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GENERAL MATHEMATICAL MODEL OF AUTONOMOUS UNDERWATER OBJECT MOTION

In this paper an autonomous moving underwater object was considered. Underwater autonomous moving objects are very complex dynamic systems controlled with computers using complex control algorithms. Controlling such dynamic objects is very demanded specially regarding environmental conditions (water waves, currents and so on). On this basis, all hydrodynamic forces and their torques acting to the moving object were identified. The position of the object was unambiguously determined in any time instance by defining coordinate systems. On the basis of known relations of hydrodynamics and system energetic balances, self-mobile underwater object and water, the general mathematical model of dynamics of underwater object is reached. General mathematical model defined in this way is prepared for computer algorithm of object control.

Keywords: *underwater object, rudder, hydrodynamic forces, general mathematical model*

1. INTRODUCTION

Underwater autonomous objects are very complex technical systems, controlled by computers, installed into them. Control of such objects is required and system synthesis includes obtaining qualified general mathematical models [1], [3], [7]. Analysis of performances of well controlled self-mobile underwater objects inevitably requires detailed knowledge of its dynamics, acoustic sensors, installations for measuring specific values, directly or indirectly defining dynamics, knowledge of failures and working conditions in such risk environment [1], [4], [5].

The basis of the general mathematical model object is analysis of conditions and energetic balances, prevailing during moving of object throughout the water. On the basis of known relations of hydrodynamics and system energetic balances, autonomous underwater object and ambient water, the general mathematical model of dynamics of underwater object is reached. By solving so obtained system of complex equations within a time unit, actual condition of underwater object in space has been provided.

2. INPUT AND OUTPUT VARIABLES

Underwater autonomous object is multi-variable system regarding of control. In general, number of input variables is different and depends on type and degree of control [2], [10]. Considered underwater object includes three groups of control elements, namely: rudder (horizontal and vertical), ailerons and drive.

Rudders are classic control elements presenting rigid parts which are fitted at the end parts (abaft) of underwater object. They are fitted horizontally and vertically (mutually perpendicular) to the object plane. Object rudder deflection towards either side, in relation to the plane of symmetry, generates forces and torques, under effect of which output control values are changed. In horizontal plane, horizontal rudders are to be fitted, one on every side, in relation to vertical plane. Input variables of object dynamic model, are marked, as follows: deflection of direction rudder $\delta_{RV}[^0]$ and deflection of depth rudder $\delta_{RH}[^0]$. It is assumed that deflection of vertical rudders towards port side is positive deflection, considering direction of moving of the object. With horizontal rudders, positive rudder deflection is downwards.

Ailerons are controls, intended for limitation of inclination angle in vertical- transversal object plane. When they are turned over they produce force and torque, with acting of which, stabilization of object is performed in transversal vertical plane within desired limits. Deflection of one rigid control part occurs in the opposite side in relation to the other side. Deflection of ailerons in the model of underwater object shall be marked as $\delta_e[^0]$. Positive deflection is deflection of port side downwards, and starboard side upwards, taking into account direction of object moving.

Drive is a control equipment. By operating it produces driving force, thus enabling moving of object in direction of its longitudinal axis. Basic measure of driving force is number of revolutions of propeller $n[^0/_s]$. Vector of input variables is defined by the following formula:

$$\vec{u}(t)=[n\ \ \delta_{RV}\ \ \delta_{RH}\ \ \delta_e]^T \tag{1}$$

Output variables of underwater object[1] unambiguously determine the position in water space. They represent acting measure of input variables, unwanted outside and disturbance forces. Output values are:

course $\psi[^0]$, trim angle $\Theta[^0]$, inclination angle $\varphi[^0]$, $p[^0/_s]$ $q[^0/_s]$ and $r[^0/_s]$angle speeds

V_x, V_y and V_z object speeds on adequate axes and object coordinates in water space x, y i z

Course of object $\psi[^0]$ represents traditionally navigational quantifier, twist measure of object in a horizontal plane and is defined by object longitudinal twist angle from the actual meridian plane in own position. Positive angle is twisting starboard, in relation to object motion direction or clockwise direction.

Trim angle $\Theta[^0]$ is a measure applied to express twisting of an object in vertical *x-y* plane. This plane is situated in object longitudinal and is perpendicular to horizontal plane *x-y*, where object longitudinal is situated, too. These two planes are symmetry planes of object. Positive angle of trim twist is in "shallow waters" or motion in vertical plane in counter clock direction.

Inclination Angle $\varphi[^0]$ is measured by deviation of transverse object axis in transverse vertical plane from the horizontal plane. Positive twist is deflection of z axe in relation to x-y plane starboard.

Object Speeds represent progress of object centre of gravity in directions of relevant axes of the inertial coordination system. For correct definition of object moving, total speed is to be regarded, the magnitude of which is obtained by formula: $V=\sqrt{V_x^2+V_y^2+V_z^2}$, and direction by sum of vector component speeds.

Position Coordinates of an object in space are indicated by achieved mass centre levels in inertial coordinate system. Magnitude of vector $\vec{R}$ is calculated by the formula: $R=\sqrt{x^2+y^2+z^2}$

Output vector has twelve variables, given by relation:

$$\vec{y}=[V_x\ V_y\ V_z\ p\ q\ r\ \Psi\ \Theta\ \varphi\ x\ y\ z]^T \tag{2}$$

[1] Object is double axis symmetric body.

3. GENERAL MATEMATICAL MODEL OF UNDERWATER OBJECT MOTION

For solving object control problems, different coordinate systems should be applied. Proper choice of coordinate systems enables getting adequate form for determining general mathematical model of underwater object motion.

This work deals with inertial (fixed), bound and object speeds systems. Figure 1 shows three coordinate systems, which will be applied in this article.

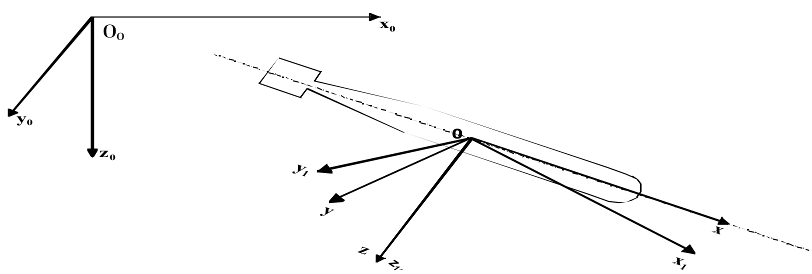

Figure 1. Outlook of inertial, bound and speed coordinate system

Rectangular coordinate system $O_0x_0y_0z_0$ is fixed in space. Axis O_0x_0 lies in direction of starting motion of object, O_0z_0 is directed vertically downwards to centre of gravity of the Earth and axis O_0z_0 is perpendicular to plane $O_0x_0z_0$. Disregarding rotation of the Earth, system is considered inertial. In mentioned coordinate system, path of object mass gravity centres is indicated.

Bound coordinate system *Oxyz* is firmly bound to object mass centre. *Oz* axis lies in symmetry plane, where *Ox* axis is directed forward and *Oy* axis is perpendicular to plane *x-z* and directed starboard. Position of bound coordinate system towards inertial, is determined by Euler angles ψ,Θ,φ.

Speed coordinate system is bound to path of object and is used for defining and studying hydrodynamic forces and torques, acting to the object, and it is defined by system $O_1x_1y_1z_1$. Position of bound coordinate system, in relation to inertial, is defined by mutual position of relevant axes.

Principle assumption for determination of general mathematical model of object motion is, that its motion may be precisely presented by model of rigid body in almost stationary current field of water. In addition to stated assumptions, following assumptions are to be introduced, namely: motion of surrounding water of object occurs only due to moving of object; motion of water is non-eddying; infinite liquid is available around the object; object has two mutually perpendicular symmetry planes; in relation to direction of moving, fore, aft, upper and lower part of object may be differed; object presents rigid body with six freedom degrees of moving; object system and surrounding water have the same freedom degrees as rigid body; dynamic coordinate system is firmly connected to the object. Hydrodynamic coefficients of forces and torques depend on Reynolds number, velocity vector, acceleration, angle speed and command areas deflection; objects are shown with point of mass centre and hydrodynamic effects on object hull, while in motion, result from forces and torques, occurred due to motion of water and outer forces and torques of such forces, considering non inertial forces.

Moving throughout the water, object seems to be complex hydrodynamic system. Solving of equations system of object dynamics includes knowledge of right side of the considered formula [8]. Determination of forces and torques, acting on object, is not an easy task. It is necessary that all forces and their torques are identified and then determined.

It is evident in hydrodynamics that in external forces, effecting rigid body, all forces of non-inertial character are classified as well as those forces where inertial acting of water is disregarded.

All those forces may be divided into the following groups: viscosity, resulting hydrodynamic force $\vec{F}_H (X_H, Y_H, Z_H)$ affecting object hull, acting centre is in the centre of displacement mass $c_B(x_{CB}, y_{CB}, z_{CB})$,

propulsion driving force $T = X_p$ which acting is in direction of axis x and torque of propeller rotation $\Delta\vec{Q} = -\vec{K}_{px}$,

control forces occurred due to rudder deflection Y_{RV} and Z_{RH} and ailerons X_e, and hydrostatic buoyancy forces $\vec{F}_B$

as well as various disturbance forces $\vec{F}_A (X_A, Y_A, Z_A)$.

Undesired hydrodynamic forces, which may not be deflected are in collision with motion of object through water. The forces appeared by deflection of rudder and ailerons are in collision with such forces, in order to neutralize the previous ones. However, since their phenomenon is delayed, disturbed object motion has appeared. Phenomenon of undesired forces results in creation of turning torques, trying to turn the object to the direction of the forces. Autonomous object is to be capable to resist such disturbance forces and to create such forces and torques which will comeback system into the equilibrium state. Such capability is characterized as object stability. According to the forces impacting the object, we differ two types of stability, i.e. static and dynamic.

Static stability understands rest of object afloat and action of two forces: gravity force $\vec{G}$ and buoyancy force $\vec{G}_B$.

On the basis of mentioned general characteristics of forces and torques, affecting the object, the equation of object motion in bound coordinate system *Oxyz*, was given in the previous part.

In case the coordinate start of bound coordinate system lies in the object mass centre: $x_G = y_G = z_G = 0$ and in case that coordinate axes are selected to be principal inertia axes: $I_{xy} = I_{yz} = I_{xz} = 0$, then general formula of equations of object motion has the form:

$$\begin{bmatrix} (m+\lambda_{11})\cdot\dot{V}_x-(m+\lambda_{22})\cdot V_y\cdot r-\lambda_{26}\cdot r^2+(m+\lambda_{33})\cdot V_z\cdot q+\lambda_{35}\cdot q^2 \\ (m+\lambda_{22})\cdot\dot{V}_y+\lambda_{26}\cdot\dot{r}+(m+\lambda_{11})\cdot V_x\cdot r-(m+\lambda_{33})\cdot V_z\cdot p-\lambda_{35}\cdot p\cdot q \\ (m+\lambda_{33})\cdot\dot{V}_z+\lambda_{35}\cdot\dot{q}-(m+\lambda_{11})\cdot V_x\cdot q+(m+\lambda_{22})\cdot V_y\cdot p+\lambda_{26}\cdot p\cdot r \\ (I_{xx}+\lambda_{44})\cdot\dot{p}-(m+\lambda_{22})\cdot V_x\cdot V_z-\lambda_{26}\cdot V_z\cdot r+(m+\lambda_{33})\cdot V_y\cdot V_z+\lambda_{35}\cdot V_y\cdot q- \\ (I_{yy}+\lambda_{55})\cdot q\cdot r-\lambda_{35}\cdot V_z\cdot r+(I_{zz}+\lambda_{66})\cdot q\cdot r+\lambda_{26}\cdot V_y\cdot q \\ (I_{yy}+\lambda_{55})\cdot\dot{q}+\lambda_{35}\cdot\dot{V}_z+(m+\lambda_{11})\cdot V_x\cdot V_z-(m+\lambda_{33})\cdot V_x\cdot V_z-\lambda_{35}\cdot V_x\cdot q+ \\ (I_{xx}+\lambda_{44})\cdot p\cdot r+(I_{zz}+\lambda_{66})\cdot p\cdot r-\lambda_{26}\cdot V_y\cdot p \\ (I_{zz}+\lambda_{66})\cdot\dot{r}+\lambda_{26}\cdot\dot{V}_y-(m+\lambda_{11})\cdot V_x\cdot V_y+(m+\lambda_{22})\cdot V_x\cdot V_y+\lambda_{26}\cdot V_x\cdot r- \\ (I_{xx}+\lambda_{44})\cdot p\cdot q+(I_{zz}+\lambda_{55})\cdot p\cdot q+\lambda_{35}\cdot V_z\cdot p \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \\ F_z \\ \\ M_x \\ \\ M_y \\ \\ M_z \end{bmatrix} \quad (3)$$

where I_{xx}, I_{yy}, I_{zz} are torques of inertia of the object hull, m is object mass, λ_{ik} are associated masses, V_x, V_y, V_z are projections of vector object speed, p, q, r are projections of vector object angle speed.

Right sides of the equation show outer forces and torques acting on object:

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} -X_H \\ Y_H \\ Z_H \end{bmatrix} + (W-F_B)\cdot\begin{bmatrix} -\sin\Theta \\ \sin\varphi\cdot\cos\Theta \\ \cos\varphi\cdot\cos\Theta \end{bmatrix} + \begin{bmatrix} T \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ Y_{RV} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ Z_{RH} \end{bmatrix} + \begin{bmatrix} -X_e \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} K_H \\ M_H \\ N_H \end{bmatrix} + \begin{bmatrix} (-W\cdot z_G+F_B\cdot z_{CB})\cdot\sin\varphi\cdot\cos\Theta \\ (-W\cdot x_G-F_B\cdot x_{CB})\cdot\cos\varphi\cdot\cos\Theta+(-W\cdot z_G-F_B\cdot z_{CB})\sin\Theta \\ (W\cdot x_G+F_B\cdot x_{CB})\cdot\cos\varphi\cdot\cos\Theta \end{bmatrix} +$$

$$+\begin{bmatrix} K_{px} \\ M_T \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ N_{RV} \end{bmatrix} + \begin{bmatrix} 0 \\ M_{RH} \\ 0 \end{bmatrix} + \begin{bmatrix} K_e \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} K_A \\ M_A \\ N_A \end{bmatrix} \quad (5)$$

where:

X_H, Y_H, Z_H – longitudinal, side and transverse hydrodynamic force of object hull,
$W = m \cdot g$ – object gravity force,
F_B – hydrostatic buoyancy force,
φ, Θ – angle of side inclination and object trim,
T – object driving force,
X_e – ailerons resistance force,
Y_{RV} – direction rudder deflection force (vertical rudder),
Z_{RH} – horizontal rudders force,
X_A, Y_A, Z_A – total external disturbance forces,
K_H, M_H, N_H – total hydrodynamic torque of inclination, pitching and turning,
K_{px} – propeller rotation torque,
M_T – propeller propulsion torque,
K_e – ailerons torque,
N_{RV} – vertical rudders torque,
M_{RH} – depth rudders torque,
K_A, M_A, N_A – additional total disturbance torques.

4. CONCLUSION

In this work general mathematical model of autonomous underwater object has been considered taking into account all essential factors and variables, defining object as dynamic system in ambient of surrounding water. According to object and ambient characteristics, general object model has been derived. It was necessary to determine adequate dynamic object model. Object motion was treated in 6 degrees of freedom with 6 independent coordinates necessary to determine object position and orientation. In this way elaborated general mathematical object model is the first step in defining its dynamics. Logic continuation of this work is to determine associated water masses and coefficients of forces and torques in various practical situations so the considered general underwater object model could be applicable in real situations for the purpose of computer controlled algorithms of object control [6]. Some of the possible practical implementations of such defined model can be adaptive autopilots for object tracking control, model-based dynamic positioning of underwater remotely operated vehicles [9] and others.

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Sažetak

OPĆI MATEMATIČKI MODEL GIBANJA AUTONOMNOG PODVODNOG OBJEKTA

U radu je razmatran pokretni autonomni podvodni objekt. Podvodni autonomni objekti su kompleksni dinamički sustavi upravljani računalima ugrađenim u njima s kompleksnim algoritmima upravljanja. Upravljanje takvim objektima je veoma zahtjevno, posebice zbog teških uvjeta okruženja (valovi, struje i sl.). Temelj općeg matematičkog modela objekta čini analiza uvjeta i energetskog balansa objekta pri gibanju u moru. Na bazi toga prepoznate su sve utjecajne hidrodinamičke sile i njihovi momenti koji djeluju na objekt u gibanju, pri čemu su korišteni zakoni hidrodinamike podvodnog objekta u moru za dobivanje matematičkog modela u dinamičkim uvjetima. Definiranjem koordinatnih sustava omogućeno je da se položaj objekta u svakom momentu jednoznačno određuje. Ovako definiran opći matematički model pripremljen je za rješavanje problema upravljanja pomoću računala i odgovarajućih upravljačkih algoritama.

Ključne riječi: *podvodni objekt, kormilo, hidrodinamičke sile, matematički model*

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