CODEN STJSAO ISSN 0562-1887
ZX470/1329 UDK 532.542:519.63:621.311.21:519.876.5(043)

Numerical Simulation of Hydraulic Transients in Rijeka HPP

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Original scientific paper

In this work a new approach considering numerical modelling of waterhammer dissipation and attenuation is proposed. The classical Allievi liquid flow model has been extended with the Brunone unsteady friction model which includes separation of the local and convective unsteady friction factor. A widely-adopted approach in solving this model is treatment of the unsteady friction term as a classical source term, while the authors propose a non-conservative formulation, due to the existence of a non-conservative source term. The second order flux limited scheme is applied to the proposed formulation and is applied to simulations of transient flow in Rijeka HPP.

Keywords
Hydroelectric power plants
Hyperbolic conservation laws
Hydraulic transients
Pipe flow
Unsteady flow
Unsteady friction
Water hammer

Ključne riječi
Hidroelektrane
Hidraulički tranzijenti
Hidraulički udar
Hiperbolički zakoni čuvanja
Nestacionarno strujanje
Nestacionarno trenje
Strujanje u cjevovodima

Received (primljeno): 2007-06-25
Accepted (prihvaćeno): 2007-12-01

Numeričke simulacije hidrauličkih tranzijenata u HE Rijeka

Izvornoznanstveni članak

Predložen je novi pristup numeričkom modeliranju disipacije i prigušivanja tlačnog udara. Klasični Allievijev model je proširen Brunoneovim modelom nestacionarnog trenja koji uključuje separaciju lokalnog i konvetivnog člana nestacionarnog faktora trenja. Općenito prihvaćen pristup rješavanju takvog modela podrazumijeva modeliranje člana nestacionarnog trenja klasičnim izvornim članom, dok su autori predložili nekonzervativnu formulaciju zbog postojanja nekonzervativnog izvornog člana. Provedene su numeričke simulacije tranzijentnog strujanja u HE Rijeka korištenjem predložene formulacije i fluks limitirajuće sheme drugog reda. Postignuti numerički rezultati su pokazali dobro slaganje sa izmjerenim podacima za dva različita režima rada promatranog sustava.

1. Introduction

During simulation of the transient pipe flow, friction losses are often estimated using expressions derived from steady or quasi-steady state flow conditions. However, as shown by a number of researchers (Zielke [19], Eichinger and Lein [7], Pezzinga [13], Brunone [3], [4], Axworthy et al. [1]), the steady friction approximation yields an insufficient amount of dumping and significant discrepancies in the phase shift of head traces between experimental and computed data. Although a number of unsteady friction models have been developed, the Brunone model [4] based on a model developed by Greco and Golia [8] is widely used, due to its simplicity and compatibility with experimental results. This paper focuses on a modified instantaneous acceleration-based (MIAB) formulation of an unsteady friction model [17] based on the Brunone model [4]. The MIAB formulation relates the unsteady friction part to an instantaneous local acceleration and instantaneous local convective acceleration and uses Vardy's analytically deduced shear decay coefficient [16] to compute the Brunone unsteady friction coefficient [3-4], which varies with time and space [2], [5]. The authors have written the system in a non-conservative formulation and then applied a second order flux limited upwind scheme to the aforementioned system, which has been successfully tested on results of experimental measurements [14]. Finally, the proposed formulation and its numerical implementation are compared with results of measurements conducted in Rijeka HPP, Croatia.

Symbols/Oznake

A	- pipe cross section area, m^2 - površina poprečnog presjeka cijevi
A	- Jacobi flux matrix - Jacobi matrica fluksa
C_{cfl}	- Courant Friedrichs Lewy number - Courant Friedrichs Lewy broj
c	- wave propagation velocity, ms^{-1} - brzina propagacije
D	- diameter of circular cross-section, m - promjer cijevi kružnog poprečnog presjeka
D_e	- equivalent diameter of circular cross-section, m - ekvivalentni promjer cijevi kružnog poprečnog presjeka
e	- absolute roughness of a pipe, m - apsolutna hrapavost
ε	- relative roughness of a pipe - relativna hrapavost cijevi
f_s	- DarcyWeisbach friction factor - DarcyWeisbachov faktor trenja
$\mathbf{f}$	- flux vector in hyperbolic conservation law - vektor fluksa hiperboličkog zakona očuvanja
g	- gravity, ms^{-2} - gravitacija
g	- source vector vector in hyperbolic conservation law - izvorni član hiperboličkog zakona očuvanja
H	- piezometric head, m - piezometrička visina
i	- location of grid point in finitedifference approximation - pozicija točke u mreži u konačno različnoj aproksimaciji
I	- identity matrix - jedinična matrica
J_s	- steady friction term - član stacionarnog trenja
J_u	- unsteady friction term - član nestacionarnog trenja
k	- Brunone friction factor - Brunoneov factor trenja
L	- pipe length, m - duljina cijevi
L	- matrix determining particular upwind scheme - matrica koja određuje određenu upwind shemu
$\mathsf{l}^{(p)}$	- left eigenvector in p^{th} characteristic field - lijevi svojstveni vektor u p-tom karakterističnom polju
m	- number of equations in the system - broj jednadžbi u sustavu
n	- time level in finite difference approximation, i.e. number of time steps - vremenski nivo u konačno različnoj aproksimaciji, tj. broj vremenskih koraka
N	- number of computational cells - broj numeričkih ćelija
p	- number of characteristic field - broj karakterističnog polja
Q	- discharge, m^3s^{-1} - protok
Re	- Reynolds number - Reynoldsov broj
$\mathbf{R}$	- right eigenvector matrix - matrica desnih svojstvenih vektora
$\mathbf{r}^{(p)}$	- right eigenvector in p^{th} field - desni svojstveni vektor u p-tom karakterističnom polju
t	- time, s - vrijeme
Δt	- time step, s - vremenski korak
$\mathbf{u}$	- state vector of hyperbolic conservation law - vektor stanja hiperboličkog zakona očuvanja
v	- speed, ms^{-1} - brzina
x	- space coordinate, m - prostorna koordinata
Δx	- cell width, m - širina ćelije
z	- measurement height of given level, m - mjerena visina od zadanog nivoa
$\lambda^{(p)}$	- eigenvalue in p^{th} characteristic field - vlastita vrijednost u p-tom karakterističnom polju
$\mathbf{\Lambda}$	- eigenvalue matrix - matrica svojestvenih vrijednosti
φ	- flux limiter function - funkcija fluks limitera
Φ_A	- discharge's algebraic sign - algebarski predznak protoka

2. Mathematical model

The governing equations for a one dimensional unsteady pipe flow [6] are:

$$H_t + \frac{c^2}{gA} Q_x = 0\,, \tag{1}$$

$$Q_t + gAH_x + J_s + J_u = 0\,, \tag{2}$$

where t denotes time, x a space coordinate along the conduit length, H a piezometric head, Q average discharge, A pipe cross-section area, c wave speed, g acceleration of gravity, J_s and J_u denote head losses per unit length due to steady and unsteady friction losses, respectively. The steady friction losses are calculated as follows:

$$J_s = \frac{f_s Q|Q|}{2DA}\,, \tag{3}$$

where f_s is the Darcy-Weisbach friction factor and D the pipe diameter. The unsteady friction term J_u is defined as:

$$J_u = \frac{k}{2}\left(Q_t + c\,\Phi_A |Q_x|\right), \tag{4}$$

where Φ_A equals sign(Q) and k is the Brunone friction coefficient [3] defined as:

$$k = \frac{\sqrt{C^*}}{2}\,, \tag{5}$$

where:

$$C^* = \begin{cases} 0{,}0476 & \text{laminar flow} \\ \dfrac{7{,}41}{Re^{\log\left(14{,}3/Re^{0{,}05}\right)}} & \text{turbulent flow} \end{cases} \tag{6}$$

where Re is the Reynolds number.

Furthermore, considering the form of (4), the unsteady friction model can be split into two parts [17], yielding:

$$J_u = \frac{1}{2}\left(k_P Q_t + k_A c\,\Phi_A |Q_x|\right). \tag{7}$$

This approach introduces two unsteady friction coefficients, k_A and k_P, which require calibration and, in general, are likely to be both frequency and time dependent. The coefficient k_A represents the Brunone friction coefficient (5) and k_P was determined through a simple calibration procedure, and defined as $3/2k_A$.

Taking into account terms (3) and (7), it is obvious that the system (1) and (2), written in the aforementioned form, is not in the classical form of a conservation law, from which implies that implementation of the standard computational methods developed for hyperbolic conservation laws could be inadequate.

Therefore, a new approach which includes the appropriate corrections of numerical schemes is needed to solve as accurately as possible the considered system of equations.

The Allievi model defined with (1) and (2), taking into an account only steady friction term J_s in (3), can be written in vector form as:

$$\mathbf{u}_t + \mathbf{f}(\mathbf{u})_x = \mathbf{g}(\mathbf{u}), \tag{8}$$

where:

$$\mathbf{u} = \begin{pmatrix} H \\ Q \end{pmatrix}, \quad \mathbf{f} = \begin{pmatrix} \dfrac{c^2}{gA} Q \\ gAH \end{pmatrix}, \quad \mathbf{g} = \begin{pmatrix} 0 \\ \dfrac{-f_s Q|Q|}{2DA} \end{pmatrix}. \tag{9}$$

According to [12], the expression (8) can be written in the quasilinear form:

$$\mathbf{u}_t + \mathbf{A}\mathbf{u}_x = \mathbf{g}(\mathbf{u})\,, \tag{10}$$

where $\mathbf{A}$ is the Jacobi matrix with (i,j) entry given by $\partial f_i/\partial u_j$ with eigenvalues and right eigenvectors being:

$$\lambda^{(1),(2)} = \mp c, \mathbf{r}^{(1),(2)} = \begin{pmatrix} \dfrac{\lambda^{(1),(2)}}{Ag} \\ 1 \end{pmatrix}. \tag{11}$$

The above system is hyperbolic and can be solved by using a standard numerical scheme.

However, The Allievi model defined with (1) and (2), taking into account steady friction term in **(3)** and unsteady friction term in (7) can be written as:

$$\mathbf{u}_t + \mathbf{f}_2(\mathbf{u})_x = \mathbf{g}_2(\mathbf{u}, \mathbf{u}_x)\,, \tag{12}$$

where:

$$\mathbf{u} = \begin{pmatrix} H \\ Q \end{pmatrix}, \quad \mathbf{f}_2 = \begin{pmatrix} \dfrac{c^2}{gA} Q \\ \dfrac{2gA}{2+k_P} H \end{pmatrix},$$
$$\mathbf{g}_2 = \begin{pmatrix} 0 \\ -\dfrac{1}{2+k_P}\left(\dfrac{Q|Q|}{DA} f_s + k_A c\,\Phi_A \mathrm{sign}(Q_x) Q_x\right) \end{pmatrix}. \tag{13}$$

The source term in (12) contains the state vector derivative with respect to space component, i.e. part of the source term becomes the non-conservative product [11]. This means that the system cannot be written in the form of (10) without introducing the modified Jacobi matrix as explained in [11]. In order to correctly evaluate the characteristic fields in the numerical approximation, the term $\mathbf{g}_2$ is rewritten as:

$$\mathbf{g}_2(\mathbf{u}, \mathbf{u}_x) = \mathbf{B}(\mathbf{u})\mathbf{u}_x + \tilde{\mathbf{g}}(\mathbf{u})\,, \tag{14}$$

where:

$$\mathbf{B}(\mathbf{u}) = \begin{pmatrix} 0 & 0 \\ 0 & \dfrac{k_A c\,\Phi_A \mathrm{sign}(Q_x)}{2+k_P} \end{pmatrix}, \tag{15}$$

and

$$\tilde{\mathbf{g}}(\mathbf{u}) = \begin{pmatrix} 0 \\ -\dfrac{1}{2+k_{\mathrm{p}}} \dfrac{Q|Q|}{DA} f_{\mathrm{s}} \end{pmatrix}. \tag{16}$$

If $\mathbf{A}$ denotes the Jacobian matrix of the flux $\mathbf{f}_2$, i.e. $\mathbf{f}_2(\mathbf{u})_{\mathrm{x}}=\mathbf{A}\mathbf{u}_{\mathrm{x}}$, then by introducing the modified Jacobian matrix:

$$\tilde{\mathbf{A}} = \mathbf{A} - \mathbf{B}, \tag{17}$$

the system (12) can finally be written in a non-conservative formulation as:

$$\mathbf{u}_{\mathrm{t}} + \tilde{\mathbf{A}}(\mathbf{u})\mathbf{u}_{\mathrm{x}} = \tilde{\mathbf{g}}(\mathbf{u}), \tag{18}$$

with eigenvalues and right eigenvectors being:

$$\lambda^{(1),(2)} = \frac{1}{2}\frac{c}{2+k_{\mathrm{p}}}\left(\Phi_{\mathrm{A}}\operatorname{sign}(Q_{\mathrm{x}}) \mp \sqrt{k_{\mathrm{A}}^2 + 8\left(2+k_{\mathrm{p}}\right)}\right),$$
$$\mathbf{r}^{(1),(2)} = \begin{pmatrix} \dfrac{\lambda^{(1),(2)}}{Ag} \\ 1 \end{pmatrix}. \tag{19}$$

In conclusion, the implementation of standard computational methods developed for hyperbolic conservation laws for solving the Allievi model defined by (1) and (2) along with terms **(3)** and (7) can be justified only if the above mentioned model is written in the non-conservative formulation (18).

3. Numerical model

In this section the authors present a short overview of Roe's formulation of finite volume first order and second order flux limited upwind numerical schemes. All the details about the subject can be found in [10] and [12], as well in many other numerous references about these classical numerical schemes.

In the case of a one-dimensional problem the domain of length $[0,L]$ is discretized in N cells with cell centers $x_{\mathrm{i}}=i\Delta x$, $i=0,...,N$, where Δx is cell width and $[x_{\mathrm{i-1/2}}, x_{\mathrm{i+1/2}}]$ is the i^{th} cell. The general finite volume numerical scheme for solving the system (8) can be written in the following form:

$$\mathbf{u}_i^{n+1} = \mathbf{u}_i^n - \frac{\Delta t}{\Delta x}\left(\mathbf{f}_{i+1/2}^n - \mathbf{f}_{i-1/2}^n\right) + \Delta t \mathbf{g}_i^n, \tag{20}$$

where $\mathbf{u}_i^n$ is the numerical approximation of the exact solution's mean value in i^{th} cell in time $t^n=n\Delta t$, $n=0,1,...,$; $\mathbf{f}_{i+1/2}^n$ represents the numerical approximation of flux on the cell edge $x_{i+1/2}$ in time t^n, and $\mathbf{g}_i^n$ is the numerical approximation of the source term in i^{th} cell in time t^n.

In the upwind schemes the numerical flux is evaluated as [10]:

$$\mathbf{f}_{i+1/2}^n = \frac{1}{2}\left(\mathbf{f}_i^n + \mathbf{f}_{i+1}^n\right) - \frac{1}{2}\mathbf{R}_{i+1/2}^n \left|\mathbf{\Lambda}_{i+1/2}^n\right| \mathbf{L}_{i+1/2}^n \mathbf{R}_{i+1/2}^{n^{-1}}\left(\mathbf{u}_{i+1}^n - \mathbf{u}_i^n\right). \tag{21}$$

Here $\mathbf{f}_i^n=\mathbf{f}(\mathbf{u}_i^n,x_i)$, $\mathbf{R}_{i+1/2}^n = \left[\mathbf{r}^{(1)},\ldots,\mathbf{r}^{(m)}\right]$ is the right eigenvector matrix and $\Lambda_{i+1/2}^n = \mathbf{diag}\left[\lambda^{(1)},\ldots,\lambda^{(p)}\right]$ is a diagonal eigenvalues matrix corresponding to some numerical approximation of the flux Jacobian matrix on the cell edge $x_{i+1/2}$ in time t^n and m denotes the number of equations in the conservation law system. In the case of the first order scheme matrix $\mathbf{L}_{i+1/2}^n$ is the identity matrix $\mathbf{I}$ and in the case of the flux limited scheme:

$$\mathbf{L}_{i+1/2}^n = \mathbf{I} - \operatorname{diag}\left(\phi\left(\theta^{(p)}\right)\left(1 - |\lambda^{(p)}|\frac{\Delta t}{\Delta x}\right)\right)_{i+1/2}^n. \tag{22}$$

Here φ is the flux limiter function which can take various forms [12], and van Albada, van Leer, Minmod and Superbee (Tb. 1) are common.

Table 1. Flux limiter functions

Tablica 1. Fluks limiter funkcije

Flux limiter/ Fluks limiter	$\varphi(\vartheta)$
Minmod	$\max(0,\min(1,\vartheta))$
Superbee	$\max(0,\min(2\vartheta,1),\min(\vartheta,2))$
van Leer	$(\lvert\vartheta\rvert+\vartheta)/(1+\vartheta)$
van Albada	$(\vartheta^2+\vartheta)/(1+\vartheta^2)$

The term $\theta^{(p)}$ is defined with the following expression:

$$\theta_{i+1/2}^{(p),n} = \frac{\left(\mathbf{u}_{i-\operatorname{sgn}\left(\lambda_{i+1/2}^{(p),n}\right)+1} - \mathbf{u}_{i-\operatorname{sgn}\left(\lambda_{i+1/2}^{(p),n}\right)}\right)\mathbf{l}_{i+1/2}^{(p),n}}{\left(\mathbf{u}_{i+1}^n - \mathbf{u}_i^n\right)\mathbf{l}_{i+1/2}^{(p),n}}, \tag{23}$$

where $\mathbf{l}_{i+1}^{(p)}$ are the left eigenvectors of the Jacobian matrix and $p=1,...,m$. The second order flux limited scheme is hereafter referred to as the *flux limited* scheme.

The described numerical schemes are designed to solve the standard form of hyperbolic balance laws defined in (8). In order to solve the non-conservative system defined in (18) by using numerical schemes in the described form, flux $\mathbf{f}$ and source term $\mathbf{g}$ are taken as defined by (12). However, some corrections of the numerical scheme due to the non-conservative product appearing in $\mathbf{g}_2$ must be done. These corrections are included in the numerical scheme through the Jacobian matrix $\tilde{\mathbf{A}}$, which is used in the numerical scheme instead of $\mathbf{A}$ [9]. This will affect appropriate evaluation of the propagation velocities and correctly perform the upwinding technique in the numerical model.

4. Numerical simulations of hydraulic transients in Rijeka HPP

The results of the derived numerical model were compared with measurements of hydraulic transients in Rijeka HPP for two different operating regimes.

The hydraulic system of Rijeka HPP consists of accumulation, pressurized conduit system, surge tank and two power generation units, each consisting of a Francis turbine with pressure regulating valve (PRV). Fig. 1. shows a schematic of the conduit profile and cross sections.

The accumulation has a maximum operating pool elevation of 229,50 m a. s. l. The conduit consists of a 3.117 m circular concrete headrace tunnel with lined and unlined reaches of different crosssection sizes 3,16 m in equivalent diameter, 140 m of reinforced concrete lined pipe 2,8 m in diameter, 42,94 m of steellined pipe 2,36 m in diameter and a 776 m long penstock reducing from 2,3 to 2,2 m.

The surge tank is an orifice surge tank with two horizontal galleries 30,7 m in length and is located downstream from the headrace tunnel.

Two Francis turbines and PRVs are located at the downstream end of the penstock. PRVs are provided to reduce maximum pressures during the transient state.

Details of the system and field tests for different operating conditions can be found in [14].

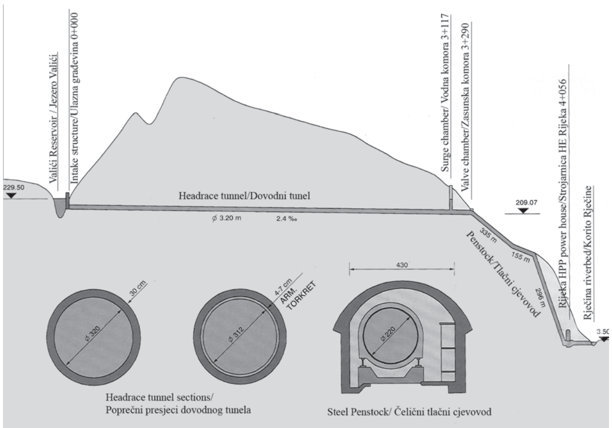

Figure 1. Longitudinal cross-section of Rijeka HPP conduit system

Slika 1. Uzdužni poprečni presjek sprovodnog aparata HE Rijeka

Table 2 presents the data used for the transformation of the Rijeka HPP conduit into a dynamically equivalent pipe system and its characteristic parameters.

Table 2. Rijeka HPP pipeline characteristics
Tablica 2. Karakteristike cjevovoda HE Rijeka

Pipeline / Cjevovod	***L*** **(m)**	D_e **(m)**	***c*** **(m/s)**	$\varepsilon=e/D$	f_s
Feed tunnel	3117	3,16	1280	$7{,}3\cdot10^{-4}$	0,0186
Concrete pipe	140	2,8	1220	$8{,}88\cdot10^{-4}$	0,0194
Steel pipe I	42,94	2,36	1320	$0{,}85\cdot10^{-4}$	0,0132
Steel pipe II	480	2,3	1340	$0{,}87\cdot10^{-4}$	0,0132
Steel pipe III	296	2,2	1340	$0{,}91\cdot10^{-4}$	0,0133

The accumulation and intake structure are represented by a reservoir boundary condition [4], with pressure drop coefficient calculated according to measurements of pressure drop at the upstream end of the headrace tunnel as presented in Figure 2.

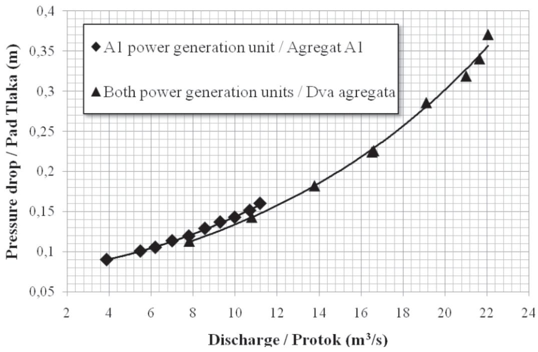

Figure 2. Pressure drop measurements at the headrace tunnel inlet

Slika 2. Mjerenja pada tlaka na ulazu u dovodni tunel

The surge tank is modeled as an orifice surge tank [18]. The orifice is 1.6 m in diameter, orifice coefficient is set at 0.85 m/(m^3/s)^2 and crosssection areas are presented in Fig. 3.

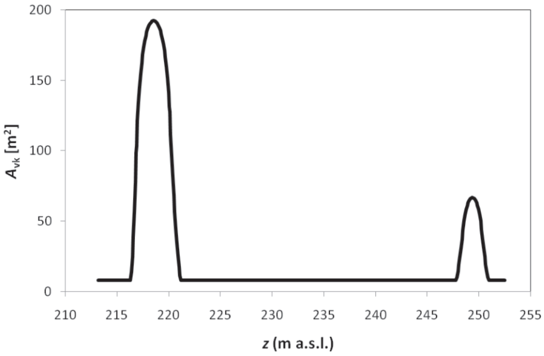

Figure 3. Surge tank cross-section area

Slika 3. Površine poprečnih presjeka vodne komore

Turbines and PRVs are modeled as single discharge boundary condition described in [18] due to the fact that the PRV characteristics were not measured, although the discharge was measured for a wide range of guide vane openings, as seen in Figure 4.

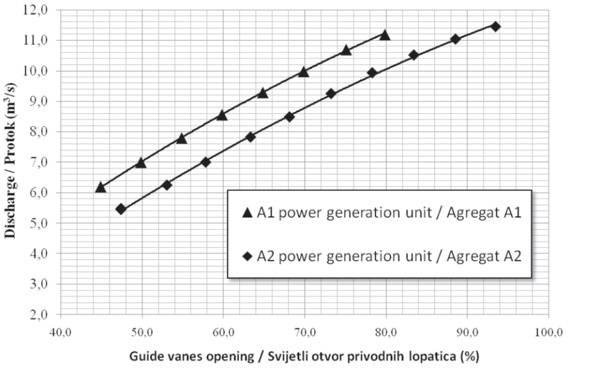

Figure 4. Measured discharge for various guide vane openings

Slika 4. Mjereni protok za različite otvore privodećih lopatica

All numerical simulations were conducted with non-conservative formulation of Allievi model and flux limited scheme with the choice of Minmod flux limiter, C_{cfl}=0,85 and Δx=10 m.

4.1. Numerical simulation of hydraulic transients for 0-75-0 % load of generator unit A1

First, the operating regime of 0-75-0 % load of generator unit is considered. According to measurements of the guide vanes and PRV opening for power generation unit A1[14], the discharge function is reconstructed with respect to discharge measurements (Figure 4) and presented in Fig. 5. Water in the hydraulic system was initially at rest with the initial water level of accumulation and surge tank at 225,91 m a.s.l. Opening of guide vanes and generator synchronization lasted 210 s, while the closing started at t=1024 s and lasted for 83,2 s.

Computational results of the transient pressure at the downstream end of the penstock and at the surge tank are compared with measured data and are presented in Figs. 6-7, which are compatible with measurements. Phase shift discrepancies, as well as prediction of maximum heads in the system can be attributed to inability to properly model PRV characteristics in moments of opening and closing of guide vanes.

Additionally, the results of standard formulation (9) and non-conservative formulation of Allievi model (12) are compared. As expected, the effect of unsteady friction is negligible for slowly varying transients, due to the immense influence of boundary conditions to the solution. However, results obtained by the non-conservative formulation correspond somewhat better with measured data than results obtained by the standard formulation when waterhammer head fluctuations are considered, as seen in Fig 8. Generally, the computational results of unsteady friction model (12) for unsteady laminar and low Reynolds number flows are compatible with measured data [3], [15], [17] and [19]. However, in case of unsteady high Reynolds number flows, computational results of unsteady friction model (12) are more appropriate than computational results obtained by the standard formulation of Allievi model (9).

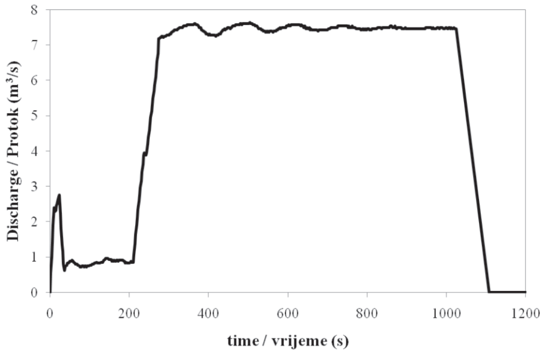

Figure 5. Discharge during hydraulic transients measurement of unit A1

Slika 5. Protok tijekom mjerenja hidrauličkih tranzijenata jedinice A1

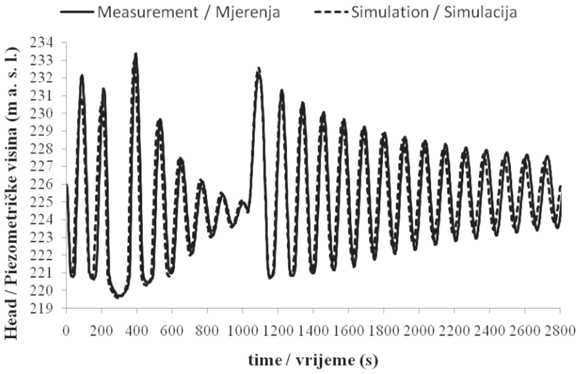

Figure 6. Comparison of computed and measured water level in surge tank for 0-75-0 % operating regime

Slika 6. Usporedba proračunatih i mjerenih razina vodnih lica u vodnoj komori za režim rada 0-75-0 %

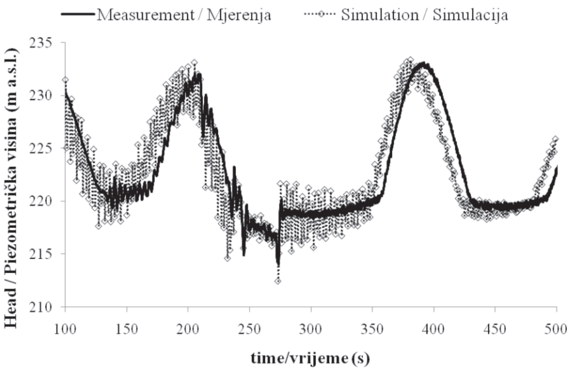

Figure 7. Comparison of computed and measured head traces at the downstream end of the penstock for 0-75-0 % operating regime

Slika 7. Usporedba proračunatih i mjerenih piezometričkih visina na nizvodnom kraju tlačnog cjevovoda za režim rada 0-75-0 %

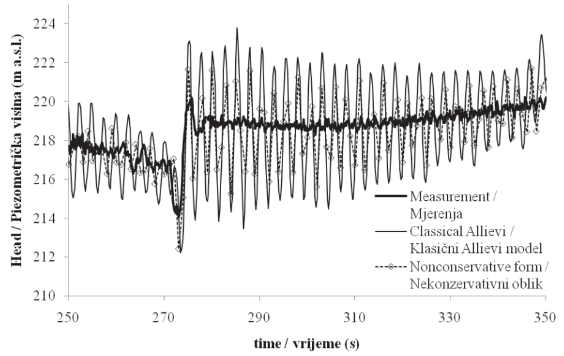

Figure 8. Comparison of different computed models and measured head traces at the downstream end of the penstock for 0-75-0 % operating regime

Slika 8. Usporedba različitih proračunatih modela i mjerenih piezometričkih visina na nizvodnom kraju tlačnog cjevovoda za režim rada 0-75-0 %

4.2. Numerical simulation of hydraulic transients for 55-0% load of both generator units

The second numerical test considers the operating regime of 55-0 % load of both generator units. Similarly, as in the first test, based on the measurements of the guide vanes and PRV opening for both power generation units [14], the discharge function is reconstructed and presented in Figure 9. Initial, a steady discharge was set to 13,86 m^3/s, and the accumulation water level was 224.41 m a.s.l.

The numerical results are compared with measured data and presented in Figures 10-11, The maximum water level obtained in the surge tank was estimated correctly, while the maximum pressure at the end of the penstock was underestimated by the numerical simulation. These differences may be explained with the same arguments as in the first test. Finally, the differences between measured and computed data for minimal surge tank head amplitudes during mass oscillations could be attributed to accuracy of the surge tank model, which was not based on measurements, but reconstructed from the project documentation.

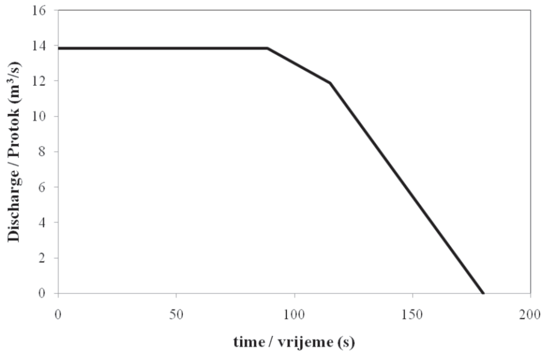

Figure 9. Discharge during hydraulic transients measurement of units A1 and A2

Slika 9. Protok tijekom mjerenja hidrauličkih tranzijenata jedinica A1 i A2

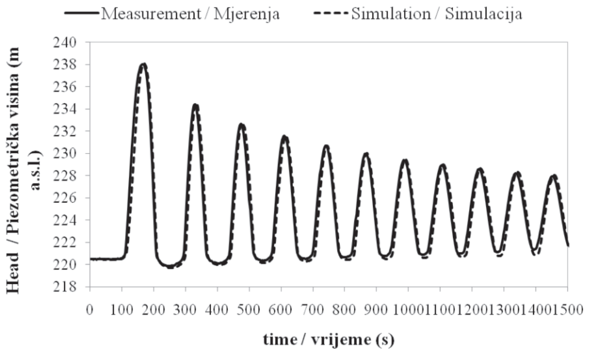

Figure 10. Comparison of computed and measured water level in surge tank for 55-0 % operating regime

Slika 10. Usporedba proračunatih i mjerenih razina vodnih lica u vodnoj komori za režim rada 55-0 %

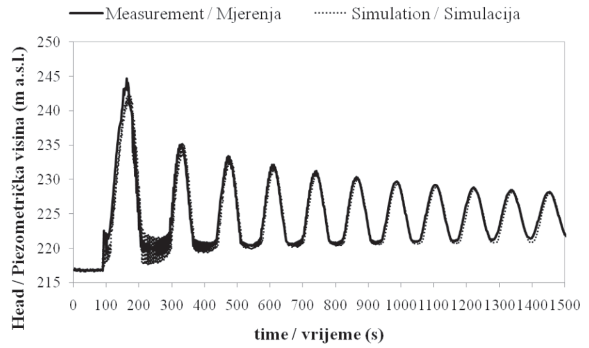

Figure 11. Comparison of computed and measured head traces at the downstream end of the penstock for 55-0 % operating regime

Slika 11. Usporedba proračunatih i mjerenih piezometričkih visina na nizvodnom kraju tlačnog cjevovoda za režim rada 55-0 %

5. Conclusion

In this work the one dimensional hydraulic transients model extended with MIAB formulation of unsteady friction model [17] is considered and used for numerical simulations of hydraulic transients in Rijeka HPP. In order to approximate correctly the considered model with the numerical scheme, the model is first written in a non-conservative form in order to appropriately include the non-conservative product appearing in an unsteady friction term. Then the second order flux limited scheme is used in numerical approximations.

The proposed computational scheme is applied to predict the transient behavior of the flow in the Rijeka HPP in Croatia. Detailed information on the experimental data for the surge tank characteristics, pipe material, diameter and roughness conditions were reported. Computational results with measured data at the downstream end of the penstock and at the surge tank are presented. In spite of approximations involved in the power generation units modeling, it is seen that the general patterns of the computed transient pressure histories correspond quite well with the measured prototype data, including prediction of pressure wave oscillations. It is worth noting that the effect of unsteady friction is negligible for a slow varying transients. However, the results obtained by the non-conservative formulation correspond somewhat better with measured data than results obtained by the classical Allievi model, when waterhammer head fluctuations are considered.

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