau)\to(Y,\sigma)$ be a function and $\{A_i: i\in\Omega\}$ a cover of X by δ-open sets of (X,τ). Then f is (δ,β)-irresolute, if $f_{A_i}: A_i\to Y$ is (δ,β)-irresolute for each $i\in\Omega$.*

Proof. Let V be any β-open set of Y. Since f_{A_i} is (δ,β)-irresolute, $(f_{A_i})^{-1}(V)=f^{-1}(V)\cap A_i$ is δ-semiopen in A_i and hence, by Lemma 4, $(f_{A_i})^{-1}(V)$ is δ-semiopen in X for each $i\in\Omega$. Therefore, $f^{-1}(V)=X\cap f^{-1}(V)=\cup\{A_i\cap f^{-1}(V): i\in\Omega\}=\cup\{f^{-1}_{A_i}(V): i\in\Omega\}$ is δ-semiopen in X. Hence f is (δ,β)-irresolute. □

Theorem 7. *The following statements hold for functions $f: X\to Y$ and $g: Y\to Z$:*
(i) If f is (δ,β)-irresolute and g is β-irresolute, then $g\circ f: X\to Z$ is (δ,β)-irresolute.
(ii) If f is (δ,β)-irresolute and g is (δ,β)-irresolute, then $g\circ f: X\to Z$ is (δ,β)-irresolute.

Proof. (i) Let W be any β-open subset in Z. Since g is β-irresolute, $g^{-1}(W)$ is β-open in Y. Since f is (δ,β)-irresolute, $f^{-1}(g^{-1}((V)))=(g\circ f)^{-1}(V)$ is δ-semiopen in X. Therefore, $g\circ f$ is (δ,β)-irresolute.
(ii) It follows immediately from (i) since every (δ,β)-irresolute function is β-irresolute. □

Recall that Lee et al. [9] define the δ-semifrontier of A denoted by $\delta\text{-}sfr(A)$ as $\delta\text{-}sfr(A)=\delta Cl_S(A)\backslash\delta Int_S(A)$, equivalently $\delta\text{-}sfr(A)=\delta Cl_S(A)\cap\delta Cl_S(X\backslash A)$.

Theorem 8. *The set of all points $x\in X$ at which $f:(X,\tau)\to(Y,\sigma)$ is not (δ,β)-irresolute is identical with the union of δ-semifrontiers of the inverse images of β-open subsets of Y containing $f(x)$.*

Proof. Necessity. Suppose that f is not (δ,β)-irresolute at a point x of X. Then, there exists a β-open set $V\subset Y$ containing $f(x)$ such that $f(U)$ is not a subset of V for every $U\in\delta SO(X,x)$. Hence we have $U\cap(X\setminus f^{-1}(V))\neq\emptyset$ for every $U\in\delta SO(X,x)$. It follows that $x\in\delta Cl_S(X\setminus f^{-1}(V))$. We also have $x\in f^{-1}(V)\subset\delta Cl_S(f^{-1}(V))$. This means that $x\in\delta\text{-}sfr(f^{-1}(V))$.
Sufficiency. Suppose that $x\in\delta\text{-}sfr(f^{-1}(V))$ for some $V\in\beta O(Y,f(x))$ Now, we assume that f is (δ,β)-irresolute at $x\in X$. Then there exists $U\in\delta SO(X,x)$ such that $f(U)\subset V$. Therefore, we have $x\in U\subset f^{-1}(V)$ and hence $x\in\delta Int_S(f^{-1}(V))\subset X\backslash\delta\text{-}sfr(f^{-1}(V))$. This is a contradiction. This means that f is not (δ,β)-irresolute at x. □

Recall that a topological space (X, τ) is called β-T_2 [11] (resp. δ-semi T_2 [7]) if for any two distinct points x and y in X there exist $U \in \beta O(X, x)$ and $V \in \beta O(X, y)$ (resp. $U \in \delta SO(X, x)$ and $V \in \delta SO(X, y)$) such that $U \cap V = \emptyset$.

Theorem 9. *If $f : X \to Y$ is a (δ, β)-irresolute injection and Y is β-T_2, then X is δ-semi T_2.*

Proof. Suppose that Y is β-T_2. Let x and y be distinct points of X. Then $f(x) \neq f(y)$. Since Y is β-T_2, there exist disjoint β-open sets V and W containing $f(x)$ and $f(y)$, respectively. Since f is (δ, β)-irresolute, there exist δ-semiopen sets G and H containing x and y, respectively, such that $f(G) \subset V$ and $f(H) \subset W$. It follows that $G \cap H = \emptyset$. This shows that X is δ-semi T_2. □

Lemma 5. *If A_i is a δ-semiopen set of X_i (i=1, 2), then $A_1 \times A_2$ is δ-semiopen in $X_1 \times X_2$*

Proof. By Theorem 2.25 of [9]. □

Theorem 10. *If $f : X \to Y$ is a (δ, β)-irresolute and Y is β-T_2, then $E = \{(x, y) : f(x) = f(y)\}$ is δ-semiclosed in $X \times X$.*

Proof. Suppose that $(x, y) \notin E$. Then $f(x) \neq f(y)$. Since Y is β-T_2, there exist $V \in \beta O(Y, f(x))$ and $W \in \beta O(Y, f(y))$ such that $V \cap W = \emptyset$. Since f is (δ, β)-irresolute, there exist $U \in \delta SO(X, x)$ and $G \in \delta SO(X, y)$ such that $f(U) \subset V$ and $f(G) \subset W$. Set $D = U \times G$. By Lemma 5 $(x, y) \in D \in \delta SO(X \times X)$ and $D \cap E = \emptyset$. This means that $\delta Cl_S(E) \subset E$ and therefore E is δ-semiclosed in $X \times X$. □

Definition 3. *For a function $f : X \to Y$, the graph $G(f) = \{(x, f(x)) : x \in X\}$ is called (δ, β)-closed if for each $(x, y) \in (X \times Y)\backslash G(f)$, there exist $U \in \delta SO(X, x)$ and $V \in \beta O(Y, y)$ such that $(U \times V) \cap G(f) = \emptyset$.*

Lemma 6. *A function $f : X \to Y$ has a (δ, β)-closed graph $G(f)$ if for each $(x, y) \in (X \times Y)\backslash G(f)$, there exist $U \in \delta SO(X, x)$ and $V \in \beta O(Y, y)$ such that $f(U) \cap V = \emptyset$.*

Proof. It is an immediate consequence of Definition 3 and the fact that for any subsets $U \subset X$ and $V \subset Y$, $(U \times V) \cap G(f) = \emptyset$ if and only if $f(U) \cap V = \emptyset$. □

Theorem 11. *If $f : X \to Y$ is (δ, β)-irresolute and Y is β-T_2, then $G(f)$ is (δ, β)-closed in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y)\backslash G(f)$. It follows that $f(x) \neq y$. Since Y is β-T_2, there exist disjoint β-open sets V and W in Y containing $f(x)$ and y, respectively. Since f is (δ, β)-irresolute, there exists $U \in \delta SO(X, x)$ such that $f(U) \subset V$. Therefore $f(U) \cap W = \emptyset$ and $G(f)$ is (δ, β)-closed in $X \times Y$. □

Theorem 12. *If $f : X \to Y$ is a (δ, β)-irresolute injection with a (δ, β)-closed graph, then X is δ-semi T_2*

Proof. Let x and y be any distinct points of X. Then since f is injective, we have $f(x) \neq f(y)$. Then we have $(x, f(y)) \in (X \times Y)\backslash G(f)$. Since $G(f)$ is (δ, β)-closed, there exist $U \in \delta SO(X, x)$ and $V \in \beta O(Y, f(y))$ such that $f(U) \cap V = \emptyset$. Since f is (δ, β)-irresolute, there exists $G \in \delta SO(Y, y)$ such that $f(G) \subset V$. Therefore, we have $f(U) \cap f(G) = \emptyset$ and hence $U \cap G = \emptyset$. This shows that X is δ-semi T_2. $\square$

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