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INVESTMENT DECISION MAKING

ABSTRACT

Investment evaluation is the control of the planning and implementation of investment activities with regard to the objectives to be achieved. In this paper I assume the objective to be efficient outcome and profit maximization. This means that investment evaluation puts normative assessments into the context of planning and management and hence into the context of intentional action and cycles of action. Here not only the assessment of facts and scenarios is important but also the, more or less implicit, causal chains which connect activities with investment results and finally with goal achievement. The model for investment evaluation I propose has two money holders who must decide how to invest their money in two investment funds (financial intermediaries) that, in turn, will use the money to bid to acquire ownership in two projects. The general case when the number of money holders, the number of funds, and the number of investments are arbitrary may be handled in a similar manner to the development below, but at a cost of greater complexity. As a result no mechanism to achieve the maximum outcome is present and different methods to find optimal structure under uncertainty and different cost structures are discussed.

JEL: C30, C53, C70

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1. Introduction

The model for investment evaluation I propose has two money holders who must decide how to invest their money in two investment funds that, in turn, will use the money to bid to acquire ownership in two projects. Importantly, the profitability of each project depends on the specific joint ownership structure that results from the money which each MIF[1] receives, as the funds are assumed to have different management capabilities.

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[1] Money Investment Fund

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I start by assuming that $N > 0$ money points are owned by the population which consists of two individuals, I_1 and I_2. Money holder I_l , l = 1, 2, has $V_l > 0$ money points where $V_1 + V_2 = N$. The number of money points held by each individual may differ to allow the possibility of pre-auction trading. Each I_l must decide independently on the number of money points to invest in each of two money funds, $F_j, j = 1, 2$. The number of money points that I_l, chooses to allocate to F_1 is denoted by x, x$\in [0,V_1]$ with the remaining V_1 – x money points being allocated to F_2.

Similarly, I denote by y, y $\in [0,V_2]$, the money points investment of I_1 in F_2 , with V_2 - y being invested in F_2 . As a consequence of investing its money points in this manner I_1 acquires the proportion $\frac{x}{x+y}$ of the profit of F_1, and $\frac{V_1-x}{(V_1-x)+(V_2-y)}$ of the profit of F_2. Correspondingly, I_2 acquires the proportion $\frac{y}{x+y}$ of the profit of F_1 and $\frac{V_2-y}{(V_1-x)+(V_2-y)}$ of the profit of F_2 . The general case when the number of money holders, the number of funds, and the number of investments are arbitrary may be handled in a similar manner to the development below, but at a cost of greater complexity.

At the outset, neither F_j has any money. In order to attract money from the I_l, each F_j reveals information useful to the I_l. I assume that this information relates to the cost structure of the F_j. Specifically, I assume that each F_j announces that its costs will be a fixed proportion of the revenues it will earn by investing the money points that it will acquire. This assumption is equivalent to the assumption that the profit of the F_j is equal to $\sigma_j R_j(x + y)$ where σ_j is constant, $\sigma_j \in [0,1]$, j = 1, 2, and R_j: $\Re_+ \to \Re_+$ is the revenue received by F_j as a result of the bidding game in which, using money points acquired from the I_l, F_1 and F_2, compete to acquire share in the projects offered for financing. The σ_j can be thought of as the proportion of revenue that the F_j promise to distribute to the share holders. $R_1(x + y)$ depends on $x+ y$ since this is the number of money points available to F_1 for investment in projects. Similarly, $R_2(x + y)$ has the same dependence since the total number of money points, N , is fixed.

Thus I_1, receives m_1: $\Re_+^2 \to \Re_+$ where

$$m_1(x,y) = \frac{x}{x+y}\sigma_1 R_1(x+y) + \frac{V_1-x}{(V_1-x)+(V_2-y)}\sigma_2 R_2(x+y)$$

Thus I_2, receives m_2: $\Re_+^2 \to \Re_+$ where

$$m_2(x,y) = \frac{y}{x+y}\sigma_1 R_1(x+y) + \frac{V_2-y}{(V_1-x)+(V_2-y)}\sigma_2 R_2(x+y)$$

I_1 chooses x to maximize m_1 and I_2 chooses y to maximize m_2. I refer to the problem of simultaneously maximizing m_1 and m_2 as the money investment problem (MIP). In what follows, I take x and y to be continuous over their respective ranges.

The $R_j(x + y)$ are determined by the following process. With $N_1 = x + y$ and $N_2 = N - N_1$ respectively, F_1 and F_2 play a non-cooperative game in which they submit bids to acquire shares in company i, i = 1, 2. Each F_j submits a money point bid of a_{ij} in company i where $a_{ij} \geq 0$ and $\sum_i a_{ij} = N_j$ As a consequence of the bidding, each F_j receives the proportion $p_{ij} = \frac{a_{ij}}{\sum_j a_{ij}}$ of π_i , the profit of project i. I assume that the $\pi_i : \Re_+^2 \to \Re_+$, i = 1, 2, depend on p_{ij} , j =1,2, that is, I assume that the F_j have different skills in managing and restructuring the projects in which they have acquired share, and that the impact of their skills on the profit of a given project depends on the proportion of ownership that they achieve in the project as a result of the bidding game.

Furthermore, I assume, for tractability, that the profit functions $\pi_i(p_{i1,} p_{i2})$can be reasonably approximated by a first-order Taylor expansion. It follows, since $p_{i1} + p_{i2} = 1$for i = 1, 2, that:

$$\pi_1(p_{11,} p_{12}) = \pi_1(0,1) + p_{11}\left[\frac{\partial \pi_1(0,1)}{\partial p_{11}} - \frac{\partial \pi_1(0,1)}{\partial p_{12}}\right]$$

$$\pi_2(p_{21}, p_{22}) = \pi_2(1,0) + p_{22}\left[\frac{\partial \pi_2(1,0)}{\partial p_{22}} - \frac{\partial \pi_2(1,0)}{\partial p_{21}}\right]$$

In the remainder of the paper I use the notation $k_{12} = \pi_1(0,1)$; $k_{21} = \pi_2(1,0)$;

$$\Delta_1 = \frac{\partial \pi_1(0,1)}{\partial p_{11}} - \frac{\partial \pi_1(0,1)}{\partial p_{12}} \quad ; \quad \Delta_2 = \frac{\partial \pi_2(1,0)}{\partial p_{22}} - \frac{\partial \pi_2(1,0)}{\partial p_{21}} \quad ; \text{ and } r_i = \frac{\Delta_i}{k_{ij}} \text{ for } j \neq i.$$

In summary, I assume that the profit function can be written as;

$$\pi_i = k_{ij} + p_{ii}\Delta_i = k_{ij}(1 + p_{ii}r_i) \text{ for } i \neq j,\ i,j=1,2$$

The parameter k_{12} represents the profit that project 1 would make if it were totally purchased by F_2. The parameter k_{21} has a similar interpretation. The parameter Δ_1 represents the difference in the differential advantage (disadvantage) that F_1 has over F_2, in managing project 1. The parameter Δ_2 has a similar interpretation. Thus, π_1 is modeled as the sum of the value that would occur if F_2 were to manage project 1 exclusively plus the improvement, (deterioration) when ownership is shared with F_1. The profit π_2 has a similar interpretation. Notice that if F_1, and F_2, have the same differential impact on π_1 , the value of the profit function would be the same regardless of how ownership were shared.

I note that since the p_{ii} depend on $x + y$, the π_i depend on $x + y$ also. For subsequent use, I define $\pi_i(z) = k_{ij} + z\Delta_i$. Thus, after having submitted their bids, F_j receives the revenue.

$p_{1j}\pi_1(p_{11}) + p_{2j}\pi_2(p_{22})$

The revenue accruing to F_j at the Nash equilibrium of the bidding game is what I call $R_j(N_1)$ and thus the profit available for distribution to the I_l by F_j is $\sigma_j R_j(N_1)$ where $N_1 = x + y$.

I assume that both I_l share the same information set concerning the projects and skill levels, as well as the reasoning and characteristics of the F_j. Since the $R_j(N_1)$, the results of the bidding game between the F_j, are required by the I_l to solve their problem, I investigate this bidding game first.

2. The money fund problem

I now formalize the non-cooperative bidding game played by the F_j. Given N_1 and N_2, and given the bids of $F_{j'}$, $j' \neq j$, F_j must choose its bids to maximize its profit. Since, by earlier assumption, its profit is a fixed multiple of its revenue, F_j's bids must satisfy

$$\max_{a_{1j}, a_{2j}} \sum_i p_{ij}\pi_i(p_{ii})$$

subject to $a_{ij} \geq 0$ and $\sum_i a_{ij} = N_j$ and where $p_{ij} = \dfrac{a_{ij}}{\sum_j a_{ij}}$. I refer to these programs as the money fund problem (VFP).

The Lagrangian for F_1 *is:*

$$L_1 = p_{11}\pi_1(p_{11}) + p_{21}\pi_2(p_{22}) - \lambda_1(a_{11} + a_{21} - N_1)$$

with first-order conditions:

$$\frac{\partial L_1}{\partial a_{11}} = 0 = \frac{(1 - p_{11})}{a_{1\bullet}}\pi_1 + p_{11}\frac{(1 - p_{11})}{a_{1\bullet}}\Delta_1 - \lambda_1 = \frac{(1 - p_{11})}{a_{1\bullet}}(\pi_1 + p_{11}\Delta_1) - \lambda_1 \qquad (1)$$

$$\frac{\partial L_1}{\partial a_{21}} = 0 = \frac{(1 - p_{21})}{a_{2\bullet}}\pi_2 - p_{21}\frac{p_{22}}{a_{2\bullet}}\Delta_2 - \lambda_1 = \frac{p_{22}}{a_{2\bullet}}(\pi_2 - \Delta_2 + p_{22}\Delta_2) - \lambda_1 \qquad (2)$$

$$\frac{\partial L_1}{\partial \lambda_1} = 0 = a_{11} + a_{21} - N_1 \qquad (3)$$

where

$a_{i\bullet} = \sum_j a_{ij}$

Similarly, the Lagrangian for F_2 is:

$L_2 = p_{12}\pi_1(p_{11}) + p_{22}\pi_2(p_{22}) - \lambda_2(a_{12} + a_{22} - N_2)$

with first-order conditions:

$$\frac{\partial L_2}{\partial a_{12}} = 0 = \frac{(1-p_{12})}{a_{1\bullet}}\pi_1 - p_{12}\frac{p_{11}}{a_{1\bullet}}\Delta_1 - \lambda_2 = \frac{p_{11}}{a_{1\bullet}}(\pi_1 - \Delta_1 + p_{11}\Delta_1) - \lambda_2 \qquad (4)$$

$$\frac{\partial L_2}{\partial a_{22}} = 0 = \frac{(1-p_{22})}{a_{2\bullet}}\pi_2 - p_{22}\frac{(1-p_{22})}{a_{2\bullet}}\Delta_2 - \lambda_2 = \frac{(1-p_{22})}{a_{2\bullet}}(\pi_2 + P_{22}\Delta_2) - \lambda_2 \qquad (5)$$

$$\frac{\partial L_2}{\partial \lambda_2} = 0 = a_{12} + a_{22} - N_2 \qquad (6)$$

Before presenting the solution to the VFP, I provide the following lemma. Recall that $r_i = \frac{\Delta_i}{k_{ij}}$ and $\pi_i(z) = k_{ij} + z\Delta_i$.

Lemma 1

Let $k_{ij} > 0$ for $i \neq j$ and let $r_i \in (-1,1]$. For any $\alpha \in [0,1]$, there exists a unique set of values $z_1^*, z_2^*, \Theta^* \in [0,1]$ that simultaneously satisfy.

(i) $1 - z_1 = \Theta\frac{1 + r_1 z_1}{1 + 2r_1 z_1}$

(ii) $1 - z_2 = (1-\Theta)\frac{1 + r_2 z_2}{1 + 2r_2 z_2}$

(iii) $z_1\pi_1(z_1) + (1-z_2)\pi_2(z_2) = \alpha[\pi_1(z_1) + \pi_2(z_2)]$

Theorem 1

When $k_{ij} > 0$ for $i \neq j$, and when $r_i \in (-1,1]$, there exists a Nash equilibrium of the VFP and it is unique. In particular, let z_1^*, z_2^*, and Θ^* be the solutions to the equations of l.emma 1 corresponding to $\alpha = \frac{N_1}{N}$. Then, under the stated conditions, the unique solution to equations (1)-(6), i.e., the Nash equilibrium of the VFP for i, j = 1,2 and $j \neq i$ is:

$a_{ii}^* = K^{-1}z_i^*\pi_i(z_i^*)$, $a_{ij}^* = K^{-1}(1-z_i^*)\pi_i(z_i^*)$, $\lambda_1 = K\Theta^*$, $\lambda_2 = K(1-\Theta^*)$,

$where\ K = \dfrac{\pi_1(z_1^*) + \pi_2(z_2^*)}{N}$

It is useful to highlight a result established in the proof of Theorem 1 signifying the proportion of each project owned by each fund. I do this in the next corollary. In what follows, an asterisk above any function denotes that function evaluated at the solution to the VFP presented in Theorem 1.

Corollary 1

The solution to the VFP *yields* $p_{ii}^* = z_i^*$.

Interpreting $\dfrac{\pi_i^*}{a_{i\bullet}^*}$ as the resulting value per money in project *I*, Theorem 1 establishes that these values are the same for both projects at the Nash equilibrium of the bidding game. Furthermore, this common value is equal to the economy- wide value of a money given by $\dfrac{\pi_1^* + \pi_2^*}{N}$. This common value of a money is also equal to the sum of the two shadow prices that is denoted by *K* in Theorem 1. An additional money to the system, yielding approximately the value K, would be divided between F_1 and F_2 in the amounts λ_1 and λ_2. Thus, F_1, would receive Θ^* percent of this additional amount, and F_2 the remainder, $1-\Theta^*$ where Θ^* incorporates, among other things, the relative skill levels of F_1 and F_2.

At the Nash equilibrium, a total of $K^{-1}\pi_j^* = N\dfrac{\pi_j^*}{\pi_1^* + \pi_2^*}$ money points are invested in project *j* , *j* = 1, 2, with F_j contributing z_j^* percent of these money points. I can interpret this total either as the part of the outstanding number of money points acquired by company *j* being proportional to π_j^*, or as the profit of project *j* denominated in units of economy-wide value per money . Although the money investment in project *i* depends on π_j^*, this profit cannot be known in advance since it depends on the composition of ownership resulting from the bidding game itself. Finally, the ratio $\dfrac{a_{ii}^*}{a_{ij}^*} = \dfrac{p_{ii}^*}{p_{ij}^*} = \dfrac{z_i^*}{1-z_i^*}$, $j \neq i$ depends on all the parameters of the problem including the skill levels of the F_j . I next establish the revenue that F_j receives as a consequence of the solution to the VFP. Let $\Pi^* = \pi_1^* + \pi_2^*$.

Corollary 2

At the Nash equilibrium of the VFP, *the revenue to* F_j *is equal to* $\dfrac{N_j}{N}\Pi^*$.

The solution to the VFP yields each F_j the proportion $\frac{N_j}{N}$ of the sum of the profits produced by projects 1 and 2 at the Nash solution. This establishes that $R_j(N_1) = \frac{N_j}{N}\Pi^*$ and that profit equals $\sigma_j \frac{N_j}{N}\Pi^*$. It also follows that at the Nash equilibrium, the revenue per money for each of the F_j is identical. I can now return to the problem facing the I_l, the original money holders.

3. The money investment problem

For the money investment problem (MIP) in which I_l wishes to maximize m_l, I_l must know $R_1(x+y)$ and $R_2(x+y)$. From Corollary 2 and the remarks following it, we derive $R_j = p_{1j}^*\pi_1^* + p_{2j}^*\pi_2^* = \frac{N_j}{N}\Pi^*$ where $\Pi^* = \pi_1^* + \pi_2^*$. Having assumed that each I_l has the same information concerning the bidding game played by F_1 and F_2 conditional on the funds having $N_1 = x+y$ money points, and $N_2 = N - N_1$ money points, respectively, it follows that each I_l also knows the Nash equilibrium of the VFP as presented in Theorem 1. Consequently, the respective objective functions of the I_l can be restated as:

$$m_1(x,y) = \frac{x}{x+y}\sigma_1\frac{N_1}{N}\Pi^* + \frac{V_1 - x}{(V_1 - x)+(V_2 - y)}\sigma_2\frac{N_2}{N}\Pi^*$$

and

$$m_2(x,y) = \frac{y}{x+y}\sigma_1\frac{N_1}{N}\Pi^* + \frac{V_2 - y}{(V_1 - x)+(V_2 - y)}\sigma_2\frac{N_2}{N}\Pi^*$$

Since $N_1 = x+y$ and $N_2 = N - N_1$, the last expressions can be reduced to:

$$m_1(x,y) = \left[\sigma_1\frac{x}{N} + \sigma_2\frac{V_1 - x}{N}\right]\Pi$$

and

$$m_2(x,y) = \left[\sigma_1\frac{y}{N} + \sigma_2\frac{V_2 - y}{N}\right]\Pi$$

Thus, in the money investment problem (MIP), investor I_1 seeks x^* where

$$x^* = \arg\max_{x\in[0,V_1]} m_1(x,y) \text{ subject to } y \in [0,V_2]$$

and investor I_2, seeks y^* where

$$y^* = \arg\max_{y\in[0,V_2]} m_2(x,y) \text{ subject to } x \in [0,V_1]$$

I next define an efficient allocation of money points. Let $N_1^* = \arg\max_{N_1 \in [0,N]} \Pi^*(N_1)$. Note that N_1^* is an apportionment of money points to the VPFs that achieves the maximum total profit.

Definition 1
An allocation of money points $(x, y), x \in [0,V_1], y \in [0,V_2]$, *is an efficient allocation if* $x + y = N_1^*$.
The case when $\sigma_1 = \sigma_2 = \sigma$
I continue by investigating the case in which the F_j pay out the same proportion of their revenues to the I_l; that is the case when $\sigma_1 = \sigma_2 = \sigma$. In this situation, $m_1 = \sigma \frac{V_1}{N} \Pi^*(x + y)$. Since increasing $\Pi^*(x + y)$ benefits both I_l it is in their joint interest to achieve the largest possible Π^* by their respective money investments. It follows that it is in the interest of the I_l to choose their money investments x^* and y^*, respectively, such that $x^* + y^* = N_1^*$ i.e., to choose their investments to be efficient. It also follows that there exists an infinity of equilibria to the MIP of the form (x^*, y^*) where $x^* = N_1^* - y^*$ for $x^* \in [0,V_2]$ and for $y^* \in [0,V_2]$. I summarize the previous remarks in the following theorem.

Theorem 2
When $\sigma_1 = \sigma_2 = \sigma$ *there exists an infinity of equilibria to the MIP consisting of the set of efficient allocations.*
But despite the fact that the I_l find it in their interest to have $x^* + y^* = N_1^*$, the non-cooperative nature of the Nash game offers no mechanism to cause the target N_1^* to be met. Since the target represents the division of the total number of money points in the system between the F_j that maximizes economy-wide profit, there is consequently no mechanism to achieve this efficient outcome. Thus, the failure to achieve efficiency is the result of the absence of coordination between the money holders.

Notice that this coordination failure is present even in the case in which the money point holders have identical and full information, and have as their goal the wish to allocate their money points in a manner consistent with the maximization of economy-wide profit. I now show that the introduction of uncertainty exacerbates the situation since it creates a situation in which the goal of the money holders is no longer one of maximizing total economy-wide profit; in fact, I show that the goal differs for the different money point holders.

When uncertainty is present, I must consider the investors' attitudes toward risk. To this end, I let $u_l : \Re_+ \to \Re_+$ with $u_l(m_l) = 1 - \exp(-\gamma_l m_l), \gamma_l > 0$ be the utility function of I_l. I assume that all information is known to the money point holders as before, with one exception: Δ_1 is known imperfectly. I assume that both money point holders perceive $\tilde{\Delta}_1$ as a random variable, distributed normally with mean Δ_1 (as before) and variance σ^2. I denote this density as $\phi(\Delta_1, \sigma^2)$. It follows that $\tilde{\Pi}^*$ is random since $\tilde{\Pi}^* = k_{12} + k_{21} + p_{11}\tilde{\Delta}_1 + p_{22}\Delta_2$. The expectation

of any function of $\tilde{\Delta}_1$ with respect to ϕ is denoted by E_ϕ . Thus $E_\phi \tilde{\Pi}^* = \Pi^*$ with Π^* as before. Let $N_1^*(\phi) = \max_{N_1} E_\phi \Pi^* = \max_{N_1} \Pi^*(N_1)$. I define Assumption
A to be made up of the following statements:

I_l has utility function $u_l(m_l) = 1 - \exp(-\gamma_l m_l), \gamma_l > 0$;

I_l is a von Neumann-Morgenstern expected utility of wealth maximizer;

$\sigma_1 = \sigma_2 = \sigma$;

$\tilde{\Delta}_1$ is distributed as $\phi(\Delta_1, \sigma^2)$;

All other information is known with certainty;

Both I_l have the same information; and

The funds F_j are risk-neutral.

In what follows, I let N_{11} be the target of I_1 and N_{12} be the target of I_2.

Theorem 3

In the presence of uncertainty about the difference in the differential impact of the funds' skills on the profit of company 1, and if F_1, is expected, but uncertain, to be more skilled than F_2 in managing company 1, then risk-averse money holders allocate fewer money points to F_1 compared to the certainty case, and more money points to F_2 resulting in an inefficient allocation of money points among the funds. In particular, let Assumption A hold. Let $\gamma_1 V_1 \neq \gamma_2 V_2, \Delta_1 > 0$ and $N_1^*(\phi) \in (0, N)$. Then there exists a constant c such that for $\sigma^2 \in (0, c), N_{11} \neq N_{12}, N_{11} < N_1^*(\phi)$ and $N_{12} < N_1^*(\phi)$.

I see from Theorem 3 that the immediate impact or the introduction of uncertainty regarding the relative skills of the funds on the profit of company 1 causes a shifting of money points away from F_1. As a consequence, even if $N_1^*(\phi)$ were close to N, F_2 would receive more money points as the uncertainty increases. Earlier I showed that when $\sigma_1 = \sigma_2$ and when all information was known with certainty, each I_l strove to achieve the target N_1^*, which, if achieved, would maximize the money holders' respective wealths as well as implement the efficient outcome. That is, the money holders were aiming at the right target; a coordination failure, however, prevented them from achieving it. This suggested that had a coordination mechanism existed, the efficient allocation would have been implemented. Now, with the introduction of uncertainty into the model, I see that the target at which the I_l aim is not the optimal value $N_1^*(\phi)$ and the I_l may have different targets, both unequal to $N_1^*(\phi)$. Coordination would not resolve this inefficiency. Though I introduced uncertainty only in regard to Δ_1, any broader introduction of uncertainty would have further exacerbated the problem. It is not surprising that the introduction of uncertainty results in a sub-optimal solution. However, I next show that even with certainty and with complete information, when the payouts of the funds to the I_l differ, inefficiency also results.

The case when $\sigma_1 \neq \sigma_2$

I have assumed so far that the F_j have identical cost structures. Generally, however, since the F_j are not identical, they could have different cost structures, leading them to select different percentages of their revenues to pay out, that is, $\sigma_1 \neq \sigma_2$. When $\sigma_1 \neq \sigma_2$, it is no longer true that the I_l will both benefit by seeking to maximize Π^* since the share of Π^* that I_l receives depends, in this case, on the investments *x** and *y**. Importantly, for the case $\sigma_1 \neq \sigma_2$, the optimal choices of *x** and y* by I_1, and I_2, respectively, need not always produce a division of the money points consistent with the maximization of economy-wide profit. I show these results to be true in Theorem 4, where I present the solution to the MIP when $\sigma_1 \neq \sigma_2$. To make this point as starkly as possible, I let $V_1 = V_2$.

Theorem 4

Even with certainty and even if the money holders start with the same number of money points, when the payouts of the funds differ, the unique Nash equilibrium of the MIP leads to a common inefficient target. In particular, let $\sigma_1 \neq \sigma_2$, $V_1 = V_2 = V$ *and let*

$$G(N_1) = \Pi^*(N_1) + \frac{N}{2}\left(\frac{N_1}{N} + \frac{\sigma_2}{\sigma_1 - \sigma_2}\right)\frac{d\Pi^*(N_1)}{dN_1} \text{ for } N_1 \in (0, N).$$

Then the unique Nash equilibrium of the MIP is:

$$x^* = y^* = \frac{N_1^\circ}{2},$$

where either $N_1^\circ \in (0, N)$ *and satisfies* $G(N_1^\circ) = 0$ *or* $N_1^\circ = 0$ *or* N.

b. If $\Delta_j \neq 0$ *for at least one value of j and* $N_1^\circ \in (0, N)$, *then* $N_1^\circ \neq N_1^*$.

When payouts are different, each I_l will invest $\frac{N_1^\circ}{2}$ in F_1, yielding a total of N_1° money points to F_1. Since $N_1^\circ \neq N_1^*$, N_1° will not be the efficient allocation of money points to F_1, and thus will not maximize total economy-wide profit. Additionally, whereas a coordination failure between the I_l is responsible for inefficient outcomes when $\sigma_1 = \sigma_2$, even permitting coordination when $\sigma_1 \neq \sigma_2$ would not result in an efficient outcome. That is, when $\sigma_1 \neq \sigma_2$, the goal of the money holders is not the goal of maximizing total economy-wide profit, as it was for the case when $\sigma_1 = \sigma_2$.

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Appendix: Proofs of the Model

Proof of Lemma 1

The value z_1^* is determined by equation (i). Multiplying this equation through by the denominator of the right hand side and collecting terms, it follows that z_1^* must satisfy $2r_1z_1^2 - z_1[(2-\Theta)r_1 - 1] - (1-\Theta) = 0$ for a given Θ. This convex polynomial (or concave polynomial depending on the sign of r_1 equals $-(1-\Theta)$ when $z_1 = 0$ and $\Theta(1+r_1)$ when $z_1 = 1$. Since $r_1 \in (-1,1]$, I have that $z_1^* \in [0,1]$ and is unique in this interval for any $\Theta \in [0,1]$. The unique value of $z_2^* \in [0,1]$ is established by a similar argument applied to equation (ii). I next show that equation (iii) is satisfied for $\Theta^* \in [0,1]$.

Since the z_i depend on Θ, I define $B(\Theta) = z_1\pi_1(z_1) + (1-z_2)\pi_2(z_2) - \alpha[\pi_1(z_1) + \pi_2(z_2)]$. To prove the uniqueness of Θ^* I show that $B(0) \geq 0$, $B(1) \leq 0$ with at least one of the inequalities strict, and $\frac{dB(\Theta)}{d\Theta} < 0$ for $\Theta \in (0,1)$. When $\Theta = 0$, equations (i) and (ii) yield $z_1 = (1-z_2) = 0$ and thus $B(1) = -\alpha[\pi_1(0) + \pi_2(1)]$. Therefore, $B(1) \leq 0$ and $B(1) = 0$ only if $\alpha = 0$. It then also follows that at least one of the inequalities involving $B(0)$ and $B(1)$ must be strict.

Differentiating with respect to Θ, I have:

$$\frac{dB(\Theta)}{d\Theta} = \frac{dz_1}{d\Theta}[k_{12} + 2z_1\Delta_1 - \alpha\Delta_1] - \frac{dz_2}{d\Theta}[k_{12} + 2z_2\Delta_2 - (1-\alpha)\Delta_2].$$

When $\Delta_1 \geq 0, k_{12} + 2z_1\Delta_1 - \alpha\Delta_1 \geq k_{12} - \Delta_1) = k_{12}(1-r_1) > 0$ by assumption. When $\Delta_1 < 0, k_{12} + 2z_1\Delta_1 - \alpha\Delta_1 \geq k_{12} + 2z_1\Delta_1 = k_{12}(1+2z_1r_1)$. Rearranging terms in equation (i) I see that $1 + 2z_1r_1 > 0$ for $\Theta \in (0,1)$. Therefore, the coeffiecient of $-\frac{dz_1}{d\Theta}$ is positive for all Δ_1 when $\Theta \in (0,1)$. Similarily, the coeffiecent of $-\frac{dz_2}{d\Theta}$ is positive for $\Theta \in (0,1)$. To determine the signs of these derivatives, I solve for z_i explicitly in terms of Θ and differentiate. Thus, $z_1 = \frac{1}{4r_1}\left\{(2-\Theta)r_1 - 1 + [D_1(\Theta)]^{\frac{1}{2}}\right\}$ where $D_1(\Theta) = [(2-\Theta)r_1 - 1]^2 + 8r_1(1-\Theta)$. Differentiating and collecting terms I have $\frac{dz_1}{d\Theta} = -\frac{1}{4}\left\{1 + [(2-\Theta)r_1 + 3][D_1(\Theta)]^{\frac{1}{2}}\right\}$. Therefore, $\frac{dz_1}{d\Theta} < 0$ for $\Theta \in [0,1]$ and $r_1 \in (1,-1]$. Also, $z_2 = \frac{1}{4r_2}\left\{(1+\Theta)r_2 - 1 + [D_2(\Theta)]^{\frac{1}{2}}\right\}$ where $D_2(\Theta) = [(1+\Theta)r_2 - 1]^2 + 8r_2\Theta$. It follows that $\frac{dz_2}{d\Theta} = \frac{1}{4}\left\{1 + \left[(1+\Theta)r_2 + 3\right]\left[D_2(\Theta)\right]^{-\frac{1}{2}}\right\}$ and thus $\frac{dz_2}{d\Theta} > 0$ for $\Theta \in [0,1]$ and $r_2 \in (-1,1]$. I can now conclude that $\frac{dB(\Theta)}{d\Theta} < 0$ for $\Theta \in (0,1)$ and $r_1 \in (-1,1]$.

Proof of Theorem 1

The first order conditions of equation (1)-(6) can be replaced by the following equivalent six equations:.

$$\lambda_1 + \lambda_2 = \frac{\pi_1(p_{11})}{a_{1\bullet}}$$

$$\lambda_1 + \lambda_2 = \frac{\pi_2(p_{22})}{a_{2\bullet}}$$

$$\lambda_1 + \lambda_2 = \frac{\pi_1(p_{11}) + \pi_2(p_{22})}{N}$$

$$(1 - p_{11}) = \frac{\lambda_1 a_{1\bullet}}{\pi_1(p_{11}) + p_{11}\Delta_1}$$

$$(1 - p_{22}) = \frac{\lambda_2 a_{2\bullet}}{\pi_2(p_{22}) + p_{22}\Delta_2}$$

$$N_1 = a_{1\bullet} p_{11} + a_{2\bullet}(1 - p_{22})$$

The relationship between the two sets of equations is established as follows. Equations (4) and (5) are just equations (1) and (5). Equation (1) results from summing equations (1) and (4). Equation (2) results from summing equations (2) and (5). Equation (6) is equation rewritten using the definition that $a_{ij} = a_{i\bullet} p_{ij}$ and that $p_{21} = (1 - p_{22})$. Finally, equation (3) results from summing equations (3) and (6) and imposing the requirement that $N_1 + N_2 = N$.

Solving for $a_{i\bullet}$ in equations (1) and (2) and substituting these values in equations (4) through (6), and replacing $\lambda_1 + \lambda_2$ using equation (3), yields:

$$(1 - p_{11}) = \frac{\Theta\pi_1(p_{11})}{\pi_1(p_{11}) + p_{11}\Delta_1}$$

$$(1 - p_{22}) = \frac{(1-\Theta)\pi_2(p_{22})}{\pi_2(p_{22}) + p_{22}\Delta_2}$$

$$N_1 = N\frac{p_{11}\pi_1(p_{11}) + (1-p_{22})\pi_2(p_{22})}{\pi_1(p_{11}) + \pi_2(p_{22})}$$

where $\Theta = \dfrac{\lambda_1}{\lambda_1+\lambda_2}$. Since $\pi_i(p_{ii}) = k_{ij} + p_{ii}\Delta_i$, I can divede the numerators and denominators of the right-hand-sides of the equations (4) and (5) by k_{ij} and call $p_{ii} = z_i$. With these changes, equations (4)-(6) become the equations of Lemma 1 with $\alpha = \dfrac{N_1}{N}$. It follows from this lemma that there is a unique solution $p_{ii}^* = z_i^*, i = 1,2,$ and Θ^* all in the unit interval satisfying these equations. Since the p_{ii}^* determine the $\pi_i(p_{ii}^*)$, it follows from (3) that $\lambda_1 + \lambda_2 = \dfrac{\pi_1(z_1^*) + \pi_2(z_2^*)}{N} = K$. Since $a_{ii}^* = a_{i\bullet}^* p_{ii}^*$, (1) and (2) yield $a_{ii}^* = K^{-1}z_i^*\pi_i(z_i^*)$. The values of $a_{ij}, j \neq i$, follow by subtraction. Finally, $\lambda_1 = (\lambda_1+\lambda_2)\dfrac{\lambda_1}{\lambda_1+\lambda_2} = K\Theta$ with λ_2 also following by substraction.

This unique solution maximizes the respective Lagrangians since, when $r_i \in (-1,1]$, the Hessian of each Lagrangian is strictly negative, i.e., the Lagrangians are strictly concave functions.

Proof of Corollary 2

The revenue to F_j t the Nash Equilibrium of the VFP is $p_{1j}^*\pi_1^* + p_{2j}^*\pi_2^*$. Using Corollary 1 and the fact that $p_{i1} + p_{i2} = 1$, I have that the revenue to F_1 is $z_1^*\pi_1^* + (1-z_2^*)\pi_2^*$. This in turn must equal $\dfrac{N_1}{N}(\pi_1^* + \pi_2^*)$ by the determination of Θ^* of Theorem 1. Thus the revenue for F_1 is $\dfrac{N_1}{N}\Pi^*$. Since the revenue for F_2 is $(1-z_1^*)\pi_1^* + z_2^*\pi_2^* = \pi_1^* + \pi_2^* - \left[z_1^*\pi_1^* + (1-z_2^*)\pi_2^*\right] = \Pi^*(1-\dfrac{N_1}{N}) = \dfrac{N_2}{N}\Pi^*$ the result follows.

Proof of Theorem 3

I first establish the following lemma.

Lemma 2.

Let $\Delta_j \neq 0$ for at leat one value of *j* and let $r_i \in (-1,1], i = 1,2.$ Then,

a. $\Pi^*(\Theta^*)$ is strictly concave for $\Theta^* \in (0,1)$; and

b. $\dfrac{d\Theta^*}{dN_1} < 0.$

Proof:

a. In the proof of Lemma 1 I showed that $\frac{dz_1}{d\Theta} = -\frac{1}{4}\left\{1 + [(2-\Theta)r_1 + 3][D_1(\Theta)]^{\frac{1}{2}}\right\}$. and $\frac{dz_2}{d\Theta} = \frac{1}{4}\left\{1 + [(1+\Theta)r_2 + 3][D_2(\Theta)]^{-\frac{1}{2}}\right\}$ with $D_i(\Theta) \geq 0$. Differentiating these equations with respect to Θ yields $\frac{d^2 z_i}{d\Theta} = -2r_i(1,+r_i)[D_i(\Theta)]^{-\frac{1}{2}}, i = 1,2.$

Since $\Pi^* = \pi_{11}^* + \pi_2^* = k_{12} + k_{21} + \Delta_1 z_1^* + \Delta_2 z_2^*$, I have $\frac{d^2\Pi^*}{d\Theta^{*2}} = -2\sum_{i=1}^{2}\Delta_i r_i(1+r_i)[D_i(\Theta)]^{-\frac{3}{2}}$. Since $\Delta_j \neq 0$ for some j, Δ_i and r_i are of the same sign and $r_i \in (-1,1]$, I have $\frac{d^2\Pi^*}{d\Theta^{*2}} < 0$ and the result follows.

b. From Theorem 1, Θ^* is determined to make $z_1^*\pi_1^* + (1-z_2^*)\pi_2^* = \frac{N_1}{N}(\pi_1^* + \pi_2^*)$. Implicit differentiation with respect to N_1 yields:

$$(\pi_1^* + z_1^*\Delta_1)\frac{dz_1^* d\Theta}{d\Theta^* dN_1} + (-\pi_2^* + (1-z_2^*)\Delta_2)\frac{dz_2^* d\Theta^*}{d\Theta^* dN_1} = \frac{1}{N}(\pi_1^* + \pi_2^*) + \frac{N_1}{N}(\Delta_1\frac{dz_1^* d\Theta}{d\Theta^* dN_1} + \Delta_2\frac{dz_2^* d\Theta^*}{d\Theta^* dN_1}).$$

It then follows that:

$$\left[\frac{d\Theta^*}{dN_1}\right]^{-1} = K^{-1}\left[(\pi_1^* + (z_1^* - \frac{N_1}{N})\Delta_1)\right]\frac{dz_1^*}{d\Theta^*} - (\pi_2^* + (z_2^* + \frac{N_1}{N} - 1)\Delta_2)\frac{dz_2^*}{d\Theta^2}.$$

As in the proof of Lemma 1, with $\alpha = \frac{N_1}{N}$, the expression in brackets is negative, K is positive and the result positive.

I now state the proof of Theorem 3.

Since $\sigma \ = \sigma_2$ and $\tilde{\Delta}_1$ is random,

$$m_l = \sigma\frac{V_l}{N}\tilde{\Pi}^*(x+y) = \sigma\frac{\tilde{V}_l}{N}(k_{12} + k_{21} + p_{11}^*\tilde{\Delta} + p_{22}^*\Delta_2)$$

where p_{11}^* depends on $N_1 = x + y$.
Since the F_j are risk-neutral,

$$E_\Phi(m_l) = \sigma \frac{V_l}{N}(k_{12} + k_{21} + p_{11}^*\Delta_1 + p_{22}^*\Delta_2) = \sigma \frac{V_l}{N}\Pi^*(x+y)$$

and

$$V(m_l) = E_\Phi(m_l^2) - [E_\Phi(m_l)]^2 = \left[\sigma \frac{V_l}{N} p_{11}^*\right]^2 \sigma^2.$$

By the property of the moment-generating function of the normal distribution:

$$E_\Phi[u_l(m_l)] = 1 - E_\Phi[\exp(-\gamma_l m_l)] = 1 - \exp\left[-\gamma_1(\sigma \frac{V_l}{N})(\Pi^*(x+y) - \gamma_1\sigma \frac{V_l}{2N} p_{11}^{*2}\sigma^2)\right].$$

Maximizing expected utlity ia achieved by maximizing $(\Pi^*(x+y) - \gamma_1\sigma \frac{V_l}{2N} p_{11}^{*2}\sigma^2)$. The conditions for an internal solution for the I_l are respectively given by the first-order conditions:

$$\frac{d\Pi^* d\Theta^* dN_1}{d\Theta^* dN_1 dx} - \frac{\sigma\gamma_1 V_1 p_{11}^*\sigma^2}{N}\frac{dp_{11}^*}{d\Theta^*}\frac{d\Theta^*}{dN_1}\frac{dN_1}{dx} = 0$$

and

$$\frac{d\Pi^* d\Theta^* dN_1}{d\Theta^* dN_1 dy} - \frac{\sigma\gamma_2 V_2 p_{11}^*\sigma^2}{N}\frac{dp_{11}^*}{d\Theta^*}\frac{d\Theta^*}{dN_1}\frac{dN_1}{dy} = 0$$

Since $\frac{\partial N_1}{\partial x} = \frac{\partial N_1}{\partial y} = 1$ and for $\Theta^* \in [0,1], \frac{d\Theta^*}{dN_1} < 0$ by Lemma 2, these conditions become:

$$\frac{d\Pi^*}{d\Theta^*} - \frac{\sigma\gamma_1 V_1 p_{11}^*\sigma^2}{N}\frac{dp_{11}^*}{d\Theta^*} = 0$$

$$\frac{d\Pi^*}{d\Theta^*} - \frac{\sigma\gamma_2 V_2 p_{11}^*\sigma^2}{N}\frac{dp_{11}^*}{d\Theta^*} = 0$$

Let $\Theta^{**}(\Phi) = \arg\max_{\Theta^*} \Pi^*(\Theta^*)$. Since $N_1^*(\Phi) \in (0, N)$, it follows that $\Theta^{**}(\Phi) \in (0,1)$. Thus by continuity, for σ^2 sufficiently small, there will be solutions to these equations, say $\Theta_1^*, \Theta_2^* \in (0,1)$. Since these equations are identical except for the factor $\gamma_l V_l, l = 1,2,$ and since $\gamma_1 V_1 \neq \gamma_2 V_2$, I have $\Theta_1^* \neq \Theta_2^*$. Using Lemma 2, it follows that $N_{11} \neq N_{12}$.

I next show that for σ^2 sufficiently small, $\frac{d\Theta_1^*}{d\sigma^2} > 0$. Consider the first of these equations and let $c_l = \frac{\sigma\gamma_l V_l}{N}, l = 1,2.$ Differentiating this equation implicitly with respect to σ^2 yields:

$$\frac{d\Theta^*}{d\sigma^2}\left[\frac{d^2\Pi^*}{d\Theta^{*2}} - c_1\sigma^2 p_{11}^* \frac{d^2 p_{11}^*}{d\Theta^{*2}} - c_1(\frac{dp_{11}^*}{d\Theta^*})^2\right] = c_1 p_{11}^* \frac{dp_{11}^*}{d\Theta^*}.$$

Using the definition of Π^* and rearranging terms produces:

$$\frac{d\Theta^*}{d\sigma^2} = \left[(\Delta_1 - c_1\sigma^2 p_{11}^*)\frac{d^2 p_{11}^*}{d\Theta^{*2}} + \Delta_2 \frac{d^2 p_{22}^*}{d\Theta^{*2}} - c_1\sigma^2(\frac{dp_{11}^*}{d\Theta^*})^2\right]^{-1} c_1 p_{11}^* \frac{dp_{11}^*}{d\Theta^*}.$$

Since $p_{ii}^* = z_i^*$ by Corollary 1, I have, using Lemma 2, that $\Delta_2 \frac{d^2 p_{22}^*}{d\Theta^{*2}} < 0$ and for $\Delta_1 > 0$, (or $r_1 > 0$), $\frac{d^2 p_{11}^*}{d\Theta^{*2}} < 0$. Furthermore, Lemma 1 also established that $\frac{dp_{11}^*}{d\Theta^*} < 0$. Thus, when $\sigma^2 \in \left(0, \frac{\Delta_1}{c_1}\right], \frac{d\Theta_1^*}{d\sigma^2} > 0$. Similarly, for the second equation, $\frac{d\Theta_2^*}{d\sigma^2} > 0$ when $\sigma^2 \in \left(0, \frac{\Delta_1}{c_2}\right]$. Therefore, there will be a constant $c < \min\left[\frac{\Delta_1}{c_1}, \frac{\Delta_1}{c_2}\right]$ such that for $\sigma^2 \in (0, c)$ there will both be solution to these equations and $\frac{d\Theta_1^*}{d\sigma^2} > 0$.

Returning to the necessary conditions above, and recalling that Π^* is concave (Lemma 2) and $\frac{dp_{11}^*}{d\Theta^*} < 0$ (Lemma 1), it now follows that for $\sigma^2 \in (0, c)$ each solution Θ_1^* satisfies $\Theta_l^* > \Theta^{**}(\Phi)$. Using Lemma 2 again, I have $N_{11} < N_1^*(\Phi)$ and $N_{12} < N_1^*(\Phi)$.

Proof of Theorem 4

a. Because the objective functions of the I_l are bounded with support $[0, V]$, the maximum must occur in this interval. Since $x^* = \arg\max_x \left[\sigma_1 \frac{x}{N} + \sigma_2 \frac{V - x}{N}\right]\Pi^*(x + y)$, the necessary condition for x^* to be an interior solution, given the choice of I_2 is:

$$\frac{\sigma_1 - \sigma_2}{N}\Pi^*(N_1) + \left(\frac{\sigma_1 - \sigma_2}{N}x + \sigma_2 \frac{V}{N}\right)\frac{d\Pi^*(N_1)\partial N_1}{dN_1 \partial x} = 0$$

where $N_1 = x + y$. Similarly, the first-order condition for y^* is:

$$\frac{\sigma_1 - \sigma_2}{N}\Pi^*(N_1) + \left(\frac{\sigma_1 - \sigma_2}{N}y + \sigma_2 \frac{V}{N}\right)\frac{d\Pi^*(N_1)\partial N_1}{dN_1 \partial y} = 0$$

Since $\frac{\partial N_1}{\partial x} = \frac{\partial N_1}{\partial y} = 1$, the difference of the two equations yields $x^* = y^*$. Because $N_1 = x + y$, I have that $x^* = y^* = \frac{N_1^\circ}{2}$. To determine N_1°, I note that one half the sum of the two equations, when x and y are each replaced by $\frac{N_1}{2}$, becomes $\frac{\sigma_1 - \sigma_2}{N}\Pi^*(N_1) + \left(\frac{\sigma_1 - \sigma_2}{N}\frac{N_1}{2} + \sigma_2 \frac{V}{N}\right)\frac{d\Pi^*(N_1)}{dN_1} = 0$. Since $V = \frac{N}{2}$, this 00last expression becomes $\frac{\sigma_1 - \sigma_2}{N} G(N_1) = 0.$ Thus, N_1° must satisfy $G(N_1^\circ) = 0.$ If $N_1^\circ = 0$, then $x^* = y^* = 0$, and if $N_1^\circ = N$ then $x^* = y^* = V = \frac{N}{2}$.

b. Let $\sigma_1 > \sigma_2$. From part a, N_1° satisfies $\Pi^*(N_1) + \frac{N}{2}\left[\frac{N_1}{N} + \frac{\sigma_2}{\sigma_1 - \sigma_2}\right]\frac{d\Pi^*(N_1)}{dN_1} = 0$. Because $\Pi^* > 0$, for this equation to be satisfied , $\frac{d\Pi^*}{dN_1}$ evaluated at N_1° must be negative. Since $\Delta_j \neq 0$ for some *j* by Lemma 2, N_1^* is the unique value of N_1 that maximizes $\Pi^*(N_1)$. It follows that $N_1^\circ > N_1^*$. Let $\sigma_1 < \sigma_2$. Then since $V = \frac{N}{2}$, $\left[\frac{N_1}{2} + \frac{\sigma_2 V}{\sigma_1 - \sigma_2}\right] = \frac{N}{2}\left[\frac{N_1}{N} - \frac{\sigma_2}{\sigma_2 - \sigma_1}\right]$ is negative. Therefore, for N_1° to satisfy the equation , $\frac{d\Pi^*}{dN_1}$ must be positive at N_1°. Again, by the property of N_1^*, it follows that $N_1^\circ < N_1^*$.

INVESTICIJSKE ODLUKE

SAŽETAK

Evaluacija ulaganja je kontrola planiranja i implementacije ulagačkih aktivnosti s obzirom na ciljeve koje želimo ostvariti. U ovom radu pretpostavljam cilj uspješnog rezultata i maksimizacije profita. To znači da evaluacija ulaganja stavlja određivanje normativa u kontekst planiranja i menadžmenta te stoga i u kontekst namjernog djelovanja i ciklusa djelovanja. Tu nije važna sama procjena činjenica i scenarija već i, manje ili više implicitni, slučajni lanci koji povezuju aktivnosti s rezultatima ulaganja te napokon i s postizanjem cilja. Model za evaluaciju ulaganja kojeg predlažem ima dva izvora novca koji moraju odlučiti kako uložiti svoj novac kako bi ostvarili vlasništvo u dva projekta. Uobičajeni slučaj u kojem je broj imatelja novca, broj fondova i broj ulaganja arbitraran može se obraditi na sličan način kako je niže prikazano ali uz povećanu složenost. Kao rezultat se ne dobiva mehanizam kojim bi se dobilo maksimalni rezultat te se analiziraju različite metode pronalaska optimalne strukture pod nesigurnosti i različite strukture troškova.

JEL: C30, C53, C70

Ključne riječi: *evaluacija ulaganja, maksimizacija profita, nesigurnost, neuspjeh koordinacije*