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Cesàro semiconservative FK spaces

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Abstract. In this paper we call an FK space X containing ϕ a Cesàro semiconservative space if $X^f \subset \sigma s$ holds. Therefore we give some characterizations of these spaces.

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1. Introduction

Conservative spaces play a special role in summability theory. However, in [7], Snyder and Wilansky have shown that the results seem mainly to depend on a weaker assumption and that the spaces be semiconservative. They give the definition of a semiconservative FK space and investigate the properties of this space in [7] ,[8]. In those papers, an FK space X containing ϕ is called a semiconservative space if $X^f \subset cs$ holds. This is a significant generalization of the theory.

In this paper we studied Cesàro semiconservative spaces which have weaker assumption than a semiconservative space. Here by replacing cs by σs, we give a new definition called a Cesàro semiconservative FK space.

2. Notions and definitions

Let w denote the space of all real or complex-valued sequences. It can be topologized with seminorms $p_i(x) = |x_i|$, $(i = 1, 2, ...)$, and any vector subspace of w is called a sequence space. A sequence space X, with a vector space topology τ is a K space provided that the inclusion mapping $I : (X, \tau) \to w$, $I(x) = x$ is continuous. If, in addition, τ is complete, metrizable and locally convex, then (X, τ) is called an FK space. So an FK space is a complete, metrizable local convex topological vector space of sequences for which the coordinate functionals are continuous. An FK space whose topology is normable is called a BK space. The basic properties of such spaces can be found in [8], [9] and [11].

By m and c_0 we denote the spaces of all bounded sequences and null sequences, respectively. These are FK spaces under $\|x\| = \sup_n |x_n|$. By l and cs we shall

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denote the space of all absolutely summable sequences and convergent series, respectively. The sequences spaces

$$h=\left\{x\in w:\lim_j x_j=0,\ \text{and}\ \sum_{j=1}^{\infty}j\left|\Delta x_j\right|<\infty\right\},$$

$$q=\left\{x\in w:\ \sup_j\left|x_j\right|<\infty\ \text{and}\ \sum_{j=1}^{\infty}j\left|\Delta^2 x_j\right|<\infty\right\},$$

$$\sigma b=\left\{x\in w:\sup_n\left|\frac{1}{n}\sum_{k=1}^{n}\sum_{j=1}^{k}x_j\right|<\infty\right\},$$

$$\sigma s=\left\{x\in w:\lim_n\frac{1}{n}\sum_{k=1}^{n}\sum_{j=1}^{k}x_j\ \ \text{exists}\right\},$$

and

$$\sigma_0=\left\{x\in w:\lim_n\left|\frac{1}{n}\sum_{k=1}^{n}x_k\right|=0\right\}$$

are FK spaces with the norms

$$\|x\|_h=\sum_{j=1}^{\infty}j\left|\Delta x_j\right|+\sup_j\left|x_j\right|,$$

$$\|x\|_q=\sum_{j=1}^{\infty}j\left|\Delta^2 x_j\right|+\sup_j\left|x_j\right|,$$

$$\|x\|_{\sigma b}=\sup_n\left|\frac{1}{n}\sum_{j=1}^{n}\sum_{j=1}^{k}x_j\right|,$$

and

$$\|x\|_{\sigma_0}=\sup_n\frac{1}{n}\left|\sum_{k=1}^{n}x_k\right|,$$

respectively, where $\Delta x_j=x_j-x_{j+1},\Delta^2 x_j=\Delta x_j-\Delta x_{j+1}$. The space $q\cap c_0$ is denoted by q_0. Under the norm $\|.\|_q$, q_0 is a BK space, (see [1], [2]).

Throughout the paper e denotes the sequence of ones, $(1,1,...,1,...)$; $\delta^j,(j=1,2,...)$, the sequence $(0,0,...,0,1,0,...)$ with the one in the $j-$th position. Let $\phi:=l.hull\left\{\delta^k:k\in N\right\}$ and $\phi_1=\phi\cup\{e\}$. The topological dual of X is denoted by X'. The space X is said to have AD if ϕ is dense in X and an FK space X is said to have AK or to be an AK space, if $X\supset\phi$ and for each $x\in X$, $x^{(n)}\to x$, $(n\to\infty)$, in X, where

$$x^{(n)}=\sum_{k=1}^{n}x_k\delta^k=(x_1,x_2,...,x_n,0,...).$$

In addition, an FK space is said to have a σK space if $X \supset \phi$ and for each $x \in X$,

$$\frac{1}{n}\sum_{k=1}^{n} x^{(k)} \to x, \ \ (n \to \infty).$$

Every AK space is a σK space. For example w, h, c_0, σ_0 are AK spaces while q_0, σs are σK spaces ([1], [2], [8]). In addition, every σK space is an AD space.

Let X be an FK space containing ϕ. Then $X^f = \left\{\left\{f\left(\delta^k\right)\right\} : f \in X'\right\}$. In addition,

$$X^{\beta} = \left\{x : \sum_{k=1}^{\infty} x_k y_k \text{ exists for every } y \in X\right\},$$

$$X^{\sigma} = \left\{x : \lim_n \frac{1}{n}\sum_{k=1}^{n}\sum_{j=1}^{k} x_j y_j \text{ exists for every } y \in X\right\},$$

$$X^{\sigma b} = \left\{x : \sup_n \frac{1}{n}\left|\sum_{k=1}^{n}\sum_{j=1}^{k} x_j y_j\right| < \infty \text{ for every } y \in X\right\}.$$

Let E, E_1 be sets of sequences. Then for $k = \beta, \sigma, \sigma_b$

a) $E \subset E^{kk}$,

b) $E^{kkk} = E^k$

c) if $E \subset E_1$, then $E_1^k \subset E^k$

holds. Also, if $\phi \subset E \subset E_1$, then $E_1^f \subset E^f$.

Theorem 1. *Let X be an FK space containing ϕ. Then*

i) $X^{\beta} \subset X^{\sigma} \subset X^{\sigma b} \subset X^f$,

ii) If X is a σK space, then $X^f = X^{\sigma}$,

iii) If X is an AD space, then $X^{\sigma} = X^{\sigma b}$.

Proof.

ii) Let $u \in X^{\sigma}$ and define $f(x) = \lim_n \frac{1}{n}\sum\limits_{k=1}^{n}\sum\limits_{j=1}^{k} x_j u_j$ for $x \in X$. Then $f \in X'$ by the Banach-Steinhause Theorem ([8], 1.0.4). Also

$$f(\delta^p) = \lim_n \frac{1}{n}\left(n - (p-1)\right) u_p = u_p (p < n)$$

so $u \in X^f$. Thus $X^{\sigma} \subset X^f$.

Now we show that $X^f \subset X^{\sigma}$. Let $u \in X^f$. Since X is a σK space

$$f(x) = \lim_n \frac{1}{n}\sum_{k=1}^{n}\sum_{j=1}^{k} x_j f(\delta^j) = \lim_n \frac{1}{n}\sum_{k=1}^{n}\sum_{j=1}^{k} x_j u_j$$

for $x \in X$, then $u \in X^{\sigma}$. Hence $X^f = X^{\sigma}$.

iii) Let $u \in X^{\sigma b}$ and define $f_n(x) = \frac{1}{n}\sum\limits_{k=1}^{n}\sum\limits_{j=1}^{k} x_j u_j$ for $x \in X$. Then $\{f_n\}$ is pointwise bounded, hence equicontinuous by ([8], 7.0.2). Since

$$\lim_n f_n(\delta^p) = u_p, \quad (p < n),$$

then $\phi \subset \{x : \lim_n f_n(x) \text{ exists}\}$. Hence $\{x : \lim_n f_n(x) \text{ exists}\}$ is a closed subspace of X by the Convergence Lemma, ([8],1.0.5) and ([8],7.0.3). Since X is an AD space, then $X = \{x : \lim_n f_n(x) \text{ exists}\} = \overline{\phi}$ and then $\lim_n f_n(x)$ exists for all $x \in X$. Thus $u \in X^{\sigma}$. The opposite inclusion is trivial.

i) $\overline{\phi} \subset X$ by the hypothesis. Since $\overline{\phi}$ is an AD space, then

$$X^{\sigma b} \subset \left(\overline{\phi}\right)^{\sigma b} = \left(\overline{\phi}\right)^{\sigma} \subset \left(\overline{\phi}\right)^{f} = X^f$$

by (iii) and ([8], 7.2.4).

□

Let $A = (a_{ij})$ be an infinite matrix. The matrix A may be considered as a linear transformation of sequences (x_k) by the formula $y = Ax$, where $y_i = \sum\limits_{j=1}^{\infty} a_{ij}x_j$, $(i = 1, 2, ...)$.

For an FK space (X, u), we consider the summability domain

$$X_A := \{x \in w : Ax \in X\}.$$

Then X_A is an FK space under the seminorms $p_i(x) = |x_i|, (i = 1, 2, ...)$, $h_i(x) = \sup_m \left|\sum\limits_{j=1}^{m} a_{ij}x_j\right|, (i = 1, 2, ...)$ and $(u \circ A)(x) = u(Ax)$, [8].

Recall that, given a matrix A with $l_A \supset \phi$ is called $l-$replaceable if there is a matrix $B = (b_{nk})$ with $l_B = l_A$ and $\sum\limits_{n=1}^{\infty} b_{nk} = 1$, for all $k \in N$, [6].

An FK space X containing ϕ_1 is called Cesàro conull if

$$f(e) = \lim_n \frac{1}{n}\sum_{k=1}^{n}\sum_{j=1}^{k} f\left(\delta^j\right),$$

for all $f \in X'$, [5].

In addition, an FK space X is called semiconservative if $X^f \subset cs$, this means that $X \supset \phi$ and $\sum\limits_{j=1}^{\infty} f\left(\delta^j\right)$ is convergent for each $f \in X'$, [7].

3. Cesàro semiconservative FK spaces

In this section we extend the notation of the semiconservative FK space introduced by Snyder and Wilansky [7] to the concept of Cesàro semiconservative FK Space and we investigate the properties of these spaces.

Definition 1. *An FK space X, containing ϕ, is called Cesàro semiconservative if $X^f \subset \sigma s$, where $X^f \subset \sigma s$ if and only if $e^{(k)}$ is weakly Cesàro Cauchy i.e. $\left\{ \frac{1}{n} \sum_{k=1}^{n} f(e^{(k)}) \right\}$ is convergent for each $f \in X'$ equivalently $\lim_n \frac{1}{n} \sum_{k=1}^{n} \sum_{j=1}^{k} f\left(\delta^j\right)$ exists.*

For example, c_0, σ_0 are Cesàro semiconservative FK spaces. Every semiconservative FK space is a Cesàro semiconservative FK space. But the following example shows that every Cesàro semiconservative FK space is not a semiconservative space. Before presenting this example we shall give some theorems.

Theorem 2. *If a matrix A is l-replaceable, then l_A is not a Cesàro semiconservative FK space.*

Proof. If A is $l-$replaceable, then there is $f \in l_A^{'}$ such that $f\left(\delta^j\right) = 1$ for all $j \in N$, [6]. Hence $\lim_n \frac{1}{n} \sum\limits_{k=1}^{n} \sum\limits_{j=1}^{k} f\left(\delta^j\right)$ does not exist since $\frac{1}{n} \sum\limits_{k=1}^{n} \sum\limits_{j=1}^{k} f\left(\delta^j\right) = \frac{n+1}{2}$, so l_A is not a Cesàro semiconservative space. □

Theorem 3. *If $X_{\ A}$ is a Cesàro conull FK space, then it is a Cesàro semiconservative space.*

Proof. Suppose that X_A is Cesàro conull. Then

$$f\left(e\right) = \lim_n \frac{1}{n} \sum_{k=1}^{n} \sum_{j=1}^{k} f\left(\delta^j\right),$$

for all $f \in X_A'$. Hence $X_A^f \subset \sigma s$. □

Now we present the example promised in this section.

Example 1. *Define the sequence Ax by $\left(Ax\right)_j = x_j - x_{j-1}$ $\left(x_0 = 0\right)$ if j is square, and 0 otherwise. Then l_A is a Cesàro semiconservative space but not a semiconservative space.*

Proof. l_A is a Cesàro conull FK space by $\left([5]\text{, Example 3.2}\right)$, so l_A is a Cesàro semiconservative space by Theorem 3.

Now we show that l_A is not a semiconservative space. To see this, let $B := A^T$. Then

$$\sum_{k=1}^{\infty} \left| \sum_{i=n}^{\infty} b_{ik} \right| = 1,$$

if n is a square, otherwise 0. Thus

$$\lim_n \sum_{k=1}^{\infty} \left| \sum_{i=n}^{\infty} b_{ik} \right|$$

does not exist and $B \notin \left(l^{\infty} : cs\right) = \left(l^{\beta} : cs\right)$ by $\left([8], 8.5.8\right)$ and then l_A is not a semiconservative space by $\left([8], 9.4.4\right)$. □

Theorem 4.

i) *A closed subspace Y, containing ϕ, of a Cesàro semiconservative space X is a Cesàro semiconservative space.*

ii) *An FK space Y that contains a Cesàro semiconservative space X must be a Cesàro semiconservative space.*

iii) *A countable intersection of Cesàro semiconservative spaces is a Cesàro semiconservative space.*

Proof.

i) is true since $Y^f = X^f$ (see [8], Theorem 7.2.6).

ii) holds since $Y^f \subset X^f \subset \sigma s$.

iii) First the intersection $X = \bigcap X_n$ is an FK space by ([8], Theorem 4.2.15). Every $f \in X'$ can be written $f = \sum_{k=1}^{m} g_k$, where each $g_k \in X'_n$ for some n by ([8], 4.0.3, 4.0.8).

□

Theorem 5. *z^σ is a Cesàro semiconservative space if and only if $z \in \sigma s$.*

Proof. Let z^σ be a Cesàro semiconservative space. Then $z^{\sigma f} \subset \sigma s$. Since z^σ is a σK space by [5], we have $z^{\sigma f} = z^{\sigma\sigma}$. So since $\{z\} \subset z^{\sigma\sigma} \subset \sigma s$, we get $z \in \sigma s$. Now let $z \in \sigma s$. Then $q = \sigma s^\sigma \subset z^\sigma$ [1] and hence $z^{\sigma\sigma} \subset q^\sigma = \sigma s$. Since z^σ is a σK space, then $z^{\sigma f} = z^{\sigma\sigma} \subset \sigma s$. □

Example 2. *σs is not a Cesàro semiconservative space. Because $\sigma s = e^\sigma$ and $e \notin \sigma s$.*

Theorem 6.

i) *Every Cesàro semiconservative space contains q_0.*

ii) *The intersection of all Cesàro semiconservative spaces is q_0.*

iii) *q_0 is not a Cesàro semiconservative space.*

iv) *There is no smallest Cesàro semiconservative space.*

Proof.

i) Let X be a Cesàro semiconservative space. Then $X^f \subset \sigma s \subset \sigma b = q_0^\sigma$, [1] and since q_0 is a σK space, then $X^f \subset q_0^\sigma = q_0^f$. So, since q_0 is an AD space, we obtain $q_0 \subset X$ by ([8], Theorem 8.6.1) .

ii) Let the intersection of all Cesàro semiconservative spaces be I. We get $I \subset \cap \{z^\sigma : z \in \sigma s\} = \sigma s^\sigma = q$ using Theorem 5. Also $I \subset c_0$, since c_0 is a Cesàro semiconservative space so $I \subset q \cap c_0 = q_0$. The opposite inclusion is by (i).

iii) Since $q_0^f = q_0^\sigma = \sigma b \nsubseteq \sigma s$, then q_0 is not a Cesàro semiconservative space.

iv) By (ii) and (iii).

□

Example 3. *q and σb are not Cesàro semiconservative spaces.*

Proof. q_0 and σs are closed subspaces of q and σb, respectively. Then since q_0 and σs are not Cesàro semiconservative spaces, q and σb are not Cesàro semiconservative spaces by Theorem 4 (i). □

$X^\sigma \subset \sigma s$ is not sufficent for X to be a Cesàro semiconservative space since $q^\sigma = \sigma s$. This is not surprising since this condition holds for every space containing e.

Definition 2. *An FK space is called bounded convex Cesàro semiconservative if it is a Cesàro semiconservative space and includes q.*

Since q_0 is an AD space, then $X \supset q_0$ if and only if $X^f \subset \sigma b$ by ([8], 8.6.1). Thus $X \supset q$ if and only if $X^f \subset \sigma b$ and $e \in X$, by ([8], 8.3.7). However, X is a bounded convex Cesàro semiconservative space if and only if $X^f \subset \sigma s$ and $e \in X$, also if and only if X is a Cesàro semiconservative space and $e \in X$.

The definition of a Cesàro conull FK space X in which $X \supset \phi$, can be given as follows by using Cesàro semiconservativity. A Cesàro semiconservative space X is called Cesàro conull, if

$$f(e) = \lim_n \frac{1}{n} \sum_{k=1}^{n} \sum_{j=1}^{k} f\left(\delta^j\right),$$

for all $f \in X'$. A Cesàro semiconservative space need not contain e but must contain e, if it is Cesàro conull. A Cesàro conull space is an automatically bounded convex Cesàro semiconservative space.

4. A relationship between the distinguished subsets and Cesàro semiconservative FK spaces

In this section we give the relation between the distinguished subspaces which are σF^+, σF, σB^+, σB and Cesàro semiconservative and bounded convex Cesàro semiconservative FK spaces. First we shall give the definition of the distinguished subsets σF^+, σF, σB^+, σB.

Let X be an FK space containing ϕ. Then we define

$$\sigma F^{+}(X) = \left\{ x : \lim_n \frac{1}{n} \sum_{k=1}^{n} \sum_{j=1}^{k} x_j f\left(\delta^j\right) \text{ exists for all } f \in X' \right\}$$
$$= \left\{ x : \{x_n f(\delta^n)\} \in \sigma s \text{ for all } f \in X' \right\},$$
$$\sigma B^{+}(X) = \left\{ x : \left\{ \frac{1}{n} \sum_{k=1}^{n} x^{(k)} \right\} \text{is bounded in } X \right\}$$
$$= \left\{ x : \{x_n f(\delta^n)\} \in \sigma b \text{ for all } f \in X' \right\}.$$

Also $\sigma F = \sigma F^{+} \cap X$ and $\sigma B = \sigma B^{+} \cap X$, [4].

Theorem 7. *Let X be an FK space containing ϕ and $z \in w$. Then $z \in \sigma F^{+}$ if and only if $z^{-1}.X = \{x : z.x \in X\}$ is a Cesàro semiconservative FK space, where $z.x = \{z_n x_n\}$ in particular $e \in \sigma F^{+}$ if and only if X is a Cesàro semiconservative FK space.*

Proof. Let $f \in (z^{-1}.X)'$. Then $f(x) = \alpha x + g(z.x)$, $\alpha \in \phi$, $g \in Y'$, by ([8], 4.4.10) and $f(\delta^n) = \alpha_n + g(z.\delta^n) = \alpha_n + g(z_n.\delta_n) = \alpha_n + z_n g(\delta^n)$. Hence, since $\alpha \in \phi \subset \sigma s$, then $\{f(\delta^n)\} \in \sigma s$ if and only if $\{z_n g(\delta^n)\} \in \sigma s$, i.e. $z \in \sigma F^{+}$. □

Theorem 8. *Let X be an FK space containing ϕ and $z \in w$. Then $z \in \sigma F$ if and only if $z^{-1}.X$ is a bounded convex Cesàro semiconservative FK space in particular $e \in \sigma F$ if and only if X is bounded convex Cesàro semiconservative.*

Proof. Let $z \in \sigma F$. Then $z \in X$ so $e \in z^{-1}.X$ and since $z \in \sigma F^{+}$, $z^{-1}.X$ is a Cesàro semiconservative FK space by Theorem 7. Thus $z^{-1}.X$ is a bounded convex Cesàro semiconservative FK space.

Let $z^{-1}.X$ be a bounded convex Cesàro semiconservative FK space. Then $z^{-1}.X$ is Cesàro semiconservative and $e \in z^{-1}.X$ so $z \in X$. Thus since $z \in \sigma F^{+}$ by Theorem 7 and $z \in X$ then, $z \in \sigma F$. □

Theorems 9 and 10 have already been obtained by Buntinas [1] but here we present their alternate proofs.

Theorem 9. *Let X be an FK space containing ϕ and $z \in w$. Then $z \in \sigma B^{+}$ if and only if $z^{-1}.X \supset q_0$, in particular $e \in \sigma B^{+}$ if and only if $X \supset q_0$.*

Proof. Let $f \in (z^{-1}.X)'$. Then $f(\delta^n) = \alpha_n + z_n g(\delta^n)$ by ([8], 4.4.10). Thus, since $\alpha \in \phi \subset \sigma s$, then $z \in \sigma B^{+}$ if and only if $\{z_n g(\delta^n)\} \in \sigma b$, i.e. $z \in \sigma B^{+}$. □

Theorem 10. *Let X be an FK space containing ϕ and $z \in w$. Then $z \in \sigma B$ if and only if $z^{-1}.X \supset q$, in particular $e \in \sigma B$ if and only if $X \supset q$.*

Proof. Let $z \in \sigma B$. Then $z \in X$ so $e \in z^{-1}.X$ and $z \in \sigma B^{+}$. Thus $z^{-1}.X \supset q$ by Theorem 9. Let $z^{-1}.X \supset q$, then $z^{-1}.X \supset q_0$ and $e \in z^{-1}.X$. Thus, since $z \in \sigma B^{+}$ by Theorem 9 and $z \in X$, then $z \in \sigma B$. □

Theorem 11. *Let X be an FK space containing ϕ. Then X is a Cesàro semiconservative space if and only if $\sigma F^+ \supset q$.*

Proof. Let X be a Cesàro semiconservative FK space. Then $e \in \sigma F^+$ by Theorem 7. Since $e \in \sigma F^+ = X^{f\sigma}$ [4], then $X^f \subset X^{f\sigma\sigma} \subset \{e\}^\sigma$ and so $q = \{e\}^{\sigma\sigma} \subset X^{f\sigma} = \sigma F^+$. Let $\sigma F^+ \supset q$. Then $e \in \sigma F^+$ and so X is a Cesàro semiconservative FK space by Theorem 7. □

Theorem 12. *Let Y be a Cesàro semiconservative FK space and Z an AD space. Suppose that for an FK space X, $X \supset Y.Z$. Then $X \supset Z$, where $Y.Z = \{y.z : y \in Y, z \in Z\}$.*

Proof. Let $z \in Z$. Then, since $X \supset Y.Z$, $z^{-1}.X \supset Y$. Thus, since Y is a Cesàro semiconservative space, then $z^{-1}.X$ is a Cesàro semiconservative space by Theorem 4 (ii) and so $z \in \sigma F^+$ by Theorem 7. Hence $Z \subset \sigma F^+ = X^{f\sigma}$ [4]. Thus $X^f \subset X^{f\sigma\sigma} \subset Z^\sigma \subset Z^f$ and so $Z \subset X$ by ([8], 8.6.1). □

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