ANALITIČKI PRORAČUN OSOVINSKIH STRUJA KOD MOTORA S HOMOGENIM JARMOM

ANALYTICAL CALCULATION OF SHAFT CURRENTS IN THE MOTORS WITH HOMOGENEOUS YOKE

Željko Hederić – Marinko Barukčić – Krešimir Miklošević,
Osijek, Hrvatska

U literaturi koja se bavi problemom osovinskih struja uglavnom su obrađivane metode dijagnostike motora, koje samo promatraju i analiziraju frekvencije harmonika karakterističnih za osovinske struje. Analitički proračun amplituda tih struja nije dosad obrađivan za motore s homogenim jarmom, a da bi se mogle provesti pravilne mjere zaštite, bitno je odrediti točno parametre koji utječu na te struje. Osovinske struje su bitna komponenta struja koje električki oštećuju ležajeve i obično predstavljaju dominantan doprinos. Osobito je to važno za niskonaponske asinkrone motore, koji su prema podacima IEC-a motori s najvećom ekspanzijom primjene u industriji i za koje je u radu objašnjen analitički proračun osovinskih struja.

Literature dealing with the matter of shaft currents mostly elaborates on the methods of motor diagnostics which only observe and analyse harmonics frequencies characteristic for shaft currents. The analytic calculation of the amplitudes of those currents has not been analysed so far for homogenous yoke motors and, in order for correct protection measures to be undertaken, it is necessary to determine exactly the parameters which influence those currents. Shaft currents are an important component of the currents which electrically damage the bearings and usually represent the dominant contribution. This is especially important for low-voltage induction motors which, according to IEC's data, are the motors with the widest application range in the industry. In this work, the analytic calculation of shaft currents is explained for those motors.

Ključne riječi: analitički proračun; asinkroni motor; homogeni jaram; osovinska struja
Key words: analytical calculation; induction motor, homogenous yoke; shaft current

1 UVOD

Suvremene analize pouzdanosti asinkronih motora pokazuju da većinu kvarova predstavljaju kvarovi na ležajevima. Električki uzročnici oštećenja ležajeva su struje koje teku kroz ležajeve i dovode do njihove erozije. One se razlikuju, kako po načinu nastanka, tako i prema njihovom trajanju tijekom rada motora i mogu biti: osovinske struje (induktivne) i ležajne struje (kapacitivne). Kapacitivne struje karakterizirane su probojima izolacije maziva ležaja, kratko traju i imaju velike iznose, te stvaraju oštećenja u obliku nasumičnih kratera po obodu ležajne košuljice (u praksi poznata pod engl. nazivom: *pitting*). Induktivne struje su relativno puno manjeg iznosa u odnosu na kapacitivne, ali su trajno prisutne u radu motora. Njihovim djelovanjem javljaju se oštećenja u obliku zareza, koji su pravilno raspoređeni po obodu ležajne košuljice (u praksi poznata pod engl. nazivom: *fluting*). Oba tipa struja djeluju erozivno na ležajnu košuljicu, zbog čega dolazi do mehaničkog oštećenja ležajnih kuglica ili valjaka, čije raspadanje uzrokuje pojavu povećanih vibracija i daljnjeg oštećenja ostalih dijelova motora.

Teorijske postavke nastanka osovinskih struja, kod asinkronih motora s homogenim statorskim jarmovima (bez zračnih raspora), u literaturi koja se bavi tim problemom nisu do kraja objašnjene. Uglavnom su obrađene metode dijagnostike motora s kojima se promatraju i analiziraju frekvencije harmonika karakterističnih za osovinske struje, te se samo ukazuje na stupanj oštećenja ležaja [1]. Da bi se mogle provesti pravilne mjere zaštite, bitno je odrediti točno parametre koji utječu na osovinske struje. S tim je ciljem objašnjen postupak analitičkog proračuna osovinskih struja za dva osnovna uzroka osovinskih struja: magnetska nesimetrija i ekscentrični položaj rotora u provrtu statora (statička i dinamička ekscentričnost i ovalnost). Pri tome nije pravljena razlika između kaveznih ili kliznokolutnih motora, jer se izvor osovinske struje prvenstveno tražio u djelovanju statorskog namota.

2 UZROCI NASTAJANJA OSOVINSKIH STRUJA

Uzroci nastajanja struja koje teku kroz osovinu i osovinskih električnih napona su različiti [2], [3], [4] i [5] mogu se podijeliti u nekoliko osnovnih grupa: magnetske nesimetrije, kružni magnetski tok u jarmu, elektrostatski naboji i naponi na rotorskom namotu. Pojave osovinskih napona, te struja kroz ležajeve i druge dijelove u dodiru s osovinom postaju to kompliciranije i opa-

1 INTRODUCTION

Up-to-date analyses of induction motor's reliability show that defects on the bearings are the most usual defects. Electrical causes of bearings defects are currents which flow through the bearings and cause their erosion. These differ both according to the source of their occurrence and their duration during motor operation, and they can be: shaft currents (inductive) and bearing currents (capacitive). Capacitive currents are characterized by ruptures of the insulation of bearing lubricants, they have short durations and high rates, and cause damages in the form of sporadic craters along the rim of the bearing sleeve (known in the practice under the English name: *pitting*). Inductive currents are relatively of much lesser rates in relation to the capacitive ones, but they are constantly present in engine operation. Their impact causes damages in the form of cuts regularly spread along the rim of the bearing sleeve (known in practice under the English name: *fluting*). Both current types have erosive effects on the bearing sleeve which causes mechanic damage of bearing balls and cylinders, the disintegration of which causes the occurrence of increased vibrations and further damage to the other motor parts.

Theoretical postulates about the occurrence of shaft currents in induction motors with homogenous stator yokes (without air gaps) are not fully explained in the literature concerned with that problem. The main subject of such literature are motor diagnostics methods used to observe and analyze harmonic frequencies characteristic for shaft currents, and it only points to the level of bearing damage [1]. In order to be able to implement appropriate protection measures, it is necessary to determine accurately the parameters influencing shaft currents. With that in mind, the procedure has been explained of shaft currents analytical calculation for two main causes of shaft currents: magnetic asymmetry and eccentric rotor position in the stator bore (static and dynamic eccentricity and ovality). Thereat no difference is made between squirrel-cage and slip-ring motors because the source of the shaft current was searched for in the impact of the stator winding.

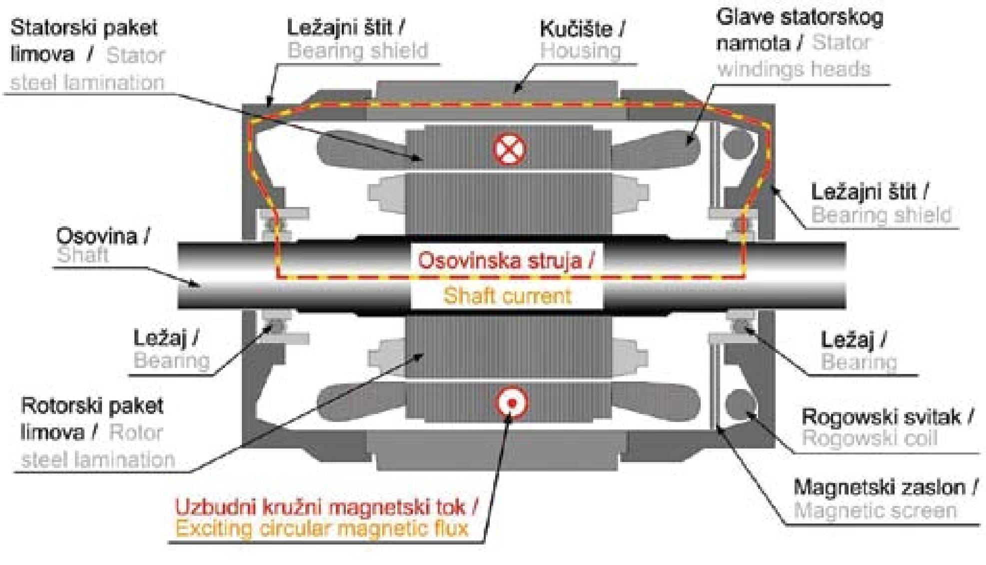
2 CAUSES OF OCCURRENCE OF SHAFT CURRENTS

The causes of the occurrence of currents flowing through the shaft and of the shaft electric voltages are different [2], [3], [4] and [5] and can be divided into several main groups: magnetic asymmetries, circular magnetic flux in the yoke, electrostatic charges and voltages on the rotor winding. Occurrences of shaft tensions and currents through the bearings and other parts touching the shaft become all the more complicated and dangerous as power and disnije što se više povećavaju snaga i dimenzije elektromotornog pogona. Napone na osovini, ili bolje rečeno uzduž osovine, može prouzrokovati promjenljiv (izmjenični) magnetski tok koji obuhvaća osovinu, magnetski tok kroz samu osovinu, te protjecanje struje kroz osovinu.

Za pojavu kružnog magnetskog toka koji obuhvaća osovinu, te dovodi do nastajanja osovinskih struja, kod asinkronih motora s homogenim statorskim jarmovima i sinusnim napajanjem, veliki doprinos daje mehanička ekscentričnost rotora u statoru (nejednak zračni raspor po obodu), a nužan uvjet je nelinearnost krivulje magnetiziranja željeza. Na slici 1 prikazan je strujni krug kojim se zatvaraju osovinske struje: osovina – prvi ležaj – prvi ležajni štit – kućište – drugi ležajni štit – drugi ležaj – osovina. Osovinska struja prolazi kroz oba ležaja i oštećuje ih. Za mjerenje osovinskih struja koristi se svitak Rogowskog koji se smješta u ležajni štit motora, koncentrično oko osovine. Radi sprječavanja pogreške mjerenja uslijed rasipnih magnetskih tokova glava statorskih namota, između svitka Rogowskog i glava namota postavlja se magnetski zaslon [6].

mensions of the electromotor drive increase. Tensions on the shaft or, better yet, along the shaft, can be caused by changeable (alterative) magnetic flux which encompasses the shaft, magnetic flux through the shaft itself and the flow of the current through the shaft.

A great contribution to the occurrence of the circular magnetic flow which encompasses the shaft and brings about the occurrence of shaft currents in induction motors with homogenous stator yokes and sinuous power supply is provided by the mechanic eccentricity of the rotor in the stator (unequal air gap along the rim) and a required condition is the nonlinearity of the magnetization curve for iron. Figure 1 shows the electric circuit in which shaft currents are closed: Shaft - first bearing - first bearing shield - bearing box - second bearing shield - second bearing - shaft. The shaft current passes through both bearings and damages them. For the measurement of shaft currents, a Rogowski coil is used which is placed in the motor bearing shield, concentrically around the shaft. For the purpose of preventing measurement errors due to magnetic fluxes leakage of the heads of stator windings, between the Rogowski coil and the winding heads, a magnetic screen is placed [6].



Slika 1 — Prikaz strujnog kruga zatvaranja osovinskih struja
Figure 1 — Presentation of the electrical circuit of shaft current closure

3 POSTUPAK ANALITIČKOG IZRAČUNA OSOVINSKIH STRUJA

Postupak analitičkog izračuna osovinskih struja treba započeti određivanjem magnetske indukcije u zračnom rasporu, čemu jasno prethodi

3 PROCEDURE OF ANALYTIC CALCULATION OF SHAFT CURRENTS

Procedure for the analytic calculation of shaft currents should be initiated by determining the magnetic induction in the air gap, which is clearly conpoznavanje struje magnetiziranja. Ako se pretpostavi κ-ti harmonik magnetske indukcije u zračnom rasporu:

ditioned by knowing the magnetization current. If the κ-harmonic of magnetic induction in the air gap is assumed:

$$b_\kappa(x,t) = B_\kappa \cos(\kappa x - \omega_\kappa t - \varphi_\kappa) \ , \tag{1}$$

gdje je:

b_κ – harmonik magnetske indukcije u zračnom rasporu, T,
B_κ – amplituda harmonika magnetske indukcije u zračnom rasporu, T,
κ – oznaka harmonika,
x – obodni kut kojim je definiran položaj na obodu zračnog raspora, rad,
ω_κ – kružna frekvencija harmonika, s^{-1},
φ_κ – fazni kut harmonika, rad,

on uzrokuje, pod pretpostavkom da se silnice zatvaraju preko jarma statora, κ-ti harmonik magnetske indukcije u jarmu statora:
gdje je:

where it is as follows:

b_κ – magnetic induction harmonic in the air gap, T,
B_κ – amplitude of magnetic induction harmonics in the air gap, T,
κ – harmonics symbol,
x – circumferential angle which defines the position on the rim of the air gap, rad,
ω_κ – circular harmonics frequency, s^{-1},
φ_κ – harmonics phase angle, rad,

it causes, under the assumption that magnetic field lines close through the stator yoke, κ-harmonic of magnetic induction in the stator yoke:
where it is as follows:

$$b_{\mathrm{y}\kappa}(x,t) = B_{\mathrm{y}\kappa} \cos(\kappa x - \omega_\kappa t - \varphi_\kappa) \ , \tag{2}$$

b_κ – harmonik magnetske indukcije u zračnom rasporu, T,
y – oznaka za jaram.

Integriranjem poluvala harmonika magnetske indukcije u zračnom rasporu (b_κ) dobiva se amplituda κ-tog harmonika magnetske indukcije u jarmu:

b_κ – magnetic induction harmonic in the air gap, T,
y – yoke symbol.

By integrating the semi-wave of the magnetic induction harmonic in the air gap (b_κ) the amplitude of the κ-harmonic of magnetic induction in the yoke is obtained:

$$B_{\mathrm{y}\kappa} = \frac{R}{h_\mathrm{y}} \int_{\frac{\pi p}{2\kappa}} b_\kappa(x,t) \, \mathrm{d}x \ , \tag{3}$$

gdje je:

$B_{y\kappa}$ – amplituda κ-tog harmonika magnetske indukcije u jarmu, T,
R – srednji radijus jarma statora, m,
h_y – visina jarma statora, m,
p – broj pari polova.

Omjer p / κ u granicama integrala je zbog toga što svaki harmonik ima drugi broj pari polova, odnosno periodu. Na osnovi postupka objašnjenog u [2], može se definirati funkcija κ-tog harmonika jakosti polja u jarmu u ovisnosti o κ-tom harmo-

where it is as follows:

$B_{y\kappa}$ – amplitude of κ-harmonic of magnetic induction in the yoke, T,
R – average stator yoke radius, m,
h_y – stator yoke height, m,
p – number of pole pairs.

Ratio p / κ is within the limits of the integral because each harmonic has a different number of pole pairs, that is, different periods. Based on the procedure explained in [2], the function of the κ-harmonic of field intensity in the yoke depending niku magnetske indukcije i rezultantne permeabilnosti:

on the κ-harmonic of magnetic induction and resulting permeability can be defined.

$$h_{y\kappa}(x,t)=\frac{b_{y\kappa}(x,t)}{\mu_{\mathrm{res}}(x,t)}\ , \tag{4}$$

gdje je:

$h_{y\kappa}$ – harmonik jakosti polja u jarmu, A/m,
μ_{res} – rezultantna permeabilnost, Vs/Am.

Pri tome se rezultantna permeabilnost rotora mora računati iz rezultantne indukcije u jarmu i iz karakteristike magnetiziranja željeza paketa limova motora.

Rezultantna magnetska indukcija u jarmu predstavlja sumu svih harmonika:

where it is as follows:

$h_{y\kappa}$ – yoke field intensity harmonic, A/m,
μ_{res} – resulting permeability, Vs/Am.

Thereat the resulting rotor permeability must be calculated from the resulting induction in the yoke and from the magnetizing curve of electrical steel of the motor stack lamination.

Resulting magnetic induction in the yoke represents the sum of all harmonics:

$$b_{\mathrm{yres}}(x,t)=\sum_{\nu} B_{y\nu}\sin(\nu x-\omega_{\nu}t-\varphi_{\nu}) \tag{5}$$

gdje je: ν – oznaka harmonika.

Sljedeći korak je aproksimacija krivulje magnetiziranja željeza paketa limova motora polinomom, odnosno definiranje rezultantne jakosti magnetskog polja kao funkcije rezultantne magnetske indukcije preko polinoma koji ima samo neparne potencije:

where it is as follows: ν – harmonics symbol.

The next step is the approximation of the magnetizing curve of motor stack lamination by polynomial, that is, the definition of resulting intensity of the magnetic field as a function of the resulting magnetic induction through the polynomial which has odd exponents only:

$$h_{\mathrm{yres}}(x,t)=\frac{1}{\mu_1}b_{\mathrm{yres}}(x,t)+\frac{1}{\mu_3}b_{\mathrm{yres}}^{3}(x,t)+\frac{1}{\mu_5}b_{\mathrm{yres}}^{5}(x,t)+\ldots., \tag{6}$$

pri čemu koeficijenti polinoma $(1\,/\,\mu_l)$ prema [6] moraju biti pozitivni realni brojevi.

Ako se funkcija rezultantne jakosti polja podijeli s rezultantnom magnetskom indukcijom, na desnoj strani se dobiva polinom, čiji su koeficijenti numerički jednaki koeficijentima polinoma koji aproksimira krivulju magnetiziranja:

whereat the polynomial coefficients $(1\,/\,\mu_l)$ according to [6] have to be positive real numbers.

If the function of the resulting field intensity is divided by the resulting magnetic induction, on the right side a polynomial is obtained and its coefficients are numerically equal to the coefficients of the polynomial which approximates the magnetization curve:

$$\frac{1}{\mu_{\mathrm{res}}}=\frac{h_{\mathrm{yres}}(x,t)}{b_{\mathrm{yres}}(x,t)}=\frac{1}{\mu_1}+\frac{1}{\mu_3}b_{\mathrm{yres}}^{2}(x,t)+\frac{1}{\mu_5}b_{\mathrm{yres}}^{4}(x,t)+\ldots\,.. \tag{7}$$

Budući da (7) mora vrijediti i za ukupno polje i za svaki pojedini harmonik, slijedi:

Since (7) needs to be valid both for the overall field and for each particular harmonic, it follows:

$$h_{y\kappa}(x,t) = B_{y\kappa}\sin(\kappa x - \omega_\kappa t - \varphi_\kappa)\cdot$$

$$\left\{\frac{1}{\mu_1} + \frac{1}{\mu_3}\left[\sum_\nu B_{y\nu}\sin(\nu x - \omega_\nu t - \varphi_\nu)\right]^2 + \frac{1}{\mu_5}\left[\sum_\nu B_{y\nu}\sin(\nu x - \omega_\nu t - \varphi_\nu)\right]^4 + \ldots\right\}. \tag{8}$$

Ovako opisana aproksimacija omogućava da se poznavanjem harmonika indukcije u zračnom rasporu, može vrlo jednostavnim polinomom izračunati harmonike jakosti polja u jarmu statora.

Za procjenu da li κ -ti harmonik magnetskog polja uzrokuje osovinski napon, mora se detaljnije proučiti linijski integral protjecanja:

Approximation described as above makes it possible, by knowing the induction harmonic in the air gap, to calculate, by virtue of a very simple polynomial, the field intensity harmonics in the stator yoke.

In order to assess whether the κ-harmonic of the magnetic field causes the shaft voltage, the line integral of the magnetomotive force should be examined:

$$\Theta_\kappa = R\int_0^{2\pi} h_{y\kappa}(x,t)\,\mathrm{d}x\ . \tag{9}$$

Da bi postojala osovinska struja jasno je da protjecanje mora biti različito od nule, tj. mora postojati kružni magnetski tok. Ako se pretpostavi linearna magnetska karakteristika, za koeficijente polinoma vrijedi: $\mu_3 = \mu_5 = \ldots = \infty$, odnosno za bilo koju vrijednost od κ uvijek vrijedi $\Theta \equiv 0$. S druge strane, prema zakonu protjecanja slijedi da $\Theta \neq 0$ uvjetuje postojanje kružne komponente protjecanja u zračnom rasporu provrta.

Ovakvo razmatranje potvrđuje prethodnu konstataciju da je magnetska nesimetrija, kod motora s homogenim jarmom, nužan uvjet postojanja kružnog toka, a time i osovinske struje. Nakon što se odredi protjecanje, množenjem s vodljivošću jarma, dobiva se kružni magnetski tok koji inducira osovinski napon (Faradayev zakon). Tada je, određivanje impedancije strujnog kruga, kojim se može zatvoriti osovinska struja, posljednji problem koji se javlja u postupku određivanja osovinske struje.

Za pravilan analitički izračun osovinskih struja potrebno je izvesti doprinos svih nesimetrija: geometrijskih i magnetskih. Geometrijske nesimetrije uzrokovane su nesavršenošću kružnog oblika ležaja, provrta statora i oboda rotora, kao i izobličenjem osovine. Ovisno o stupnju nesavršenosti definiraju se pojmovi ekscentričnosti i ovalnosti. Nadalje je opisan pojednostavljen izvod utjecaja ekscentričnog položaja centričnog rotora u centričnom statoru.

U [5] se pojam statičke ekscentričnosti definira uz pojavu kada se os rotora ne poklapa sa osi provrta statora (na slici 2 označena sa M) i mi-

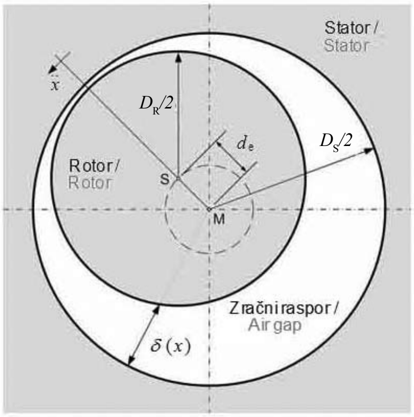
It is obvious that the occurrence of the shaft current requires the magnetomotive force to be different than zero, that is, there has to be a circular magnetic flux. If a linear magnetic characteristic is assumed, the following applies for the polynomial coefficients: $\mu_3 = \mu_5 = \ldots = \infty$, that is for any value of κ, $\Theta \equiv 0$ always applies. On the other hand, according to the Ampere law it follows that $\Theta \neq 0$ conditions the existence of a circular flux component in the air gap of the bore.

This kind of consideration proves the previous conclusion that magnetic asymmetry in motors with homogenous yoke is a required condition for the existence of a circular flux, and thus of a shaft current. After the determination of the flow, by multiplication with yoke conductivity, a circular magnetic flux is gained and it induces the shaft voltage (the Faraday Law). Then the determination of the impedance of electrical circuit in which shaft current can be closed, is the last problem which occurs in the process of determination of the shaft current.

For a correct analytic calculation of shaft currents, it is necessary to derive the contribution of all asymmetries: geometrical and magnetic. Geometric asymmetries are caused by the imperfection of the round shape of the bearing, stator bore and rotor rim, as well as by the disfigurement of the shaft. Depending on the level of imperfection, the notions of eccentricity and ovality are defined. A simplified derivation of the impact of the centric rotor's eccentric position in the centric stator is further described.

In [5], the notion of static eccentricity is defined for the event when the rotor axis does not overlap the stator bore axis (marked with M in Figure 2) and reruje u provrtu statora, odnosno rotor se vrti oko točke koja je fiksna u prostoru (na slici 2 označena sa S). U koliko se točka S vrti oko točke M (slika 2) ovisno o vremenu, tada se definira pojam dinamičke ekscentričnosti. U slučaju statičke ekscentričnosti širina zračnog raspora je isključivo funkcija obodnog kuta (x), dok je u slučaju dinamičke ekscentričnosti ona funkcija i obodnog kuta (x) i vremena (t).

sts in the stator bore, that is, the rotor spins around a point which is fixed in space (marked with S in Figure 2). If point S spins around point M (Figure 2) depending on time, then the notion of dynamic eccentricity is defined. In case of static eccentricity, the width of the air gap is exclusively a function of the circumferential angle (x), while in case of dynamic eccentricity, it is a function both the circumferential angle (x) and time (t).



Slika 2 — Prikaz ekscentričnog pomaka rotora u provrtu statora

Figure 2 — Presentation of the eccentric rotor shift in the stator bore

Pri pisanju izraza za vodljivost zračnog raspora za ekscentrični rotor, obično se zanemaruje utjecaj nazubljenja paketa i zasićenja u željezu paketa. Općenito se za veličinu zračnog raspora može napisati funkcija:

When writing expressions for the air gap conductivity for the eccentric rotor, the impact of the serration of the stack and the stack iron saturation is usually disregarded. In general, the function can be written for the size of the air gap:

$$\delta(x,t) = \delta_a \left[1 + \varepsilon \cdot \cos(x - \omega_\varepsilon t - \varphi_\varepsilon)\right] , \tag{10}$$

gdje je:

δ – veličina zračnog raspora, m,
δ_a – srednja vrijednost zračnog raspora, m,
ε – ekscentricitet.

Pri tome je srednja vrijednost zračnog raspora:

where it is as follows:

δ – air gap size, m,
δ_a – mean value of air gap size, m,
ε – eccentricity.

Thereat, mean value of air gap size is:

$$\delta_a = \frac{D_S - D_R}{2} , \tag{11}$$

dok za ekscentricitet vrijedi:

while the following applies for eccentricity:

$$\varepsilon = \frac{d_{\mathrm{e}}}{\delta_{\mathrm{a}}} \quad , \tag{12}$$

gdje je:

d_e – ekscentrični pomak rotora, m (razmak točaka S i M na slici 2),
D_S – promjer statora motora, m,
D_R – promjer rotora motora, m.

Kružna frekvencija (ω_ε) veličine zračnog raspora ovisna je o tome radi li se o statičkom ekscentricitetu $\omega_\varepsilon = 0$ ili o dinamičkom ekscentricitetu ($\omega_\varepsilon = (1 - s)\ \omega/p$), tj. ovisna je o broju pari polova (p) i o klizanju (s), odnosno o brzini vrtnje rotora. Općenito se vodljivost zračnog raspora definira kao omjer permeabilnosti zraka i veličine zračnog raspora:

where it is as follows:

d_e – eccentric rotor shift, m (distance between the points S and M in Figure 2),
D_S – motor stator diameter, m,
D_R – motor rotor diameter, m.

Circular frequency (ω_ε) of the air gap size depends on whether it is about static eccentricity $\omega_\varepsilon = 0$ or dynamic eccentricity ($\omega_\varepsilon = (1 - s)\ \omega/p$), that is, it depends on the number of pole pairs (p) and the slip (s), that is, on the rotor spinning speed. Generally, the conductivity of the air gap is defined as the ratio of air permeability and air gap size:

$$\lambda(x,t) = \frac{\mu_0}{\delta(x,t)} \quad , \tag{13}$$

gdje je:

λ – vodljivost zračnog raspora, Vs/Am^2,
μ_0 – permeabilnost zraka, Vs/Am.

Uz pojednostavljenje, prikladniji oblik vodljivosti zračnog raspora određen je prema [5]:

where it is as follows:

λ – air gap conductivity, Vs/Am^2,
μ_0 – air permeability, Vs/Am.

With a simplification, a more adequate form of conductivity of air gap is determined according to [5]:

$$\lambda(x,t) = \Lambda_{\mathrm{o}} \left[1 + \sum_{\gamma=1,2,\ldots}^{\infty} k_\gamma \cdot \cos\ \left[\gamma (x - \omega_\varepsilon t - \varphi_\varepsilon) \right] \right] \quad , \tag{14}$$

gdje se vodljivost idealnog zračnog raspora Λ_0, Vs/Am^2 i koeficijent ekscentričnosti (k_γ) računaju preko sljedećih izraza:

where the conductivity of the ideal air gap Λ_0, Vs/Am^2 and the eccentricity coefficient (k_γ) are calculated through the following expressions:

$$\Lambda_{\mathrm{o}} = \frac{\mu_0}{\delta_{\mathrm{a}}} \frac{1}{\sqrt{1-\varepsilon^2}} \quad , \tag{15}$$

$$k_\gamma = 2 \frac{\left(1 - \sqrt{1-\varepsilon^2}\right)^2}{\varepsilon^\gamma} \quad . \tag{16}$$

Pošto se magnetska indukcija računa množenjem protjecanja s vodljivošću zračnog raspora, vidljivo je iz izraza za vodljivost (14) da će tako dobivena indukcija imati veliki broj harmonika.

Since the magnetic induction is calculated by multiplying the magnetomotive force with the conductivity of the air gap, the conductivity expression (14) reveals that induction achieved in that way will have a large number of harmonics.

Radi jednostavnijeg objašnjenja na primjeru motora s $p = 1$ pari polova, bit će ukratko opisan postupak izvođenja harmonika za različite vrste geometrijske nesimetrije (ekscentričnost i ovalnost). Prema [6], vodljivost zračnog raspora pri statičkoj i dinamičkoj ekscentričnosti, može se definirati kao:

For the purpose of a simpler explanation, the procedure of derivation of harmonics for different types of geometric asymmetries (eccentricity and ovality) will be briefly described on the example of the motor with $p = 1$ pole pairs. According to [6], air gap conductivity at static and dynamic eccentricity can be defined as:

$$\lambda(x,t) = \Lambda_\mathrm{o}\left[1 + k_\mathrm{se}\cos(x - x_\mathrm{os}) + k_\mathrm{de}\cos(x - \omega(1-s)t - x_\mathrm{od})\right], \tag{17}$$

gdje prema (16) izvedeni koeficijenti imaju indekse:

se – statička ekscentričnost,
de – dinamička ekscentričnost.

Početna vrijednost obodnih kutova za statičku i dinamičku ekscentričnost označena je indeksima:

os – statička ekscentričnost,
od – dinamička ekscentričnost.

Harmonik protjecanja (za jednopolni stroj) uz amplitudu prema (9) ima izraz:

where according to (16) the derived coefficients have indices:

se – static eccentricity,
de – dynamic eccentricity.

Initial value of circumferential angles for static and dynamic eccentricity is marked by indices:

os – static eccentricity,
od – dynamic eccentricity.

Magnetomotive force harmonic (for a one-pole machine) with the amplitude according to (9) has the expression:

$$\vartheta_1(x,t) = \Theta_1\cos(x - \omega t - \varphi_1), \tag{18}$$

pa uz statički ekscentricitet:

so with static eccentricity:

$$\lambda_\mathrm{se}(x,t) = \Lambda_\mathrm{o}\left[1 + k_\mathrm{se}\cos\,(x - x_\mathrm{os})\right], \tag{19}$$

slijedi izraz za harmonik ($\nu = p = 1$) magnetske indukcije uslijed statičke ekscentričnosti:

the expression for the magnetic induction harmonic ($\nu = p = 1$) due to static eccentricity follows:

$$b_\mathrm{1se} = \vartheta_1\lambda_\mathrm{se} = \Theta_1\Lambda_\mathrm{o}\left[\cos(x - \omega t - \varphi_1) + k_\mathrm{se}\cos(x - x_\mathrm{os})\cdot\cos(x - \omega t - \varphi_1)\right], \tag{20}$$

gdje je indeks 1 oznaka za harmonik. Doprinos statičkog ekscentriciteta harmoniku magnetske indukcije predstavlja drugi član zagrade, te slijedi:

where index 1 is the symbol for the harmonic. The contribution of the static eccentricity to the magnetic induction harmonic represents the second term in the brackets, and it follows:

$$b_\mathrm{1ss} = \Theta_1\Lambda_\mathrm{o}\frac{k_\mathrm{se}}{2}\left[\cos(xt + \varphi_1 - x_\mathrm{os}) + \cos(2x - \omega t - \varphi_1 - x_\mathrm{os})\right]. \tag{21}$$

Prema [2] za harmonike vrijedi:

According to [2], the following applies for the harmonics:

$$\kappa = 2 \cdot g \cdot \nu \ , \tag{22}$$

gdje je koeficijent g predstavlja cijele brojeve:

where the coefficient g represents whole numbers:

$$g = \pm 1, \ \pm 2, \ \ldots \ . \tag{23}$$

Sada iz argumenta protjecanja $(x - \omega t)$ prema izrazu (5) slijedi:

Now, from the magnetomotive force operator $(x - \omega t)$ according to the expression (5) it follows:

$$\nu = 1 \ \rightarrow \ \omega_\nu = \omega \ , \tag{24}$$

a iz doprinosa statičkog ekscentriciteta harmoniku magnetske indukcije $(2x - \omega t)$ slijedi:

and from the static eccentricity contribution to the magnetic induction harmonic $(2x - \omega t)$, it follows:

$$\kappa = 2 \ \rightarrow \ \omega_\kappa = \omega \ . \tag{25}$$

Iz (22), uvrštavanjem vrijednosti harmonika $(\nu = 1, \kappa = 1)$, slijedi da je $g = 1$, pa se nadalje, prema [2], računa kružna frekvencija harmonika uslijed statičke ekscentričnosti:

From (22), by inserting the harmonics value $(\nu = 1, \kappa = 1)$, follows that $g = 1$, therefore according to [2], the circular harmonic frequency due to static eccentricity is further calculated:

$$|\omega_o| = |\omega_\kappa - 2 \cdot g . \omega_\nu| = |-\omega| = \omega \ . \tag{26}$$

Analognim postupkom, za dinamički ekscentricitet prema izrazu (17) određuje se vodljivost zračnog raspora:

By analogous procedure, for dynamic eccentricity according to the expression (17), air gap conductivity is determined:

$$\lambda_{\mathrm{de}}(x,t) = \Lambda_{\mathrm{o}}\left[1 + k_{\mathrm{de}} \cos\left(x - \omega(1-s) - x_{\mathrm{od}}\right)\right] . \tag{27}$$

Kružna frekvencija harmonika uslijed dinamičke ekscentričnosti prema izrazu (26) je:

Circular harmonics frequency due to dynamic eccentricity according to the expression (26) is:

$$|\omega_{\mathrm{o}}| = s\omega \ . \tag{28}$$

Treba naglasiti da je, za razliku od kružne frekvencije harmonika uslijed statičke ekscentričnosti, kružna frekvencija harmonika zbog dinamičke ekscentričnosti ovisna o klizanju, odnosno o brzi-

It should be pointed out that, unlike the circular harmonics frequency due to static eccentricity, circular harmonics frequency due to dynamic eccentricity depends on the slip, that is, on the ni vrtnje motora. Na osnovi prethodnog izvoda mogu se izvesti harmonici magnetske indukcije u zračnom rasporu koji zbog geometrijske nesimetrije uzrokuju pojavu osovinskih struja. Detaljniji pregled izraza za vodljivosti i protjecanja, te izvodi harmonika za najčešće oblike geometrijske nesimetrije (ekscentričnost, ovalnost), kao i pregledne tablice usporedbi s mjerenjima dani su u [5].

U tablicama 1 i 2 dani su prikazi raspodjele harmonika magnetske indukcije po uzrocima nastajanja za motor sa $p = 2$ pari polova, na kojem su vršena mjerenja. Za svaku vrstu geometrijske nesimetrije, u prvim stupcima su brojevi pojavljivanja najutjecajnijih harmonika, a u trećim stupcima su harmonici čije se amplitude mogu zanemariti. U posljednja tri stupca je zbirni doprinos, prema kojem su harmonici i sortirani.

motor rotation speed. Based on the above derivation, magnetic induction harmonics in the air gap, which cause the occurrence of shaft current because of the geometric asymmetry, can be derived. A more detailed overview of the expressions for conductivity and magnetomotive force, as well as the derivations of harmonics for the most usual forms of geometric asymmetry (eccentricity, ovality), as well as the overview tables which show comparisons with measurements, are provided in [5].

Tables 1 and 2 provide presentations of the distribution of magnetic induction harmonics according to the occurrence patterns for the motor, on which measurements were done, with $p = 2$ pole pairs. For each type of geometric asymmetry, numbers of occurrence of the most influential harmonics are in the first columns and harmonics, the amplitudes of which can be ignored, are in the third columns.The aggregate contribution according to which the harmonics are sorted is in the last three columns.

Tablica 1 – Raspodjela harmonika kružnog toka prema doprinosu amplitudi osovinske struje za Y spoj motora
Table 1 – Distribution of circular magnetic flux harmonics according to contribution to the shaft currents amplitude for Y motor connection

Frekvencija [Hz] Frequency	Harmonik Harmonic	Statička ekscentričnost Static eccentricity			Dinamička ekscentričnost Dynamic eccentricity			Statička ovalnost Static ovality			Dinamička ovalnost Dynamic ovality			Suma harmonika Sum of harmonics		
50,0	ω	3	3	3				5	4	9				8	7	14
1,4	$s\omega$				3	3	5				2	1	5	5	4	10
150,0	$3\,\omega$							6	2	4				6	2	4
295,8	$(6-3s)\,\omega$										4	1		4	1	0
550,0	$11\,\omega$		2						4					0	6	0
4,2	$3\,s\omega$										2	1		2	1	0
598,6	$(12-s)\,\omega$					2						2		0	4	0
450,0	$9\,\omega$								3					0	3	0
250,0	$5\,\omega$		1						2					0	3	0
350,0	$7\,\omega$		1						2					0	3	0
650,0	$13\,\omega$		1						2					0	3	0
850,0	$17\,\omega$		1						2					0	3	0
950,5	$19\,\omega$		1						2					0	2	0
750,0	$15\,\omega$								2					0	2	0
298,6	$(6-s)\,\omega$					1						1		0	2	0
301,4	$(6+s)\,\omega$					1						1		0	2	0
595,8	$(12-3s)\,\omega$											2		0	2	0
601,4	$(12+s)\,\omega$					1						1		0	2	0
898,4	$(18-s)\,\omega$					1						1		0	2	0
901,4	$(18+s)\,\omega$					1						1		0	2	0
304,2	$(6+3s)\,\omega$											1		0	1	0
604,2	$(12+3s)\,\omega$											1		0	1	0
895,8	$(18-3s)\,\omega$											1		0	1	0

Tablica 2 – Raspodjela harmonika kružnog toka prema doprinosu amplitudi osovinske struje za D spoj motora

Table 2 – Distribution of circular magnetic flux harmonics according to contribution to the shaft currents amplitude for D motor connection

Frekvencija [Hz] Frequency	Harmonik Harmonic	Statička ekscentričnost Static eccentricity			Dinamička ekscentričnost Dynamic eccentricity			Statička ovalnost Static ovality			Dinamička ovalnost Dynamic ovality			Suma harmonika Sum of harmonics		
1,5	$s\omega$				5	6	5				9	10	9	14	16	14
50,0	ω	5	6	5				9	10	9				14	16	14
298,5	$(6-s)\,\omega$				6	2					14	4		20	6	0
250,0	$5\,\omega$	6	2					12	4					18	6	0
350,0	$7\,\omega$	4	2					8	4					12	6	0
150,0	$3\,\omega$							10	6	4				10	6	4
301,5	$(6+s)\,\omega$				4	2					4	4		8	6	0
4,5	$3\,s\omega$										4	4	4	4	4	4
450,0	$9\,\omega$													4	6	0
295,5	$(6-3s)\,\omega$										6	2		6	2	0
550,0	$11\,\omega$		3						6					0	9	0
601,5	$(12+s)\,\omega$					3						6		0	9	0
650,0	$13\,\omega$		3						6					2	6	0
595,5	$(12-3s)\,\omega$										2	6		0	7	0
598,5	$(12-s)\,\omega$					3						4		2	2	0
304,5	$(6+3s)\,\omega$										2	2		0	3	0
750,0	$15\,\omega$								3					0	3	0
850,0	$17\,\omega$		1						2					0	3	0
898,5	$(18-s)\,\omega$					1						2		0	3	0
901,5	$(18+s)\,\omega$					1						2		0	3	0
950,0	$19\,\omega$		1						2					0	3	0
604,5	$(12+3s)\,\omega$											2		0	2	0
895,5	$(18-3s)\,\omega$											1		0	1	0

4 PRIMJER ANALITIČKOG IZRAČUNA I USPOREDBA S REZULTATIMA MJERENJA

Na temelju prethodno izvedenog analitičkog proračuna osovinskih struja za motore s homogenim jarmom, na primjeru jednog četveropolnog kaveznog asinkronog motora za pogon viličara napravljen je ogledni primjer analitičkog izračuna nekih frekvencija harmonika osovinske struje. Za izračun je potrebno da se uz osnovne parametre motora (nazivni napon, frekvencija, broj pari polova, broj utora statora i rotora, faktor skraćenja namota) poznaju dimenzije motora i krivulja prvog magnetiziranja magnetskog lima paketa statora i rotora, kao i čelika osovine. Podaci motora su: spoj statorskog namota u trokut, nazivni napon 22,5 V, nazivna struja 115 A, nazivni faktor snage 0,749, nazivna frekvenci-

4 EXAMPLE OF THE ANALYTIC CALCULATION AND COMPARISON WITH THE MEASUREMENT RESULTS

Based on the analytic calculation of shaft currents for homogenous yoke motors derived above, a sample example of the analytic calculation of certain frequencies of shaft current harmonics is created on the example of a four-pole squirrel-cage induction motor for the forklift drive. The calculation requires knowing not only the basic motor parameters (nominal voltage, frequency, number of pole pairs, number of stator and rotor slots, winding shortening factor) but also motor dimensions and the magnetizing curve of electrical steel of the stack lamination of stator and rotor. Motor data are: stator winding connection in a delta configuration, nominal voltage 22,5 V, nominal current 115 A, nominal power factor ja 50 Hz, nazivna brzina vrtnje 1 455 min-^{1}, nazivni moment 31,5 Nm, broj utora statora 36, broj utora rotora 48, zračni raspor 0,4 mm.

Postupak proračuna osovinske struje treba započeti izračunom struje magnetiziranja na osnovi poznatih izraza iz teorije strojeva. Analitički je izračunata vrijednost od $I_{\mathrm{sm}} = 98{,}85$ A, iz čega slijedi da je, u odnosu na mjerenu vrijednost struje motora u praznom hodu: $I_{\mathrm{sm,mj}} = 92{,}31$ A, relativna pogreška izračuna 7,25 % Slijedi određivanje harmonika magnetske indukcije u zračnom rasporu.

Na temelju razmatranja u prethodnom poglavlju i rezultata prikazanih u tablici 2, odabrana su za izračun samo tri harmonika: glavni harmonik, treći i peti. Njihove analitički dobivene vrijednosti su: $B_{1\mathrm{p}} = 1{,}043$ T, $B_{3\mathrm{p}} = 0{,}049\,57$ T, $B_{5\mathrm{p}} = 0{,}029\,23$ T.

Ako se definira da je ovaj proračun namijenjen za spljoštenu (nesinusnu) krivulju raspodjele magnetske indukcije u zračnom rasporu, za rezultat se u treći harmonik, koji dolazi iz sustava istosmjernih struja, prema [6] mora uvesti dodatni korektivni faktor ovisno o karakteristici magnetiziranja. Sad se mogu definirati harmonici indukcije u zračnom rasporu:

0,749, nominal frequency 50 Hz, nominal rotation speed 1 455 min-^{1}, nominal moment 31,5 Nm, number of stator slots 36, number of rotor slots 48, air gap 0,4 mm.

The procedure of shaft current calculation should be initiated by calculating the magnetization current on the basis of known expressions from the machine theory. The value of $I_{\mathrm{sm}} = 98{,}85$ A, was analytically calculated, and from that it follows that, in relation to the measured current value of the idle running motor: $I_{\mathrm{sm,mj}} = 92{,}31$ A the relative calculation error is 7,25 %. Next is determination of the magnetic induction harmonics in the air gap.

Based on the considerations from the following chapter and the results shown in table 2, only three harmonics have been chosen for calculation: the main harmonic, the third and the fifth. Their analytically derived values are: $B_{1\mathrm{p}} = 1{,}043$ T, $B_{3\mathrm{p}} = 0{,}049\,57$ T, $B_{5\mathrm{p}} = 0{,}029\,23$ T.

If it is defined that this calculation is intended for the flat (non-sinuous) curve of the magnetic induction distribution in the air gap, for the result, in the third harmonic, which comes from the direct current system, according to [6] an additional corrective factor must be inserted depending on the magnetization characteristic. Now the magnetic induction harmonics in the air gap can be determined.

$$\begin{aligned}
b_2(x,t) &= B_2 \sin(2x - \omega t - \varphi_{\mathrm{m}}) \,, \\
b_6(x,t) &= B_6 \sin(6x - 3\omega t - 3\varphi_{\mathrm{m}}) \,, \\
b_3(x,t) &= B_3 \sin(3x - \omega t - \varphi_{\mathrm{m}}) \,, \\
b_{10}(x,t) &= B_{10} \sin(10x - \omega t - \varphi_{\mathrm{m}}) \,,
\end{aligned} \tag{29}$$

gdje indeksi magnetske indukcije (indeksi: 2, 6, 10) predstavljaju harmonike: ν/p. Pojavljuje se i treći harmonik (indeks 3), koji je uzrokovan istosmjernim sustavom napajanja i zasićenjem željeza [6].

Za definiranje doprinosa statičke ekscentričnosti ili ovalnosti, iznose indukcije u zračnom rasporu treba množiti s vodljivošću koja u sebi sadrži doprinos geometrijske nesimetrije. Prema postupku određivanja harmonika osovinske struje definiranim u poglavlju 3 mogu se izvesti izrazi koji ukazuju koliko te geometrijske nesimetrije doprinose harmonicima osovinske struje. U tablici 3 prikazane su kombinacije koje daju tri osnovna harmonika osovinske struje (ω, 3ω, 5ω).

where magnetic induction indices (indices: 2, 6, 10) represent the harmonics: ν/p. A third harmonic also appears (index 3) which is caused by the direct current system of power supply and iron saturation [6].

For the definition of the static eccentricity or ovality contribution, the rates of induction in the air gap should be multiplied with the conductivity which contains in it the contribution of geometric asymmetry. According to the procedure for determining the shaft current harmonics defined in Chapter 3, expressions can be derived which show how much those asymmetries contribute to the shaft current harmonics. Table 3 shows combinations which give three basic shaft current harmonics (ω, 3ω, 5ω).

Tablica 3 – Pregled amplituda harmonika osovinske struje prema doprinosu određenih harmonika indukcije
Table 3 – An overview of the shaft currents harmonic amplitudes to contribution of certain magnetic induction harmonics

Statička ekscentričnost / Static eccentricity		
ω	ω	5ω
$B_2\cdot\frac{k_e B_2}{2}$	$B_6\cdot\frac{k_e B_2}{2}$	$B_{10}\cdot\frac{k_e B_6}{2}$

Statička ovalnost / Static ovality				
ω	ω	ω	3ω	5ω
$B_{10}\cdot\frac{k_o B_6}{2}$	$\frac{k_o B_{10}}{2}\cdot\frac{k_o B_2}{2}$	$\frac{k_o B_6}{2}\cdot\frac{k_o B_2}{2}$	$\frac{k_o B_6}{2}\cdot\frac{k_o B_6}{2}$	$\frac{k_o B_{10}}{2}\cdot\frac{k_o B_6}{2}$

Pri tome je:

k_e – koeficijent statičke ekscentričnosti,
k_o – koeficijent statičke ovalnosti.

Uz zadane vrijednosti: $\varepsilon = 0{,}25$ i $\Lambda_e = 1$ Vs/Am^2, koeficijenti k_e i k_o
računaju se prema (16)
što daje: $k_e = 0{,}008\,07$, $\Lambda_o = 2$ Vs/Am^2 i $k_o = 0{,}032\,27$.

Amplitude indukcije u jarmu dobivaju se integriranjem indukcije u zračnom rasporu prema izrazu (3). Pri tome treba prvo prema tablici 3 zbrojiti sve doprinose za svaki pojedini harmonik. Kao konačan rezultat za amplitude harmonika indukcije u jarmu dobiva se: $B_{\mathrm{y1p}} = 1{,}625$ T, $B_{\mathrm{y3p}} = 0{,}060\,17$ T, $B_{\mathrm{y5p}} = 0{,}009\,19$ T.

Prema izrazu (6) određuje se jakost polja u jarmu uporabom izraza za aproksimaciju krivulje magnetiziranja prema [7], a dobiveni polinom ovisnosti o magnetskoj indukciji je: $h(b) = 146{,}629\cdot b + 17{,}112\cdot b^{11}$ A/m
.

Iz tablice 2 je očito da najveći doprinos osovinskoj struji daje harmonik B_6, pa će se daljnji proračun pojednostaviti i uzet će se samo dominantni član jakosti polja (h_{y6}). Iz (8) slijedi:

Thereat it is as follows:

k_e – static eccentricity coefficient,
k_o – static ovality coefficient.

With given values: $\varepsilon = 0{,}25$ and $\Lambda_e = 1$ Vs/Am^2, coefficients k_e and k_o
are calculated according to (16)
which gives the following: $k_e = 0{,}008\,07$, $\Lambda_o = 2$ Vs/Am^2 and $k_o = 0{,}032\,27$.

Induction amplitudes in the yoke are obtained by integrating induction in the air gap according to the expression (3). Thereat, according to Table 3, all the contributions for each particular harmonic need to be summed up first. As the final result for harmonic induction amplitudes in the yoke, the following is derived: $B_{\mathrm{y1p}} = 1{,}625$ T, $B_{\mathrm{y3p}} = 0{,}060\,17$ T, $B_{\mathrm{y5p}} = 0{,}009\,19$ T.

According to the expression (6), the the field intensity in the yoke is determined by using the expression for the approximation of the magnetization curve according to [7], and the obtained polynomial of dependency on magnetic induction is: $h(b) = 146{,}629\cdot b + 17{,}112\cdot b^{11}$ A/m
.

Table 2 clearly shows that the greatest contribution to the shaft current is given by the harmonic B_6, so that further calculation will be simplified and only the dominant term of field intensity (h_{y6}) will be taken. From (8) the following ensues:

$$h_{\mathrm{y6}}(x,t) = B_{\mathrm{y6}}\sin(6x - 3\omega t - 3\varphi_{\mathrm{m}})\left\{\frac{1}{\mu_1} + \frac{1}{\mu_{11}}\left[\sum_{\nu} B_{\mathrm{y}\nu}\sin(\nu x - \omega_\nu t - \varphi_\nu)\right]^{10}\right\}, \tag{30}$$

pri tome vrijedi:

thereat it is as follows:

$$\begin{aligned}\sum_{\nu} B_{\mathrm{y}\nu}\sin(\nu x - \omega_\nu t - \varphi_\nu) &= B_{\mathrm{y2}}(2x - \omega t - \varphi_{\mathrm{m}}) + B_{\mathrm{y6}}\sin(3x - \omega t - \varphi_{\mathrm{m}}) + \\ &+ B_{\mathrm{y6}}\sin(6x - 3\omega t - 3\varphi_{\mathrm{m}}) + B_{\mathrm{y10}}\sin(10x - \omega t - \varphi_{\mathrm{m}})\end{aligned} \tag{31}$$

Kao rješenje integrala (9) pojavljuju se samo harmonici jakosti polja koji imaju frekvenciju $(6x)$, jer svi ostali zbog periodičnosti daju rezultat jednak nuli. To znači da se iz polinoma (30) može izostaviti prvi član $(1 / \mu_1)$, kao i svi članovi zagrade koji nemaju frekvenciju $(6x)$. Uporabom Fourierovog integrala za određivanje koeficijenata Fourierovog reda za amplitudu jakosti polja iz (30) slijedi:

As the solution of the integral (9), only field intensity harmonics appear with $(6x)$ frequency because all others, due to their periodicity, give results equal to zero. That means that the first term $(1/\mu_1)$, as well as all the terms of the brackets which have no frequency $(6x)$ can be omitted from the polynomial. By using the Fourier integral for the determination of the Fourier series coefficient for the field intensity amplitude from (30) it follows:

$$H_{y6} = \frac{1}{\pi}\int_0^{2\pi}\left[\sum_\nu B_{y\nu}\sin\left(\nu x - \omega_\nu\, t - \varphi_\nu\right)\right]^{10} \cdot \sin\left(6x - 3\omega t - 3\varphi_m\right)\mathrm{d}x, \qquad (32)$$

pri čemu se promatra položaj u neutralnoj zoni između polova u jarmu, pa se uz $\omega t - \varphi_m = \pi / 2$ izraz u zagradi u (30) može reducirati u oblik nepreglednog polinoma (b_{y6}), koji je funkcija amplituda harmonika magnetske indukcije u jarmu statora. Za jakost polja u jarmu (u neutralnoj zoni između polova u jarmu) iz izraza (30) i (32) slijedi:

whereat the position is observed in the neutral zone between the poles in the yoke, so with $\omega t - \varphi_m = \pi/2$, the expression in the brackets in (30) can be reduced in the form of the illegible polynomial (b_{y6}) which is a function of the amplitudes of the magnetic induction amplitudes in the stator yoke. For the field intensity in the yoke (in the neutral zone between the poles in the yoke) from the expressions (30) and (32) it follows:

$$h_{y6}(x,t) = B_{y6}\sin\left(6x - 3\omega t - 3\varphi_m\right)\cdot b'_{y6} = H_{y6}\sin\left(6x - 3\omega t - 3\varphi_m\right). \qquad (33)$$

U uvodu je definirano da je bitno imati mogućnost uvida u to koji parametri najviše utječu na pojavu osovinskih struja, odnosno koji harmonici magnetske indukcije u zračnom rasporu. Jasno da se uporabom računala, numerički rezultat integriranja iz (32), odnosno vrijednost koeficijenta (b_{y6}) u izrazu (33), koji je potreban za daljnji proračun, može bez problema izračunati.

Prema izrazu (9) mogu se odrediti amplitude harmonika protjecanja, odnosno za amplitude harmonika kružnog magnetskog toka po jedinici površine jarma statora vrijedi izraz:

The introduction defines that it is necessary to have the possibility of insight into the parameters which have the greatest influence on the occurrence of shaft currents, that is, the magnetic induction harmonics in the air gap. It is clear that, by using a computer, the numerical integration results from (32), that is, the value of the coefficient (b_{y6}) in the expression (33), which is necessary for further calculation, can be calculated without difficulty.

According to the expression (9), flux harmonics amplitudes can be determined, that is, for the amplitudes of harmonics of circular magnetic flux per unit of stator yoke surface the following expression applies:

$$\Phi'_\nu = \Theta_\nu \cdot \mu_0 \mu_r \cdot \frac{2R\pi}{S_y}, \qquad (34)$$

gdje je:

S_y – poprečni presjek jarma, m^2,
$2R\pi$– srednja duljina jarma, m,
μ_r – relativna permeabilnost koja se dobiva iz polinoma aproksimacije krivulje magnetiziranja za uvrštenu vrijednost amplitude h_{y6}.

where it is as follows:

S_y– yoke cross cut [m^2],
$2R\pi$– average yoke length [m],
μ_r – relative permeability derived from the polynomial of the magnetization curve approximation for the inserted amplitude value h_{y6}.

Budući da su poznate sve numeričke vrijednosti, može se izračunati iznos kružnog magnetskog toka u jarmu:

Since all numeric values are known, the amount of circular magnetic flux in the yoke can be calculated:

$$\phi(t) = \Phi_{1\mathrm{p}} \sin(\omega_{1\mathrm{p}} t) + \Phi_{3\mathrm{p}} \sin(\omega_{3\mathrm{p}} t) + \Phi_{5\mathrm{p}} \sin(\omega_{5\mathrm{p}} t), \qquad (35)$$

gdje su amplitude harmonika kružnog magnetskog toka u jarmu:
$\Phi_{1\mathrm{p}} = 0{,}003\,25$ Vs, $\Phi_{3\mathrm{p}} = 0{,}000\,12$ Vs, $\Phi_{5\mathrm{p}} = 1{,}838 \cdot 10^{-5}$ Vs.

Za izračun amplituda harmonika osovinske struje upotrebom izraza:

where the amplitudes of the harmonic of the circular magnetic flux in the yoke are:
$\Phi_{1\mathrm{p}} = 0{,}003\,25$ Vs, $\Phi_{3\mathrm{p}} = 0{,}000\,12$ Vs, $\Phi_{5\mathrm{p}} = 1{,}838 \cdot 10^{-5}$ Vs.

For the calculation of the shaft current harmonics amplitudes by using the expression:

$$I_v = \frac{\omega_v \Phi_v}{Z_v}, \qquad (36)$$

potrebno je poznavati impedanciju strujnog kruga kojim se zatvara osovinska struja. Ovisno o konstrukciji motora taj podatak se može računski izvesti, ali je jednostavnije provesti mjerenja impedancije. U [6] su predočeni rezultati mjerenja za promatrani motor u kojima se impedancija zanemarivo mijenja s frekvencijom, odnosno može se aproksimirati radnim otporom iznosa 0,7 Ω.

Konačno se za harmonike osovinske struje dobivaju sljedeće vrijednosti: $I_{1\mathrm{p}} = 1{,}031$ A, $I_{3\mathrm{p}} = 0{,}114\,6$ A, $I_{5\mathrm{p}} = 0{,}029\,2$ A, što za efektivnu vrijednost osovinske struje daje 0,734 A. U odnosu na izmjerenu vrijednost, gdje je efektivna vrijednost osovinske struje 0,690 A, relativna pogreška izračuna je 6,37 %, što je vrlo dobar rezultat s obzirom da je u proračun uključeno samo nekoliko harmonika.

Mjerenja osovinskih struja na četveropolnom niskonaponskom asinkronom motoru su vršena pomoću svitka Rogowskog, koji je bio smješten oko osovine na rotoru (slika 2) pri napajanju iz mreže [8]. Za mjerenja osovinskih struja korišten je svitak Rogowskog standardnih karakteristika AmpFLEX A100 20/200A, a za prikupljanje podataka iz svitka Rogowskog korištena je AD kartica NI-DAQPad-6015.

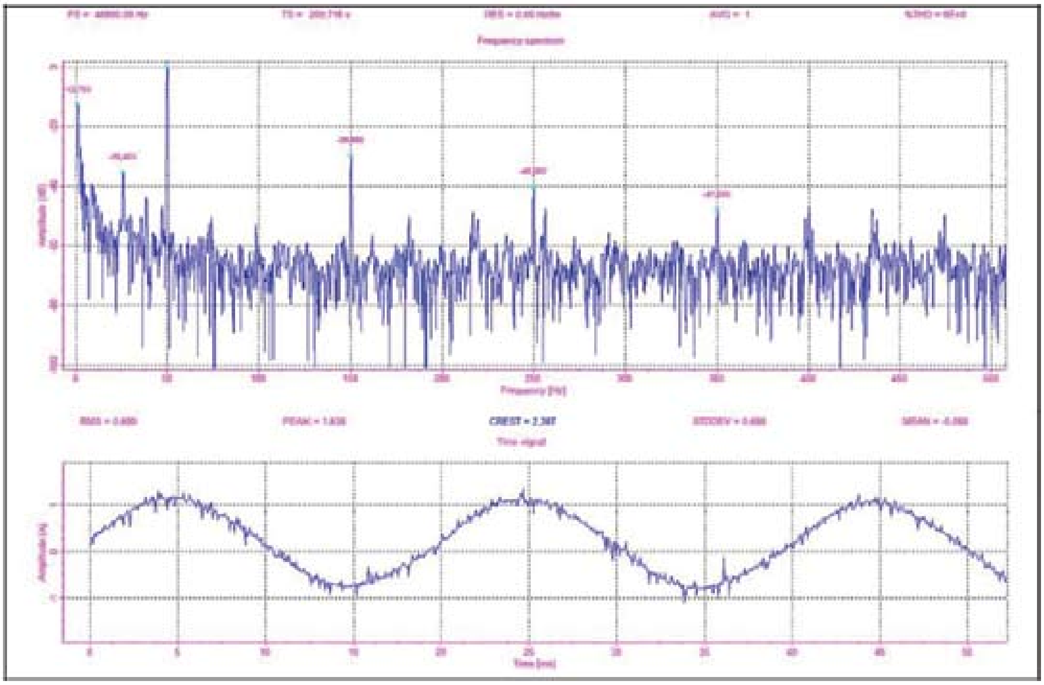
Na slici 3 je prikazan frekvencijski spektar osovinske struje i vidljivo je da su najizraženiji harmonici upravo frekvencije: ω, 3ω i 5ω. Međutim, u spektru se mogu uočiti i drugi harmonici iz tablice 2. Vremenski signal prikazan je u donjem dijelu slike 3, a predstavlja amplitudno filtriran signal. Detaljnije objašnjenje pravilnog postupka mjerenja i obrade mjerenog signala osovinske struje dano je u [8] I [9].

it is necessary to know the impendence of the electrical circuit in which shaft current can be closed. Depending on the motor construction, that data can be derived by computation, but it is simpler to undertake measurements of impendence. [6] presents the measurement results for the observed motor and in those results the impendence is insignificantly exchanged by frequency, that is, it can be approximated by ohmic resistance at the rate of 0,7 Ω.

Finally, for the shaft current harmonics, the following values are derived: $I_{1\mathrm{p}} = 1{,}031$ A, $I_{3\mathrm{p}} = 0{,}1146$ A, $I_{5\mathrm{p}} = 0{,}029\,2$ A, which gives 0,734 A for the effective value of the shaft current. In relation to the measured value, where the effective value of the shaft current is 0,690 A, the relative calculation error is 6,37 %, which is a very good result considering that only a few harmonics are included in the calculation.

Measurements of shaft currents on a four-pole low-voltage induction motor were done by virtue of a Rogowski coil which was placed around the rotor axis (Figure 2) with power supply from the network [8]. For the measurements of the shaft currents, the Rogowski coil of AmpFLEX A100 20/200A standard characteristics was used, and for the collection of data from the Rogowski coil, the AD NI-DAQPad-6015 card was used.

Figure 3 shows the frequency spectre of the shaft current and it is evident that the most prominent harmonics are exactly the following frequencies: ω, 3ω and 5ω. However, other harmonics from Table 2 can also be seen in the spectre. Time signal is shown in the lower part of Figure 3 and it represents the amplitudinally filtered signal. A more



Slika 3 — Frekvencijski spektar i vremenski dijagram mjerenja osovinske struje
Figure 3 — Frequency spectre and time diagram of shaft current measurements

5 ZAKLJUČAK

Pri napajanju niskonaponskih asinkronih motora s homogenim jarmovima mogu nastati osovinske struje, koje dovode do kvarova ležajeva. Uzrok tim strujama je uglavnom ekscentrični položaj rotora u statoru, a nužni uvjet za nastajanje je nelinearnost krivulje magnetiziranja magnetskih limova. U članku je opisan postupak analitičkog izračuna osovinske struje, te je na primjeru jednog četveropolnog asinkronog kaveznog motora za pogon viličara dana usporedba izračuna amplituda nekih značajnih harmonika osovinske struje s mjerenim vrijednostima, koja je pokazala da se takvim postupkom mogu analitički računati amplitude harmonika osovinske struje s pogreškom manjom od 10 %, što je jasno povezano s točnošću ulaznih podataka.

Međutim, nameće se zaključak da se pri projektiranju motora, ovakvim proračunom mogu definirati svi važniji harmonici osovinske struje, te se mogu izračunati njihove amplitude pri različitim stupnjevima oštećenja ležaja. Na taj način je moguće napraviti detaljni prikaz harmonika, koji se trebaju tražiti u harmonijskom spektru osovinske struje, kao i struje faza motora, te definirati na koji stupanj oštećenja ukazuju iznosi njihovih amplituda. Time se može poboljšati i dijagnostika motora.

detailed explanation of the correct measurement procedure and the analysis of the measured shaft current signal are given in [8] and [9].

5 CONCLUSION

In the supply of low-voltage induction motors with homogenous yoke, shaft currents may appear which cause damage to the bearings. The cause of those currents is mostly the eccentric position of the rotor in the stator, and a necessary condition for the occurrence of those currents is the non-linearity of the magnetic tin magnetization curve. The article describes the procedure of the analytical calculation of the shaft current, and on the example of a four-pole squirrel-cage induction motor for the forklift drive, a comparison is given of the calculation of the amplitudes of certain important shaft current harmonics with the measured values and it shows that by virtue of such a procedure, shaft current harmonics amplitudes can be analytically calculated with an error less than 10 %, which is clearly connected with the accuracy of input data.

However, the conclusion is evoked that this procedure can be used to define all important shaft current harmonics during the design of engineering motors, and their amplitudes can be calculated at different levels of bearing damage. In that way, it is possible to make a detailed presentation of the harmonics which should be searched for in the harmonic spectre of the shaft current, as well as of the motor phases current, and the level of damage, indicated by the rates of their amplitudes, can be defined. In that way, motor diagnostics can also be improved.

LITERATURA / REFERENCES

[1] BLODT M., GRANJON P., RAISON B., ROSTAING G., Models for Bearing Damage Detection in Induction Motors Using Stator Current Monitoring, Industrial Electronics, IEEE Transactions on, April 2008, Volume: 55, Issue: 4, pp. 1813–1822

[2] SEINSCH H. O., Lagerstrome bei Drehstrom Induktions-maschinen, Ursachen und Methoden ihrer Unterdrückung, Conti Elektro-Berichte 15, Jan/Juni 1969., pp. 43–51

[3] ŠTEFANKO S., BOGUT M., KURTOVIĆ I., KOVAČEVIĆ M., MOMIĆ M., The origins of Shaft Currents in Squirrel-cage Low-voltage Induction Machines, Automatika, 46 (2005), 3-4, str. 109–111

[4] FREISE W., JORDAN H., Die Ermittlung von Wellenspannung und Lagerstrom in elektrischen Maschinen unter Berücksichtigung der magnetischen Spannung im Eisen, Elektrotechnik und Maschinenbau (E&M) 80, 1963, pp. 80–84

[5] PALJAN D., Analiza utjecaja nesimetričnog zračnog raspora na elektromagnetska i akustička svojstva asinhronog motora, doktorska disertacija, FER, Zagreb, 1990.

[6] HEDERIĆ Ž., Proračun osovinskih struja niskonaponskih asinkronih motora, doktorska disertacija, ETF, Osijek, 2008.

[7] SCITOVSKI R., Krivulja magnetiziranja, Elektrotehnika 31(1988), str 31–35

[8] ŠTEFANKO S., HEDERIĆ Ž., HADŽISELIMOVIĆ M., ZAGRADIŠNIK I., Analyses of Shaft Currents in Low-voltage Induction Motor for Forklift Drive with Electronic Equipment, Informacije MIDEM, Ljubljana, 2007., vol 37, part 3, pp. 152–157

[9] ONG R.,DYMOND J. H., FINDLAY D., SZABADOS B., Shaft Current in AC Induction Machine-An Online Monitoring System and Prediction Rules, IEEE Transactions on Industry Applications, vol. 37, No. 4, July/Aug. 2001, pp. 1189–1196

Adrese autora:

Dr. sc. **Željko Hederić**
hederic@etfos
Mr. sc. **Marinko Barukčić**
Krešimir Miklošević, dipl. inž.
Sveučilište J. J. Strossmayer u Osijeku
Elektrotehnički Fakultet Osijek
Kneza Trpimira 2b
31000 Osijek
Hrvatska

Authors' Adresses:

Željko Hederić, PhD
hederic@etfos
Marinko Barukčić, MSc
Krešimir Miklošević, dipl. ing.
J. J. Strossmayer University of Osijek
Faculty of Electrical Engineering
Kneza Trpimira 2b
31000 Osijek
Croatia

Uredništvo primilo rukpois:
2009-01-20

Prihvaćeno:
2009-02-09

Manuscript received on:
2009-01-20

Accepted:
2009-02-09