

ANALIZA NAPREZANJA I DEFORMACIJE AKSIJALNOGA KOMPENZATORA STRESS AND STRAIN ANALYSIS OF AN AXIAL BELLOW

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Sažetak: U radu je dana analiza aksijalnoga kompenzatora pri povišenoj temperaturi. Analiza se vrši primjenom metode konačnih elemenata pomoću osnosimetričnih elemenata. Iznosi se teorijska osnova modeliranja kontakta u metodi konačnih elemenata.

Ključne riječi:

- aksijalni kompenzator
- metoda konačnih elemenata
- kontakt

Abstract: This paper presents an analysis of an axial bellow expansion joint at elevated temperature. The analysis is performed with the finite element method, using axisymmetric elements. The theory of contact modeling in the finite element method is given.

Keywords:

- axial bellow expansion joint
- finite element method
- contact

1. UVOD

Kompenzatori su cijevni elementi koji služe za kompenzaciju u cjevovodima kod kojih dolazi do istezanja zbog promjena u temperaturi, tlaku ili zbog fizičkih kretanja. Postoji, naravno, više vrsta kompenzatora, ovisno o načinu tj. vrsti kompenzacije. Glavni je cilj ovoga istraživanja razviti detaljnoga modela i postupka koji se mogu koristiti za bilo koju vrstu aksijalnoga kompenzatora. Poseban naglasak dan je na modeliranje kontakta.

2. DIMENZIJE I OPTEREĆENJA KOMPENZATORA

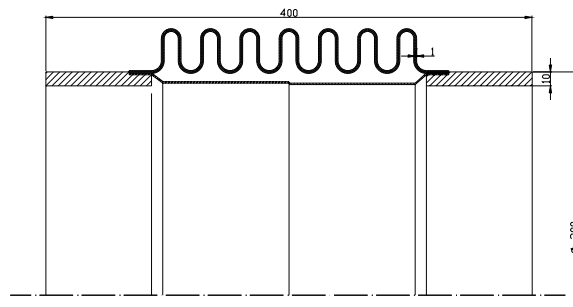
Analiza je izvršena za aksijalni kompenzator pri povišenoj temperaturi. Dimenzije kompenzatora i kompenzator prikazani su na slici 1. Kompenzator ima nazivni promjer 300 mm, ukupne je dužine 400 mm, debljina cijevi iznosi 10 mm, dok je središnji dio kompenzatora, mijeh, napravljen od dvaju slojeva debljine 1 mm. Svojstva materijala dana su u tablici 1.

1. INTRODUCTION

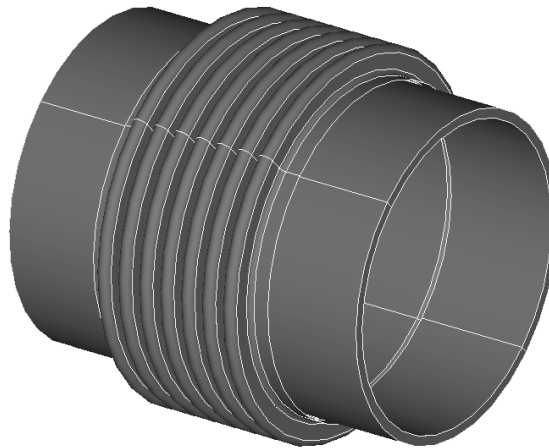
Axial bellows expansion joints are tubular elements for pipelines compensation of axial stretching due to temperature change, pressure or physical movement. There are, of course, several types of expansion joints, depending on compensation method. The main goal of this research is to develop a detailed model and procedure that can be used for any type of axial expansion joint. A special emphasis is given to contact modelling.

2. DIMENSIONS AND LOAD CONDITIONS OF AXIAL EXPANSION JOINT

Analysis was carried out for axial expansion joint at elevated temperature. Dimensions of an axial expansion joint and an expansion joint are shown in Figure 1. The axial expansion joint's diameter is 300 mm, the overall length is 400 mm, with a tube thickness of 10 mm, while the center part of the expansion joint, the bellow, is made out of two layers, each with a thickness of 1 mm. The material properties are given in Table 1.



Slika 1. Dimenzije aksijalnoga kompenzatora iz primjera
Figure 1. Measurement of an axial expansion joint from the example



Slika 2. 3D-prikaz modela aksijalnoga kompenzatora
Figure 2. 3D model of an axial expansion joint

Tablica 1. Svojstva materijala kompenzatora
Table 1. Axial expansion joint material properties

Materijal / Material:	Nehrđajući čelik/Stainless steel, AISI 304
Modul elastičnosti, E : Elasticity modulus, E :	200 [GPa]
Poissonov koeficijent, ν : Poisson's ratio, ν :	0,3
Koeficijent topl. rastezanja, α : Thermal expansion coefficient, α :	$1,875 \times 10^{-5}$ [$1/^\circ\text{C}$]
Naprezanje na granici tečenja, σ_y : Yield stress, σ_y :	300 [MPa]

Također, definirano je linearno izotropno očvršćenje, modulom očvršćavanja, $H = 1000$ MPa. Radna temperatura kompenzatora je 200 °C, unutarnji tlak $p = 6$ bar, a zadani je pomak 70 mm.

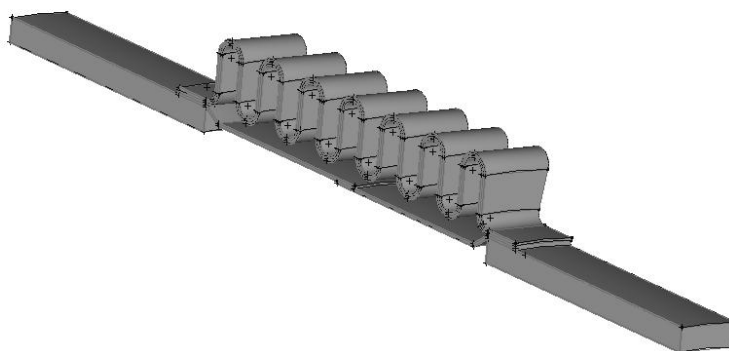
Linear isotropic hardening is defined by hardening modulus, $H = 1000$ MPa. Working temperature of the expansion joint is 200 °C, inner pressure is $p = 6$ bar, while the prescribed axial displacement is 70 mm.

3. FE MODELIRANJE AKSIJALNOGA KOMPENZATORA

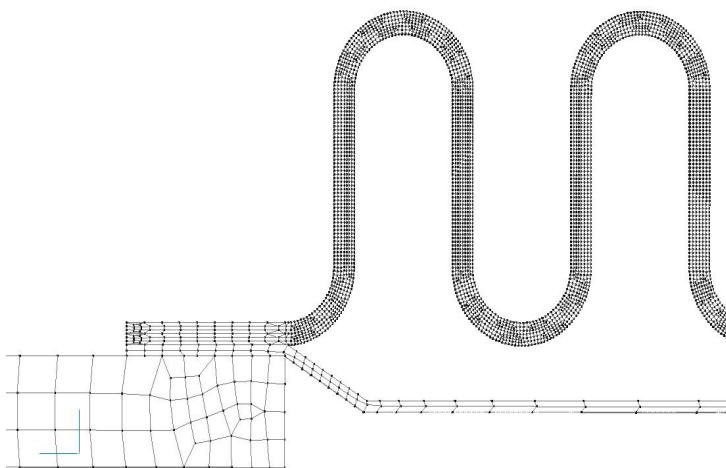
Analiza je izvršena pomoću metode konačnih elemenata, računalnim programom MSC Nastran. Problem je analiziran trodimenzionalnim osnosimetričnim konačnim SOLID elementima. To je moguće zbog konfiguracije aksijalnoga kompenzatora. Stoga se kao model i ne uzima cijeli kompenzator, nego samo dio tj. isječak kompenzatora (slika 3.).

3. FE MODELLING OF AN AXIAL EXPANSION JOINT

Analysis was carried out by the finite element method, using the computer program MSC Nastran. The problem was analyzed with three-dimensional axisymmetric SOLID elements. That is possible due to the axial bellow expansion joint configuration. Therefore, as a model, only a sector of an axial expansion joint is used, not the whole expansion joint (Figure 3).



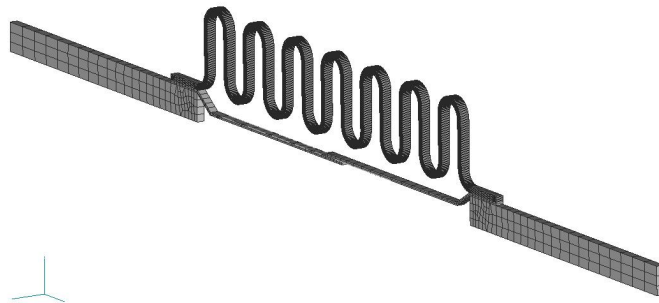
Slika 3. Osnosimetričan isječak modela kompenzatora
Figure 3. Axisymmetric section of an expansion joint model



Slika 4. Dio umreženoga 2D-modela
Figure 4. Part of a meshed 2D model

Budući da je ovo osnosimetričan problem, umreženi 2D-PLATE elementi pretvoreni su rotiranjem za $2,5^\circ$ u 3D-SOLID elemente. Tako model postaje 3D-SOLID model, tj. isječak aksijalnoga kompenzatora rotiran za kut od $2,5^\circ$ (Slika 5.). Konačan model sastoji se od 27994 čvora i 10956 konačnih elemenata.

Since it is an axisymmetric problem, 2D meshed PLATE elements are transformed into 3D SOLID elements rotating by $2,5^\circ$. Thus, the model becomes a 3D SOLID model, i.e. the sector of an axial expansion joint, rotated around its axis at an angle of $2,5^\circ$ (Figure 5.). The final model consists of 27994 nodes and 10956 finite elements.



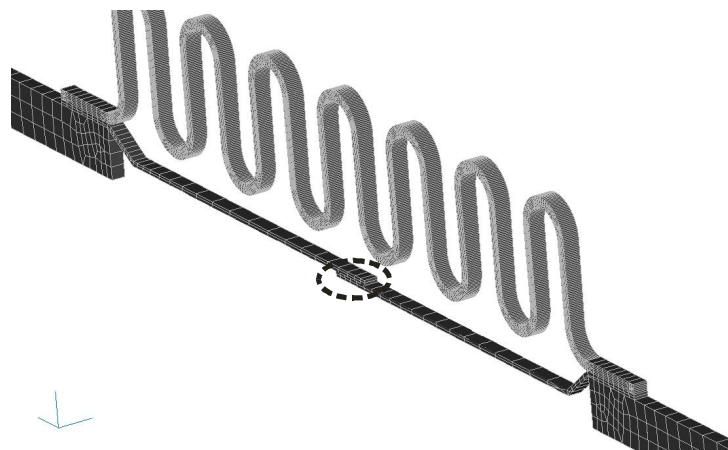
Slika 5. 3D-model isječka kompenzatora
Figure 5. 3D model of an expansion joint section

4. FE ANALIZA AKSIJALNOGA KOMPENZATORA

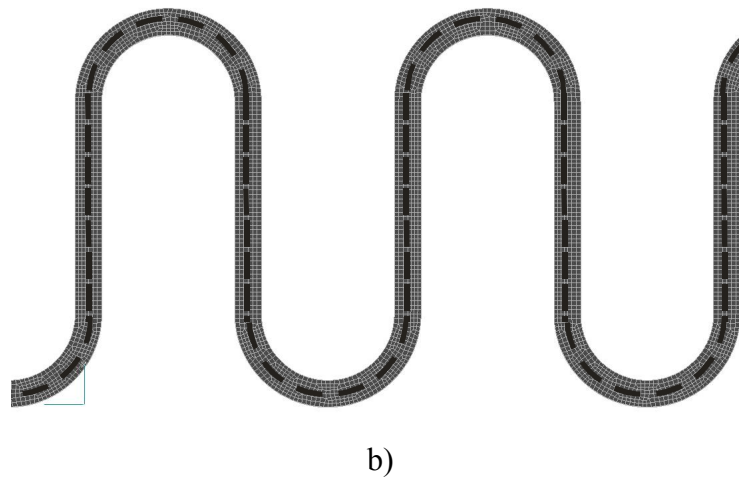
Sljedeći je korak pri konačnoelementnoj analizi aksijalnoga kompenzatora definiranje ograničenja i dozvoljenih kontakata kod kompenzatora te, naravno, definiranje opterećenja. Za ispravno modeliranje aksijalnoga kompenzatora potrebno je uvesti kontakt na dvama mjestima: na unutrašnjem dijelu kliznih cijevi (slika 6.a), što omogućuje skupljanje bez izazivanja naprezanja, te na mijehu, na granici između dvaju slojeva budući da se mijeh sastoji od dvaju slojeva lima (slika 6.b.).

4. FE ANALYSIS OF AN AXIAL EXPANSION JOINT

The next step in the finite element analysis of an axial expansion joint is the setup of the constraints and permitted contacts in the expansion joint and, of course, the load setup. For correct modelling of an axial expansion joint, it is necessary to introduce contact at two spots: at inner part of the sliding tubes (Figure 6.a), which enables contraction without stress, and at the bellow, at the boundary between the two layers, since the bellow is made out of two layers of sheet metal (Figure 6.b).



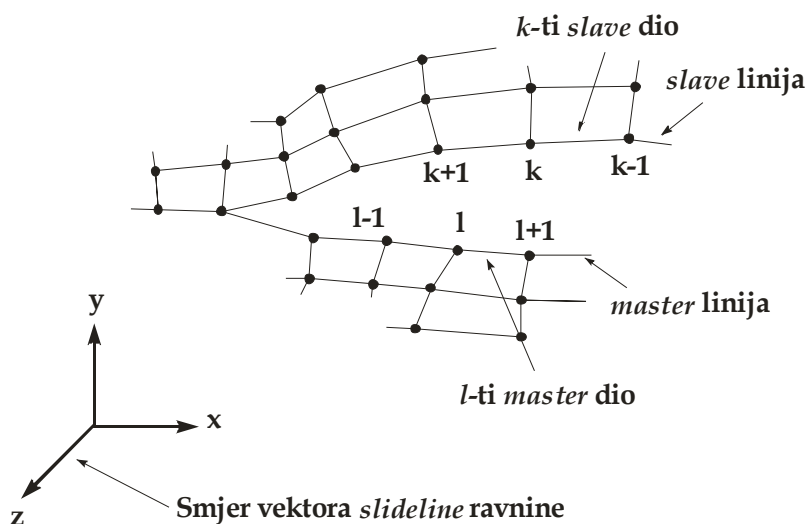
a)



Slika 6. Kontakt između: a) dijela kliznih cijevi, b) slojeva mijeha
 Figure 6. Contact between: a) section of a sliding tubes, b) bellow layers

Kontakt se definira posebnim elementom – *SLIDE LINE*. Linija kontaktnih elemenata, tj. *slide line*, odnosno kontakt definiran *slide line* elementom mora se uvijek provoditi u nelinearnoj analizi. Također, softverski je zahtjev da je *slide line* element uvijek definiran u XY ravnini, bilo lokalnoga ili globalnoga koordinatnog sustava. *Slide line* element sastoji se od *master* i *slave* čvorova. Definiranje tih čvorova, odnosno određivanje koji dio je *slave*, koji *master*, nije presudno osim u slučajevima nesimetričnog prodiranja. U tom slučaju samo *slave* čvorovi, tj. segmenti ulaze u provjeru kontakta, odnosno u provjeru prodiranja u *master* dijelove.

The contact is defined using special elements—*SLIDE LINE*. *Slide line* indicates that contact defined using the *slide line* element is always conducted through nonlinear analysis. Moreover, the software requires that the *slide line* element has to be defined in the XY plane, of either the local or the global coordinate system. The *Slide line* element consists of *master* and *slave* nodes. The definition of those nodes, i.e. defining which node is a *master* node and which is a *slave* node, is not critical, unless in the case of asymmetrical penetration. In that case, only *slave* nodes or segments enter into contact check, i.e. penetration into the *master* segment check.



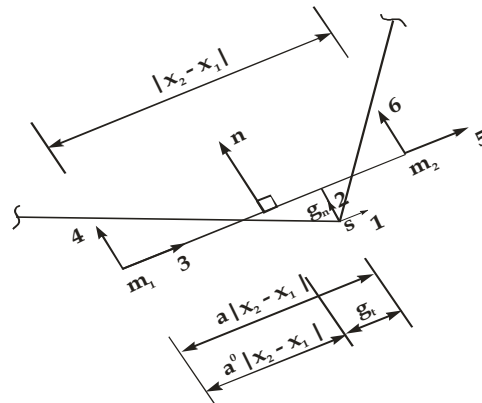
Slika 7. Metoda definiranja i označavanja master i slave čvorova u slide line elementu
 Figure 7. Master and slave node setup and marking method in Slide line element

Kontakt se određuje između *slave* čvorova i *master* linije. *Slave* čvorovi ograničeni su uvjetom klizanja po *master* liniji u slučaju kontakta te moraju ostati na toj liniji dok se ne pojavi vlačna sila koja će odvojiti kontakne površine. Okolicu *slide line* kontakta definira korisnik. Za površinu kontakta potrebno je definirati normalu *slide line* ravnine te *slave* i *master* linije. Te linije definiraju se čvorovima po određenom redoslijedu. Također, u simetričnom prodiranju, moguće je definirati kontakt između *master* čvorova i *slave* linije kao i kontakt između *slave* čvorova i *master* linije, a bez definiranja nove okolice kontakta.

MSC Nastran za rješavanje problema kontakta koristi *penalty* metodu. Kada je otkriven kontakt između *slave* čvora i *master* segmenta, automatski je kreiran tročvorni *slide line* element (slika 8.).

Contact is determined between *slave* and *master* lines. *Slave* nodes are restrained with the *master* line sliding condition in the case of contact and must stay on that line until traction force occurs, which will separate contact interface. The user defines the *slide line* contact environment. For the contact area, it is necessary to define the *slide line* plane norm and the *master* and *slave* lines. Those lines are defined with nodes in a designated order. Also, in symmetrical penetration, it is possible to define contact between *master* nodes and *slave* line as well as contact between *slave* nodes and *master* line, without defining a new contact environment.

MSC Nastran uses a penalty method for solving contact problems. When a contact between *slave* node and a *master* segment is determined, the three node *slide line* element is automatically created (Figure 8.)



- $S, m_1, m_2 =$ *slave*, *master* čvor 1, *master* čvor 2 / *slave*, *master* node 1, *master* node 2,
- $a, a^0 =$ trenutna i prethodna površinska koordinata / current and past surface coordinate,
- $g_n =$ prodor *slave* čvora u *master* segment / penetration of a *slave* node into *master* segment,
- $g_t =$ klizanje *slave* čvora po *master* segmentu / *slave* node sliding on *master* segment,
- $n =$ smjer normale *master* segmenta / normal direction of a *master* segment.

Slika 8. Tročvorni slide line element

Figure 8. Three node slide line element

Prvi čvor *slide line* elementa je *slave* čvor, drugi čvor je prvi čvor *master* segmenta, a treći čvor je drugi čvor *master* segmenta. Svaki čvor ima dva stupnja slobode. Prvi stupanj slobode za čvor je pomak u smjeru normale elementa. Smjer tangente t je smjer od *master* čvora 1 do *master* čvora 2 te je dan izrazom:

The first node of a *slide line* element is a *slave* node, the second one is a first node of a *master* segment, the third node is a second node of *master* segment. Every node has two degrees of freedom. The first DOF for the node is a displacement in the element's normal direction. The direction of the tangent t is the direction of the *master* node 1 to the *master* node 2, and is given by the term:

$$t = \frac{\mathbf{x}_2 - \mathbf{x}_1}{|\mathbf{x}_2 - \mathbf{x}_1|} = \frac{\mathbf{x}_2 - \mathbf{x}_1}{l}, \quad (1)$$

gdje su $\mathbf{x}_1 = X_1 + u_1$, $\mathbf{x}_2 = X_2 + u_2$, trenutni položaji dvaju *master* čvorova, dok je l trenutna duljina *master* segmenta. Površinska koordinata a je trenutna projekcija *slave* čvora s na *master* segmentu i dana je izrazom:

where $\mathbf{x}_1 = X_1 + u_1$, $\mathbf{x}_2 = X_2 + u_2$, are the current positions of the two *master* nodes, and while l is the current length of the *master* segment. Surface coordinate a is a current projection of a *slave* node on a *master* segment and is given by the term:

$$\mathbf{a} = \frac{\mathbf{x}_s - \mathbf{x}_1}{|\mathbf{x}_2 - \mathbf{x}_1|} \cdot \mathbf{t} = \frac{\mathbf{x}_s - \mathbf{x}_1}{l} \cdot \mathbf{t}, \quad (2)$$

gdje je \mathbf{x}_s trenutni položaj *slave* čvora.

Nastran određuje kontakt između *slave* čvora i *master* linije pomoću mjerenja udaljenosti *slave* čvora od *master* segmenta u smjeru normale. Normalni prodor g_n *slave* čvora s u *master* segment je projekcija vektora od \mathbf{x}_1 do \mathbf{x}_s u smjeru normale, dan izrazom:

$$g_n = (\mathbf{x}_s - \mathbf{x}_1) \cdot \mathbf{n}, \quad (3)$$

Prodor, tj. kontakt se otkriva negativnom vrijednošću prodora g_n . Položaj prodora dan je površinskom koordinatom (2).

U *penalty* metodi ograničenje prodora *slave* čvora u *master* segment pretpostavljeno je *penalty* izrazom u izrazu za energiju i rezultat se dobije minimiziranjem modificirane energije. Matematički, modifikacija energije Π_n zbog ograničenja prodora za *slave* čvor, dana je izrazom:

$$\Pi_n = \frac{\varepsilon_n}{2} g_n^2 \quad (4)$$

gdje je ε_n *penalty* vrijednost. Rješenje problema ograničenja dobiva se minimiziranjem modificirane energije kada $\varepsilon_n \rightarrow \infty$. Rezidualni kontakt i kontaktna krutost u konačnoelementnoj formulaciji zbog ograničenja kontakta dobiveni su prvom i drugom varijacijom energije Π_n . Matrica krutosti dobivena drugom varijacijom naziva se konzistentnom lineariziranom matricom krutosti. Vektor rezidualne sile \mathbf{G}_n i matrica krutosti kontakta \mathbf{K}_n za normalni kontakt su:

where \mathbf{x}_s is the current position of the *slave* node.

Nastran determines the contact between the *slave* node and *master* line using distance measuring of a *slave* node to the *master* segment in the normal direction. The normal penetration g_n of a *slave* node s into the *master* segment is a projection of the vectors \mathbf{x}_1 to \mathbf{x}_s in the normal direction, given by term:

Penetration, i.e. contact is determined with a negative value of the penetration g_n . The penetration position is given by surface coordinate (2).

In the *penalty* method, the penetration constraint of a *slave* node into a *master* segment is represented by the term *penalty* in the term for energy and the result is obtained by minimization of the modified energy. Mathematically, the modification of an energy Π_n because of penetration for the *slave* node, is given by the term:

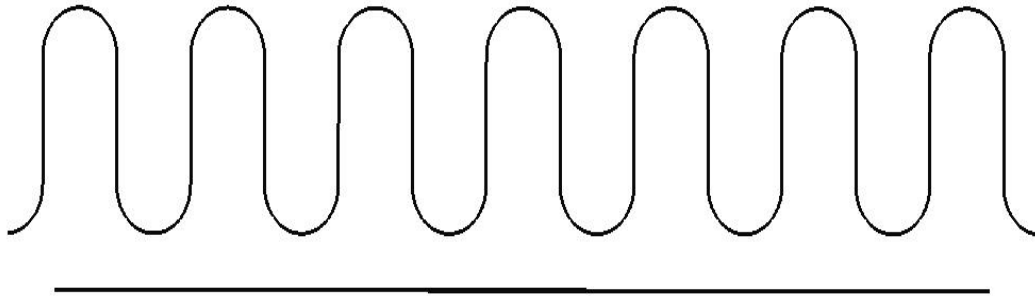
where ε_n is the *penalty* value. The solution for the constraint problem is yielded by minimization of a modified energy when $\varepsilon_n \rightarrow \infty$. Residual contact and contact stiffness in finite element formulation, because of contact, are obtained with the first and second variations of energy Π_n . The stiffness matrix obtained by second variation is called the consistent linearized stiffness matrix. Residual force vector \mathbf{G}_n and a contact stiffness matrix \mathbf{K}_n for normal contact are:

$$\mathbf{G}_n = -\varepsilon_n g_n \mathbf{N}_s \quad (5)$$

$$\mathbf{K}_n = \varepsilon_n \left[\mathbf{N}_s \mathbf{N}_s^T - \frac{g_n}{l} \left(\mathbf{T}_s \mathbf{N}^T + \mathbf{N} \mathbf{T}_s^T + \frac{g_n}{l} \mathbf{N} \mathbf{N}^T \right) \right] \quad (6)$$

Vektori N_s i T_s predstavljaju pomake po stupnjevima slobode elementa, koji odgovaraju jediničnom pomaku u smjeru normale i tangente *slave* čvora. Vektor N predstavlja pomak po stupnjevima slobode elementa *master* segmenta zbog jediničnog pomaka u smjeru normale *master* čvora 2.

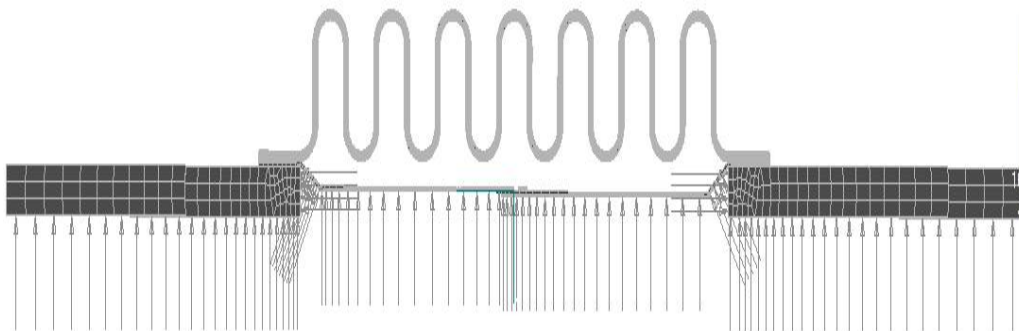
Vectors N_s and T_s represent the element's DOF displacements, which correspond to unit displacement in the normal and tangent direction of a *slave* node. Vector N represents DOF displacement of the *master* segment element because of *master* node 2 unit displacement in the normal direction.



Slika 9. Slide line elementi u analizi aksijalnoga kompenzatora
Figure 9. Slide line element in axial expansion joint analysis

U analizi s kontaktnim elementima poželjno je primijeniti prisilni pomak, koji vodi ka boljoj konvergenciji nego ako se primijene samo opterećenja. Stoga je u slučaju analize kompenzatora primijenjen pomak u tlačnom aksijalnom smjeru od 70 mm na čvorovima kraja modela. Među ostalim opterećenjima na kompenzator djeluju pritisak od 6 bar s unutrašnje strane, tj. unutar cjevovoda (slika 10.), te temperaturno opterećenje od 200 °C na cijeli kompenzator.

In contact element analysis, it is advisable to apply enforced displacement, apart from applied loads, which leads to better convergence. Hence, in expansion joint analysis, a 70 mm displacement is applied at nodes at the model's ends in the axial compression direction. Other loads that act on the expansion joint are, 6 bar pressure, from the inner side, i.e. the inside pipeline (Figure 10), and a 200 °C temperature load on the whole expansion joint.



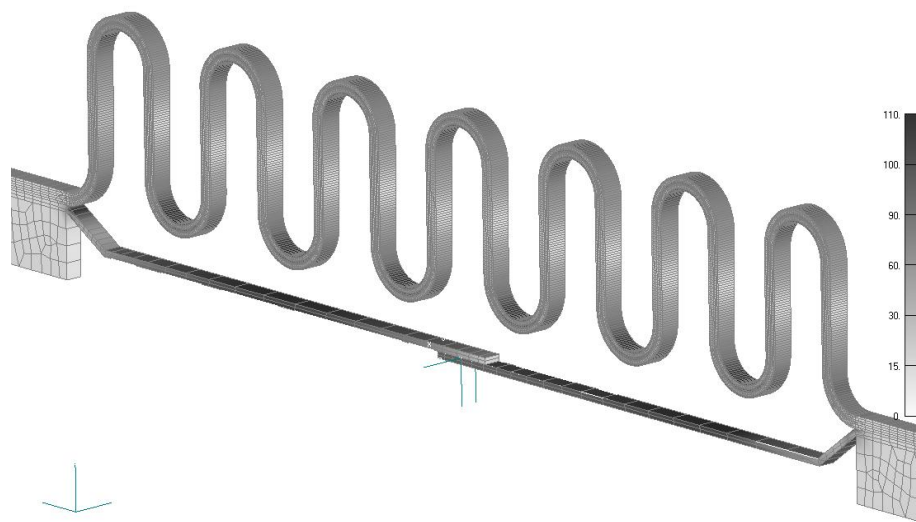
Slika 10. Pritisak u kompenzatoru
Figure 10. Pressure in expansion joint

Da bi se što bolje modelirali radni uvjeti cjevovoda, potrebno je ograničiti tj. postaviti rubne uvjete kompenzatora. Kako se radi samo o isječku, postavljeno je ograničenje, i to za sve čvorove, pomaka po osi y te rotacije oko osi x i z . Čvorovima na krajevima cijevi ograničeni su pomak i rotacija po svim osima.

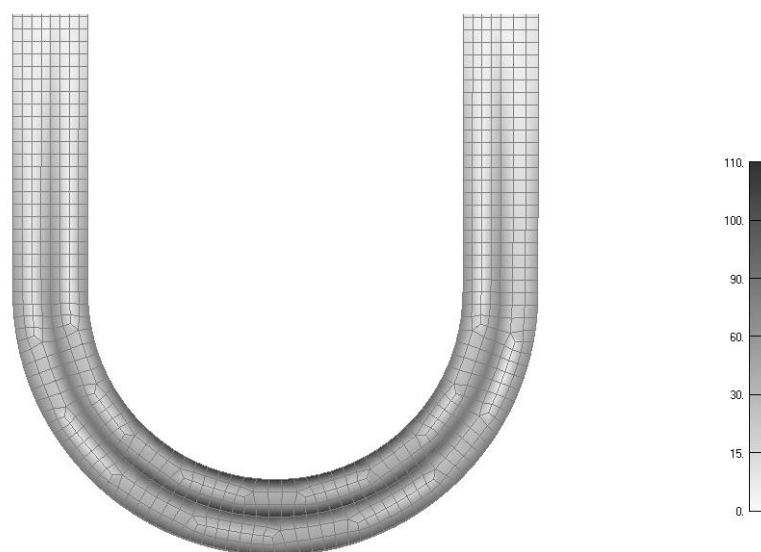
For the purpose of better modeling of working conditions of a pipeline, it is necessary to constrain, i.e. to setup the boundary conditions of an expansion joint. In the case of a sector of an expansion joint, every node has y axis displacement constrained and x and z axis rotation constrained. The nodes at the ends of a tube are fixed.

Pri nelinearnoj statičkoj analizi korišteno je 240 vremenskih inkremenata, tj. koraka povećanja. Reakcija na kraju kompenzatora, za skraćivanje od 70 mm, iznosi 1131 N, iz čega se izračunava krutost kompenzatora od 16,16 N/mm. Najveća Von Mises naprezanja iznose 103,36 N/mm². Slika 11. prikazuje raspored Von Mises naprezanja u određenom vremenskom koraku, dok je na slici 12. detaljniji prikaz Von Mises naprezanja u dijelu 2-slojnog mijeha kompenzatora. Na slici 13. prikazan je deformirani oblik kompenzatora. Bitno je napomenuti da je uz pomoć definiranoga kontakta omogućeno realno modeliranje ponašanja aksijalnoga kompenzatora u radnim uvjetima povišene temperature i radnog tlaka. Time je spriječen prodor jednog sloja mijeha u drugi te isprepletanje unutrašnjeg dijela cijevi kompenzatora.

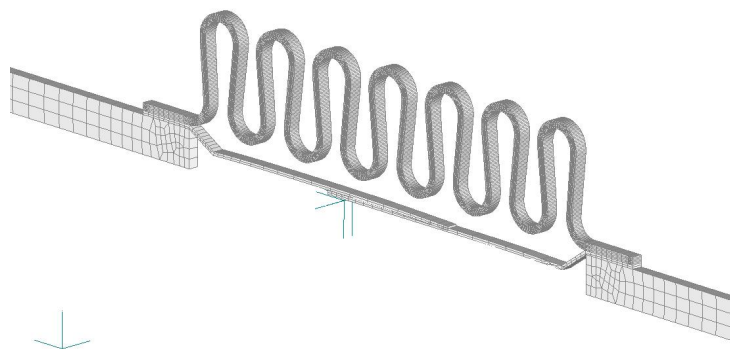
In nonlinear static analysis, 240 time increments were used. The reaction force at the end of an axial expansion joint is 1131 N, for contraction of 70 mm, which gives the stiffness of an axial expansion joint of 16,16 N/mm. The highest Von Mises stress is 103,36 N/mm². Figure 11 shows the distribution of Von Mises stress in referred time step, while Figure 12 shows a detailed overview of Von Mises stress in the section of a two-layer bellow. Figure 13 shows the deformed shape of an expansion joint. It is essential to notice that, with help from a predefined contact, realistic modeling of the axial expansion joint under working conditions, elevated temperature and working pressure, is possible. In that way, penetration of one layer of a bellow into another and interweaving of the inner part of a tube, is prevented.



Slika 12. Raspored Von Mises naprezanja
Figure 12. Von Mises stress distribution



Slika 13. Detaljni prikaz raspodjele naprezanja u dijelu mijeha kompenzatora
Figure 13. Detailed display of stress distribution in bellow part of an expansion joint



Slika 14. Deformirani oblik kompenzatora
Figure 14. Deformed shape of an expansion joint

5. ZAKLJUČAK

Na primjeru aksijalnoga kompenzatora prikazan je način modeliranja kontakta u računskim programima kao što je MSC Nastran. Kontakt se modelira korištenjem kontaktnog elementa *slide line*, koji se nadalje zasniva na *penalty* metodi. Dana su objašnjenja *slide line* elementa te *penalty* metode.

6. POPIS OZNAKA

modul očvršćivanja	H ,	MPa
tlak	p ,	bar
prodor	g_n	
modificirana energija	Π_n	
<i>penalty</i> vrijednost	ε_n	
vektor rezidualne sile	\mathbf{G}_n	
matrica krutosti kontakta	\mathbf{K}_n	

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5. CONCLUSIONS

The contact modeling method with computer software, like MSC Nastran, is presented through an example of an axial expansion joint. The contact is modeled using *slide line* contact element, which is based on the *penalty* method. Further explanations of *slide line* element and *penalty* method are given.

6. LIST OF SYMBOLS

hardening modulus
pressure
penetration
modified energy
<i>penalty</i> value
residual force vector
contact stiffness matrix

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