

## Linearizing Control Based on Adaptive Observer for Anaerobic Continuous Sulphate Reducing Bioreactors with Unknown Kinetics

M. I. Neria-González,<sup>a</sup> A. R. Domínguez-Bocanegra,<sup>a</sup> J. Torres,<sup>b</sup>  
R. Maya-Yescas,<sup>c</sup> and R. Aguilar-López<sup>a,\*</sup>

<sup>a</sup>Departamento de Biotecnología y Bioingeniería, CINVESTAV-IPN

<sup>b</sup>Departamento de Control Automático, CINVESTAV-IPN  
Av. Instituto Politécnico Nacional 2508, San Pedro Zacatenco,  
México, D.F. Mexico

<sup>c</sup>Facultad de Ingeniería Química, Universidad Michoacana  
de San Nicolás de Hidalgo, Santiago Tapia 403,  
58000, Morelia, Michoacán, Mexico

Original scientific paper  
Received: January 24, 2008  
Accepted: July 25, 2008

Anaerobic reactors are a typical example of processes that exhibit non-linear behavior and, also time varying parameters; hence their operation is known to be difficult to model and control. In contrast to modeling approaches, in practice linear controllers are widely employed for industrial processes because of their easy implementation and manipulation by plant operators; nevertheless linear approaches are not robust when the operating conditions change suddenly and/or strong disturbances are present. In order to introduce robust controllers to these processes, this paper addresses the tracking problem for the substrate (sulphate) control in a class of continuous bioreactors. An experimentally corroborated bioreactor model serves as benchmark problem for advanced non-linear analysis and control techniques; taking into account system non-linearities, stability and performance objectives over large operating regions. It is considered that, as it is common in practice, the rate of substrate consumption exhibits uncertainty. Results show that the proposed controller exhibits better dynamic performance than a classical Proportional-Integral control tuned using the methodology suggested by Internal Model Control.

*Key words:*

Observer-based controller, robust performance, sulphate reducing bioreactor, uncertain kinetics

### Target system and its control

#### Anaerobic digesters of primary sludge

Primary sludge is the name given to those solids that settle out of the wastewater in the sedimentation tanks, just after the wastewater passes through the grit chambers. Due to its source of production, this sludge has to be treated before waste is disposed. Anaerobic digestion is preferred to reduce the high organic loading of primary sludge because of several process features, such as the rapid growth of the biomass; moreover, it creates considerably less biomass than the aerobic process and converts as much of the sludge as possible to end liquid and gas products.<sup>1</sup>

Anaerobic processes are performed, mainly, by two basic types of bacteria: acid formers and methane formers. The acid formers are facultative and anaerobic microorganisms that perform hydrolysis and dissolve organic solids; soluble products are then fermented to acids and alcohols of lower mo-

lecular mass. The methane formers are strict anaerobic microorganisms that convert acids and alcohol along with hydrogen and carbon dioxide into methane.<sup>2</sup> This complex blend of microorganisms provokes the substrate consumption rate to be a known but uncertain parameter.

Operating stability of the anaerobic process is very fragile; balance among several microbial populations has to be maintained. Hydrolysis and fermentation phases have the most robust organisms that react quickly to increased food availability because of their adaptation velocity. Consequently, volatile fatty acid concentrations increase rapidly and could provoke the decrease of pH in the system.<sup>3</sup> This is kept in check by the buffering action of the system provided by carbon dioxide in the form of bicarbonate alkalinity, maintaining pH range under normal circumstances. However, there are shock-loading situations that provoke the acid concentration to overcome the buffering action and raise the pH out of the narrow acceptable limits, which can kill acetogen and methanogen microorganisms. If this situation occurs, methane produc-

\*Corresponding author: E-mail.– ral640210@yahoo.com.mx

tion stops and acid levels rise above the tolerance of acid formers; at this point the system fails.<sup>4</sup> Therefore, it is necessary to implement control systems in order to prevent system shut-down after usual feedstock disturbances.

### Development of controllers

Anaerobic digesters lead to the necessity of robust, flexible and efficient industrial operation modes, therefore corresponding control strategies play important roles. A number of papers dealing with new controllers design under the framework of gain scheduling, predictive, optimal and nonlinear control theories have been published in the open literature.<sup>5–8</sup> Unfortunately, because of their mathematical complexity most of them cannot be applied to industrial plants. For this reason, part of the control theory research has to be focused on the practical application in industrial processes; especially because there is always uncertainty in important parameters, such as the rate of substrate consumption. In order to solve this problem, control engineers have to design *ad hoc* control schemes to be able to deal with demanding operating conditions. For example, Aguilar *et al.*<sup>9</sup> and Aguilar-López and Martínez-Guerra<sup>10</sup> have proposed novel approaches to design non-linear PI and PID type controllers using sophisticated tuning techniques, which allow new friendly rules for the controller's gains and assure semi-global robust performance.

### Experimental

Microorganism: *Desulfovibrio alaskensis* 6SR was isolated from oil pipelines.<sup>11</sup> Previously, the strain was cultured in Ravot medium<sup>12</sup> over 15 days at 32 °C under an atmosphere of N<sub>2</sub>-CO<sub>2</sub> ( $\Psi_{N_2/CO_2} = 8:2$ ).

*Congenital water medium (CW)*: A sample of congenital water was obtained from an oil pipeline located in the Mexican Southeast region. Chemical determination of water: chlorides  $w = 64.0 \cdot 10^{-3}$ , sulphur  $w = 0.178 \cdot 10^{-3}$ , sulphate  $w = 0.35 - 0.40 \cdot 10^{-3}$ , pH 8.84. A 1000 mL aliquot of congenital water was saturated with N<sub>2</sub> for 1 hour and enriched with sodium lactate 6 mL, yeast extract 0.5 g, and reducing solution 5 mL (acid ascorbic  $\gamma = 1 \text{ g L}^{-1}$ , and sodium thioglycolate,  $\gamma = 1 \text{ g L}^{-1}$ ). The pH was adjusted to 7 with KOH 1 mol L<sup>-1</sup>. The CW medium was distributed in serum bottles of 60 mL using Hungate's technique;<sup>13</sup> the samples autoclaved at 120 °C for 15 min.

The cultures initiate of *Desulfovibrio alaskensis* in medium Ravot were used to inoculate 45 mL of CW medium. The culture was incubated for 20 days at 37 °C. This was used to inoculate three bottles with EC medium to different time: zero, 24

and 36 hours, respectively, and were incubated under same conditions. The bacterial growth was followed through Optical Density (OD) methodology, consume of sulphate and production of sulphide. Samples from the cultures were taken, avoiding contact with oxygen, each hour. Sulphate in the medium was measured by the turbid metric method based on the precipitation of barium.<sup>14</sup> Also, the production of sulphide was measured by a turbid metric method.<sup>15</sup>

The OD reading for cell growth was transformed into dry mass (mg mL<sup>-1</sup>) through a standard growth curve. The data were analyzed and only the points that adjusted a straight line (exponential phase) were used to determinate the growth kinetic parameters according to Monod's model.<sup>16</sup>

### Mathematical model of the bioreactor

Anaerobic digesters are large fermentation tanks provided with mechanical mixing, heating, gas collection, sludge addition and withdrawal ports, and supernatant outlets such that can be considered as continuous stirred tanks for analysis purposes. Sludge digestion and settling occur simultaneously in the tank. Sludge stratifies and forms the following layers from the bottom to the tip of the tank: digested sludge, actively digested sludge, supernatant, scum layer and gas. Higher sludge loading rates are achieved in the high-rate version, in which sludge is continuously mixed and heated, anaerobic digestion is affected by temperature, retention time, pH chemical composition of wastewater, presence of toxics, and competition between methanogenic bacteria and sulphate-reducing bacteria.<sup>17</sup> However, for control purposes a reduced order model which describes the dynamic behavior of the main state variables is adequate. Therefore, a simple mathematical model, based on classical mass balances for biomass ( $\gamma_X$ ), sulphate ( $\gamma_S$ , substrate) and sulphide ( $\gamma_P$ , product) concentration (1–6) and initial conditions under continuous operation is proposed.

Sulphate ( $\gamma_S$ )

$$\frac{d\gamma_S}{dt} = D(\gamma_{S,in} - \gamma_S) - \mu(\gamma_S) \frac{\gamma_X}{Y_{X/S}} \quad (1)$$

$$\gamma_{S_0} = 5 \text{ g L}^{-1} \quad (1a)$$

Biomass ( $\gamma_X$ )

$$\frac{d\gamma_X}{dt} = -D\gamma_X + \mu(\gamma_S)\gamma_X \quad (2)$$

$$\gamma_{X_0} = 0.12 \text{ g L}^{-1} \quad (2a)$$

Sulphide (P)

$$\frac{d\gamma_P}{dt} = -D\gamma_P + \mu(\gamma_S) \frac{\gamma_X}{Y_{X/P}} \quad (3)$$

$$\gamma_{P_0} = 0.16 \text{ g L}^{-1} \quad (3a)$$

The specific growth rate is considered that follows Monod's model<sup>16</sup>(4) and substrate and product yields are available<sup>18</sup> (5, 6).

$$\mu(\gamma_S) = \frac{0.035\gamma_S}{0.90 + \gamma_S} \quad (4)$$

$$Y_{X/S} = 0.25 \quad (5)$$

$$Y_{X/P} = 0.26 \quad (6)$$

This model was integrated from initial conditions up-to-steady state, finding the following stationary operating values:

$$\gamma_{S,eq} = 2.5 \text{ g L}^{-1}$$

$$\gamma_{X,eq} = 0.6875 \text{ g L}^{-1} \quad (7)$$

$$\gamma_{P,eq} = 2.6123 \text{ g L}^{-1}$$

Based on the bioreactor model, open-loop stability analysis was done in accordance with a linear approach, evaluating the Jacobian matrix ( $J_x$ ) on the steady operating point (eq. (8)); a commercial software was used (MathLab<sup>TM</sup>). It was found that one eigenvalue ( $\lambda_3$ ) exhibits real positive part (9), therefore this point is unstable.

$$J_x = \begin{bmatrix} -0.025 & -0.7142 & 0 \\ 0 & 0.1535 & 0 \\ 0 & 0.6785 & -0.025 \end{bmatrix} \quad (8)$$

$$\lambda_1 = -0.025$$

$$\lambda_2 = -0.025 \quad (9)$$

$$\lambda_3 = 0.15357$$

As a consequence of the positive value of  $\lambda_3$ , it is possible to note that open-loop operation would be hard to perform, because any disturbance arriving to the bioreactor would lead the process to undesirable operating points; this justifies the closed-loop operation using feedback control.

## Controller design

As first assumption, the bio-reacting mix is considered as an incompressible fluid, contained in a closed tank with the corresponding input and output lines, where the convective input and output flows are considered with the same value, in accor-

dance with the proposed mathematical model of the bioreactor; from the above the density of the bio-reacting mix is a constant, in consequence the general mass balance to include volume changes is not necessary, this leads to consider the dilution rate as control input, instead of the convective input flow. The operation task is related to the optimum sulfate consumption, this operating policy is generally applied when the degradation of sulfure compounds are required for wastewater treatment.

The model given by (1–4) is transformed into canonical control form as shown in (10). This particular equation is known as relative-degree one system.

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= Cx \end{aligned} \quad (10)$$

Here  $x = \gamma_S$ ,  $f(x) = \mu(\gamma_S) \frac{\gamma_X}{Y_{X/S}}$ ,  $g(x) = (\gamma_{S,in} - \gamma_S)$ .

The study of systems that exhibit relative-degree one is very important for many control applications, since the dynamics of a wide class of chemical reactors can be described in this form. Such systems are mathematically modeled as *affine systems* with respect to the control input.<sup>19</sup> Systems that present relative-degree one display some interesting features, such as the *equivalent dissipativeness* by means of state or output feedback. In general, it is easier to stabilize *dissipative* systems than *non-dissipative* ones.<sup>20</sup>

In what follows, non-linear systems of the form (10) will be considered. In order to stabilize the system defined by (10) via regulation of  $x$ , the following nominal I/O linearizing feedback control is proposed (11).

$$u = g^{-1}(x)[- \tau_g^{-1} \xi - f(x)] \quad (11)$$

Here  $\xi$  is the control error with respect to the set point and  $\tau_g > 0$  is a prescribed time-constant. The controller defined by (11) guarantees asymptotic stability of non-linear systems (10) with no uncertainties and perfect measurements.<sup>21</sup> Moreover, it imposes a linear behavior to the system I/O dynamics by cancelling the non-linearities.

Now, introduce the following function, which corresponds to the I/O modeling error: Due to the existence of uncertainty ( $\zeta(x, u)$ ) both, the perfect knowledge of the states ( $x$ ) and the perfect control action ( $u$ ) are impossible to reach. Therefore, it is necessary to develop an uncertainty estimator (12). Let functions  $f(x)$  and  $\Delta g(x)$  be model uncertainties related to the non-linear system, and  $\bar{g}(x)$  is a nominal value of the control input coefficient. In

the most general case, the functions  $f(x)$  and  $\Delta g(x)$  are assumed to be unknown.

$$\zeta(x, u) = f(x) + \Delta g(x)u \quad (12)$$

By introducing (12) into (11), a new representation of the system is obtained (13).

$$\begin{aligned} \dot{x} &= \zeta(x, u) + \bar{g}(x)u \\ \dot{\zeta} &= \Phi(x, u) \\ y &= h(x) = Cx \end{aligned} \quad (13)$$

In order to simplify notation, this set of equations can be written in vector notation (14).

$$\begin{aligned} \dot{\mathfrak{X}} &= \hat{\mathfrak{S}} \\ y &= C\mathfrak{X} \end{aligned} \quad (14)$$

Here  $\mathfrak{X} = \begin{bmatrix} x \\ \zeta \end{bmatrix}$ ;  $\hat{\mathfrak{S}} = \begin{bmatrix} \zeta(x, u) + \bar{g}(x)u \\ \Phi(x, u) \end{bmatrix}$ .

Since the uncertainty term,  $\zeta(x, u)$  is an unknown function of the states and the control input, the ideal control law for the regulation of  $x$  is not causal, therefore it cannot be implemented in practice. Nevertheless, there is another way to develop an input-output linearizing controller that is robust against uncertainties. The procedure described below provides a method to estimate the uncertainty term,  $\zeta(x, u)$ . Estimators or observers for states and uncertainties can play a key role during the early detection of hazardous and unsafe operating conditions. Following this spirit, several researches have been focused in the proposition of estimation methodologies for states and uncertainties for monitoring and control purposes.<sup>22–25</sup>

Now, the following state observer is proposed:

$$\begin{aligned} \dot{\hat{x}} &= \hat{\zeta}(\hat{x}, u) + \bar{g}(\hat{x})u + k_1(y - \hat{y}) \\ \dot{\hat{\zeta}} &= k_2(y - \hat{y}) \\ \dot{K} &= -\beta \text{abs}(y - \hat{y})^{1/m} \end{aligned} \quad (15)$$

By defining  $\hat{\mathfrak{X}} = \begin{bmatrix} \hat{x} \\ \hat{\zeta} \end{bmatrix}$ ,  $\hat{\mathfrak{S}} = \begin{bmatrix} \hat{\zeta}(\hat{x}, u) + \bar{g}(\hat{x})u \\ 0 \end{bmatrix}$  and

$K = [k_1, k_2]$ , (15) can be rewritten as:

$$\begin{aligned} \dot{\hat{\mathfrak{X}}} &= \hat{\mathfrak{S}} + K(y - \hat{y}) \\ \dot{K} &= -\beta \text{abs}(y - \hat{y})^{1/m} \end{aligned} \quad (16)$$

Here the dynamic equation for  $K$  is an adaptation algorithm that updates the time-varying control gain and  $\beta$  is a parameter design.

In order to prove the convergence of the proposed observer, let's consider the dynamic equation of the estimation error,  $\varepsilon = \mathfrak{X} - \hat{\mathfrak{X}}$ , as follows:

$$\begin{aligned} \dot{\varepsilon} &= \hat{\mathfrak{S}} - \hat{\mathfrak{S}} + K \cdot \varepsilon \\ \dot{K} &= -\beta \text{abs}(\varepsilon)^{1/m} \end{aligned} \quad (17)$$

Because the error is a finite quantity, there should be a constant  $L$  that:

$$AI. \quad |\hat{\mathfrak{S}} - \hat{\mathfrak{S}}| \leq L|\mathfrak{X} - \hat{\mathfrak{X}}|$$

Taking norms in both sides of (17) and applying AI it is obtained:

$$|\dot{\varepsilon}| \leq L|\varepsilon| + K|\varepsilon| \quad (18)$$

Now, to solve the system given by (17), consider the function  $\text{abs}(\varepsilon)$  as a positive continuous function in the integration interval  $[a, b]$ ; if  $M$  is the maximum of the function on the domain  $[a, b]$ , then  $\text{abs}(\varepsilon)$  is bounded, i.e.  $\text{abs}(\varepsilon) \leq M, \forall t \in [a, b]$ , hence:

$$\begin{aligned} \text{abs}(\varepsilon)^{1/n} &\leq M^{1/n} \\ n > 0 &\Rightarrow \int_a^b \text{abs}(\varepsilon)^{1/n} \leq M^{1/n}(b-a) \end{aligned} \quad (19)$$

Here,  $n$  is restricted to be a positive odd number i.e.  $n = 2p + 1, p \in Z^+$ . Therefore, for  $p$  large enough, the following limit is obtained:

$$\begin{aligned} \limsup \int_a^b \text{abs}(\varepsilon)^{1/(2p+1)} &\leq \\ &\leq \limsup M^{1/(2p+1)}(b-a) \leq (b-a) \end{aligned} \quad (20)$$

Applying the equality  $|\varepsilon| = \text{sign}(\varepsilon)\varepsilon$  to (18), another quota can be found:

$$\text{sign}(\dot{\varepsilon})\dot{\varepsilon} \leq (L - \beta(b-a))\text{sign}(\varepsilon)\varepsilon \quad (21)$$

By solving (21) it is possible to note that the error is bounded by:

$$\varepsilon \leq \varepsilon_0 \exp\left(\text{sign}(\dot{\varepsilon})^{-1} \text{sign}(\varepsilon)(L - \beta(b-a))t\right) \quad (22)$$

Therefore the estimation error will be asymptotically and exponentially stable if:

$$\beta > (b-a)^{-1}L \quad (23)$$

Now, the above observer based uncertainty estimator can be coupled with the non-ideal control law, as:

$$u = \bar{g}^{-1}(x)[- \tau_g^{-1} \xi - \hat{\zeta}] \quad (24)$$

In order to analyze the closed-loop stability of the sulphate trajectories in the reactor, the



closed-loop dynamic equation of the mass balance should be used.

$$\dot{\xi} = \tau_g^{-1} \xi + (\zeta - \hat{\zeta}) \tag{25}$$

If  $\hat{\zeta} \rightarrow \zeta$  then  $\zeta - \hat{\zeta} \rightarrow 0$ , the ideal control law is recovered together with its stability properties; otherwise, the control error is limited as  $\lim_{t \rightarrow \infty} |\zeta - \hat{\zeta}| \leq \lim_{t \rightarrow \infty} \Pi = 0$ , accordingly with the above development.

Now, consider:

**A2.** – Let  $\lambda_1, \lambda_2, \dots, \lambda_k$  be the distinct eigenvalues of the matrix  $A$ , value  $\lambda_j$  exhibits multiplicity  $n_j$  and  $n_1 + n_2 + \dots + n_k = n$ ; if there is any number  $\rho$  larger than the real part of  $\lambda_1, \lambda_2, \dots, \lambda_k$ , that is  $\rho > \max(\text{Re}(\lambda_j))$ , then there exists a positive constant  $j$  that satisfies:

$$\|\exp(At)\xi\| \leq j \exp(-\rho t) \|\xi\| \tag{26}$$

Considering assumptions **A1** and **A2**, it is possible to find a bound for the error of the closed-loop system (25).

$$\|\zeta\| \leq j \exp(-\rho t) \left[ \|\xi_0\| - \frac{j\Pi}{\rho} \right] + \frac{j\Pi}{\rho} \tag{27}$$

Evaluating the limit when  $t \rightarrow \infty$ :

$$\|\xi\| \leq \frac{j\Pi}{\rho} \rightarrow 0 \tag{28}$$

In order to measure the impact of the control error  $\xi$  on the closed-loop operation, Ogunnaike and Ray<sup>26</sup> suggested the “Integral Time-Weighted Squared Error” (ITSE) defined by (29). ITSE exhibits the advantage of heavy penalization of large errors at long time; therefore is a good measure of resilience of the controller.

$$ITSE = \int_0^{\infty} t \xi^2 dt \tag{29}$$

### Results and discussion

Firstly, validation of simulation predictions vs. experimental data show that the model proposed reproduces satisfactorily the dynamic behavior of the open-loop system (Fig. 1). Therefore, this model is used as the tool to compare the development of the closed-loop system, when the control developed and a classic PI tuned by IMC rules<sup>26</sup> are used.

Following industrial practice, the control input considered is the dilution rate (input flow) and the controlled variable is the sulphate concentration (substrate), the controller acts at  $t = 35$  h (as seen in Fig. 2). Regulation and servo-control actions are simulated by considering two set points: 1) the first one is  $\gamma_S = 2.5$  g L<sup>-1</sup> of sulphate, taken from the beginning of the simulation and 2) at  $t = 50$  h it

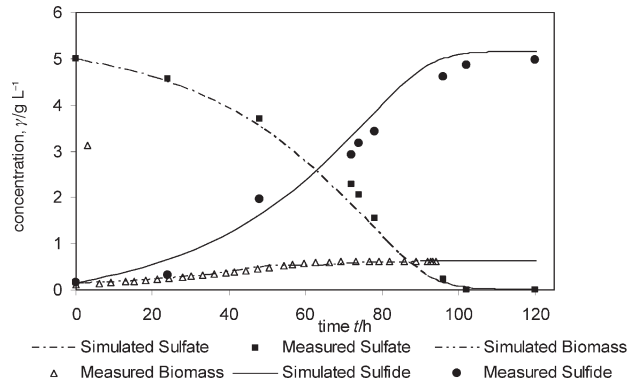


Fig. 1 – Model validation using experimental data

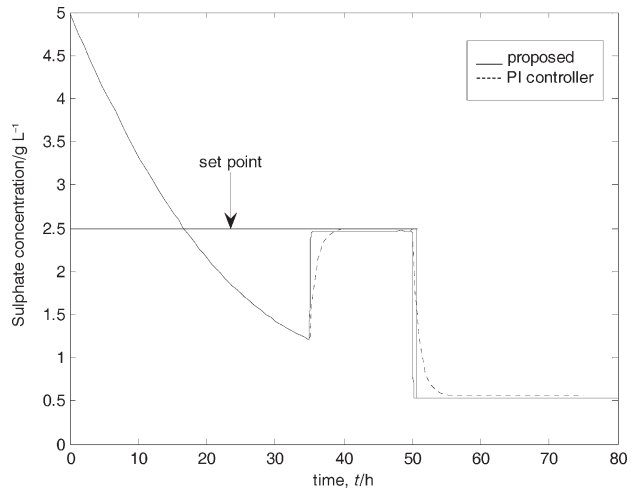


Fig. 2 – Performance of the process under the action of the controller proposed

changes to  $\gamma_S = 0.55$  g L<sup>-1</sup>. Now, tuning gain is considered as  $\tau_g = 1.0$  h; note that when the proposed controller acts at  $t = 35$  h (Fig. 2) it forces the open-loop of the sulphate concentration to reach the first set point required, without overshoots and short settling time; later, at  $t = 50$  h the set point changes and the performance is also satisfactory, in contrast classic PI controllers acts slowly in comparison with the proposed methodology.

In order to explain the success of the controller actions, gains and initial conditions shown in Table 1 are used to illustrate the observer ability to follow the trajectory of the state variables.

Table 1 – Parameters of the observer (15)

Balance	Gain	Initial condition $\gamma/g L^{-1}$
sulphate	0.01	4.70
biomass	-0.01	1.55
sulphite	0.01	0.15
uncertainty (rate of substrate up-taking)	0.01	0.28

As it is possible to note, estimation of biomass (Fig. 3), sulphite (Fig. 4) concentrations as well as uncertainty (Fig. 5) exhibit the desirable performance, achieving actual values of the state vari-

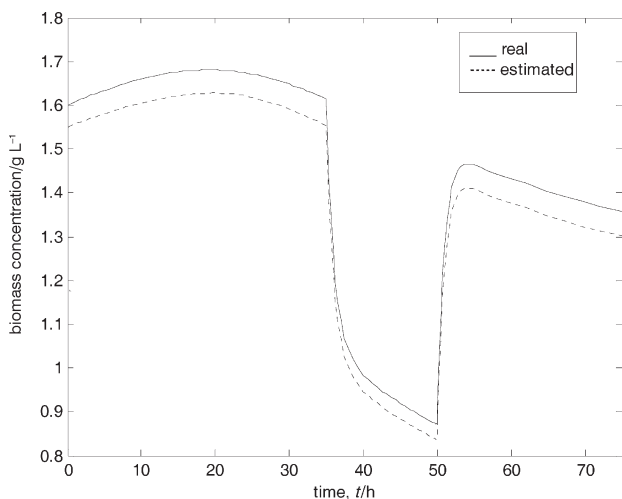


Fig. 3 – Estimation of biomass concentration ( $\gamma_X$ )

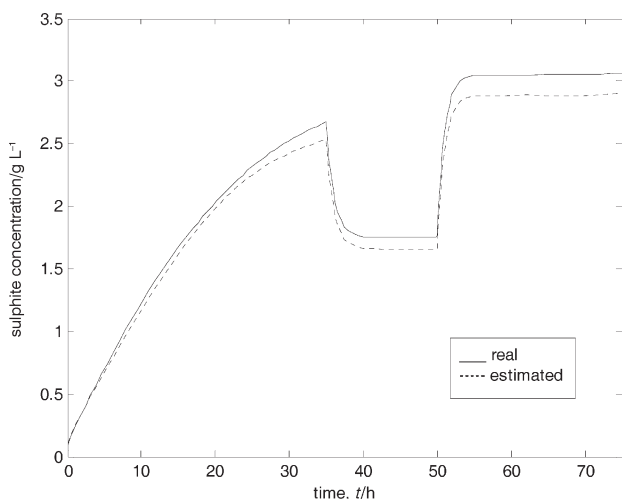


Fig. 4 – Estimation of sulphide concentration ( $\gamma_P$ )

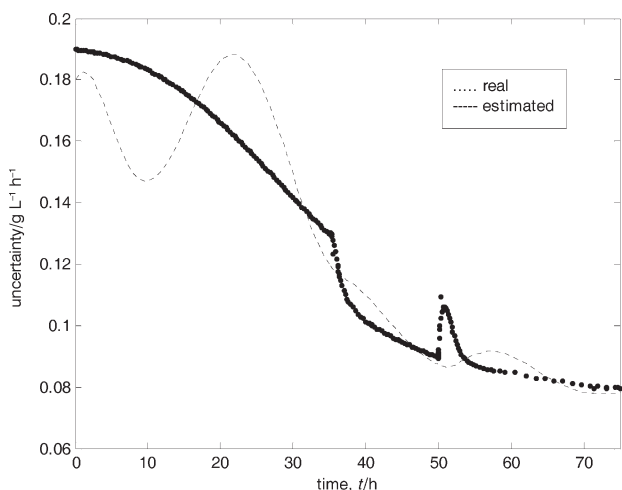


Fig. 5 – Estimation of the substrate uptake rate (uncertainty)

ables via estimation; moreover, the observer proposed follows adequately the dynamic trajectory.

For comparison purposes, simulation of the closed-loop system was performed using the controller proposed and a classical PI controller tuned by Internal Model Control (IMC) guidelines.<sup>26</sup> The corresponding tuning was done via a step disturbance of 5 % in the nominal value of the control input ( $q = 0.02 \text{ L h}^{-1}$ ). Steady-state gain is calculated as  $K = 1400 \text{ g h L}^{-1}$ , the characteristic time is  $\tau = 170 \text{ h}$ , the time delay is  $\theta = 6 \text{ h}$ , therefore the proportional gain is  $K_p = 4.857 \text{ L h}^{-1}$  and, integral time is  $\tau_I = 170 \text{ h}$  for the closed-loop time constant  $\lambda = 35 \text{ h}$ . The resilience of both controllers was evaluated using (29).

Control effort is diminished by the controller proposed with respect to the linear PI controller (Fig. 6); moreover, this new controller exhibits smooth performance in contrast to the oscillatory behavior of the linear PI one.

Summing up, the controller proposed exhibits better performance than the classical PI (Fig. 7), as

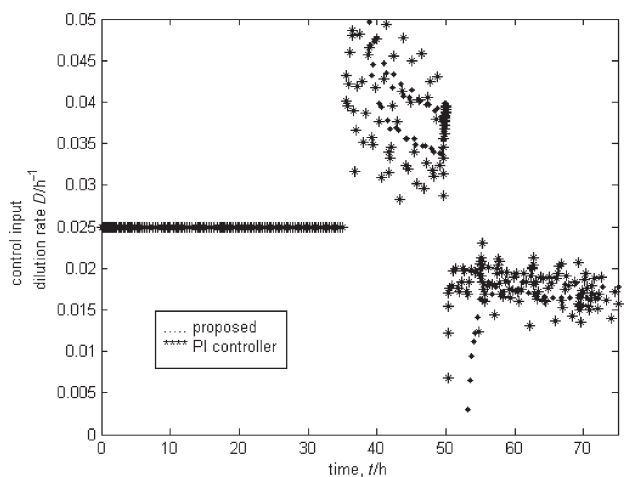


Fig. 6 – Control efforts for the linear PI control law and the proposed methodology

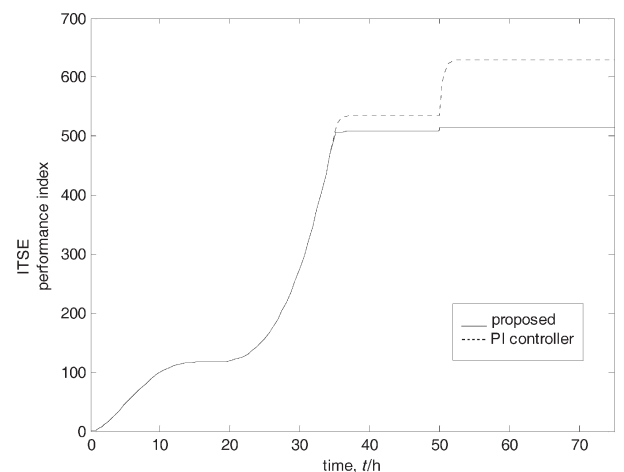


Fig. 7 – Integral time-weighted error of the controller proposed and the IMC-PI

consequence ITSE does not accumulate diversion from the desired set point after  $t = 35$  h, the instant when the control starts to act. Moreover, during the change of set point at  $t = 50$  h ITSE only accumulates a very small error for the controller proposed. On the other hand, the system controlled by the PI is able to regulate the system since the beginning, but the effort is higher; later, after the change of set point, the linear PI exhibits large accumulation of ITSE (Fig. 7). Therefore, the use of these sophisticated mathematical tools, in order to consider dynamic properties of the system, improved, notoriously, the closed-loop performance of this system.

## Concluding remarks

A mathematical model for a class of continuous anaerobic bioreactor, containing a sulphate reducing bacteria *Desulfovibrio alaskensis*, was presented in this work; the experimental validation of the kinetic model provided satisfactory description of the behavior of the bioreactor. The local stability of the selected set points was analyzed via the Lyapunov first method and both equilibrium points were unstable, therefore the bioreactor had to be controlled via an adaptive proportional control law with asymptotic and exponential stability characteristics, as proved. The proposed controller was able to track reference signals with satisfactory performance, as it was illustrated via numerical simulations; furthermore, its dynamic performance was better than the one exhibited by a classical PI control tuned by using IMC rules. It will be interesting to introduce this tuning technique to actual controllers in industrial processes.

## List of symbols

$D$	– dilution rate, $\text{h}^{-1}$
$f$	– nonlinear smooth function
$g$	– invertible nonlinear function
$J_x$	– Jacobian matrix
$K$	– observer's gain vector, $\text{h}^{-1}$
$L$	– Lipchitz constant, $\text{h}^{-1}$
$q$	– volume flow rate, $\text{L h}^{-1}$
$u$	– control input, $\text{h}^{-1}$
$t$	– time, h
$V$	– volume, L
$w$	– mass fraction, $10^{-3}$
$x$	– state variables vector, $\text{g L}^{-1}$
$y$	– measured output, $\text{g L}^{-1}$
$Y_{ij}$	– yield coefficient

## Greek letters

$\beta$	– observer's parameter, $\text{h}^{-1}$
$\varepsilon$	– estimation error, $\text{g L}^{-1}$
$\xi$	– control error, $\text{g L}^{-1}$
$\gamma_S$	– substrate concentration, $\text{g L}^{-1}$
$\gamma_P$	– sulphide concentration, $\text{g L}^{-1}$
$\gamma_X$	– biomass concentration, $\text{g L}^{-1}$
$\zeta$	– uncertain kinetic term, $\text{g h}^{-1} \text{L}^{-1}$
$\tau_q$	– control gain, h
$\mu$	– specific growth rate, $\text{h}^{-1}$
$\Psi$	– volume ratio

## Literature

- Nagpal, S., Chuichukcherm, S., Peeva, L., Livingston, A., *Biotechnol. & Bioeng.* **70** (2000) 533.
- Raskin, L., Rittmann, B., Stahl, D. A., *Applied & Env. Microbiol.* **62** (1996) 3847.
- O'Flaherty, V., Collieran, E., *Biores. Technol.* **68** (1999) 101.
- Omil, F., Bakker, C. D., Hulshoff-Pol, L. W., Lettinga, G., *Environ. Technol.* **18** (1997) 255.
- Bequette, B. W., *Ind. Eng. Chem. Res.* **30** (1991) 1391.
- Chen, C.-T., Peng, S.-T., *J. Proc. Control* **15** (2005) 515.
- Aguilar, R., Martínez, S. A., Rodríguez, M. G., Soto, G., *Chem. Eng. J.* **105** (2005) 139.
- Smets, Y., Bastin, G., Van Impe, J. F., *Biotech. Progr.* **18** (2002) 1116.
- Aguilar, R., González, J., Barrón, M. A., Martínez, R., Maya-Yescas, R., *Proc. Biochem.* **36** (2001) 1007.
- Aguilar-López, R., Martínez-Guerra, R., *Chaos, Solitons & Fractals* **33** (2007) 572.
- Neria-González, I., Wang, E. T., Ramírez, V., Romero, J. M., Hernández-Rodríguez, E., *Anaerobe* **12** (2006) 122.
- Ravot, G., Ollivier, B., Magot, M., Patel, B. K. C., Crolet, J. L., Fardeau, M. L., García, J. L., *Appl. Environ. Microbiol.* **61** (1995) 2053.
- Hungate, R. E., A roll tube method for cultivation of strict anaerobes In: Norris, J. R., Ribbons, D. W., (Eds.). *Methods in Microbiology*, Vol. 3B, Academic Press, London, 1969, p. 117–132.
- Kolmert, Å., Wikström, P., Hallberg, K. J., *Microbiol. Meth.* **41** (2000) 179.
- Cord-Ruwisch, R. J., *Microbiol. Methods.* **4** (1985) 33.
- Bailey, J. E., Ollis, D. F., *Biochemical Engineering Fundamentals*, 2<sup>nd</sup> ed. McGraw Hill BC, Singapore, 1986.
- Kaksonen, A. H., Riekkola-Vanhanen, M. L., Puhakka, J. A., *Water Res.* **37** (2003) 255.
- Rao, V. S. H., Rao, P. R. S., *Chaos, Solitons & Fractals.* **28** (2006) 1222.
- Maya-Yescas, R., Aguilar, R., *Chem. Eng. J.* **92** (2003) 69.
- Alvarez-Ramirez, J., *Chem. Eng. Sci.* **49** (1994) 1743.
- Isidori, A., *Nonlinear Control Theory*. Springer-Verlag, New York, 1995.
- Velovelu, K. C., Soh, Y. C., Cao, W., *IET Control Theory & Applications* **1** (2007) 1533.
- Bernard, O., Gouzé, J. L., *J. Proc. Control* **14** (2004) 765.
- Hoek, J., Field, D. C., Reysenbach, A. L., Iversen, L., *Microbial Ecology* **51** (2006) 470.
- Hadj-Sadok, M. Z., Gouzé, J. L., *J. Proc. Cont.* **11** (2001) 299.
- Ogunnaike, B. A., Ray, W. H., *Process, Dynamics, Modeling and Control*. Oxford University Press, New York, 1994.