

TOČNIJI PRORAČUN MAGNETSKIH GUBITAKA U ASINKRONOM MOTORU A MORE ACCURATE CALCULATION OF MAGNETIC LOSSES IN THE INDUCTION MOTOR

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U radu je načinjena usporedba rezultata izračuna gubitaka vrtložnih struja u zubima asinkronog motora analitičkom metodom i metodom konačnih elemenata. Uz pretpostavke da su permeabilnost i vodljivost lima konstantni, mogu se ukupni gubici vrtložnih struja računati kao zbroj pulzacijskih i površinskih gubitaka. Dane su upute kako se mogu računati ukupni gubici u zubima za sve oblike zuba uzimajući u obzir potiskivanje toka u limovima, prigušenje toka u zubima, te faktore obrade lima i paketa limova. Vrednovanje analitičkog modela izračuna gubitaka načinjeno je i usporedbom s mjerenjima. The work presents a comparison of results of the calculation of eddy current losses in the induction motor teeth by virtue of the analytical method and the finite element method. Under the assumption that sheet permeability and conductivity are constant, total eddy current losses can be calculated as the aggregate of pulsation and surface losses. Instructions are given regarding the manner of calculation of total losses in the teeth for all teeth forms, taking into consideration the suppression of flows in the sheets, flow attenuation in the teeth and the sheet and sheet package processing factors. Valuation of the analytical model of loss calculation was also made by virtue of comparison with the measurements.

Ključne riječi: asinkroni motor; gubici histereze; gubici vrtložnih struja; metoda konačnih elemenata; površinski gubici; pulzacijski gubici
Key words: induction motor; finite element method; hysteresis losses; pulsation losses; surface losses



1 UVOD

Elektromotorni pogoni čine oko 70 % industrijskih trošila, pa se trend povećanja energetske korisnosti elektromotornih pogona, ne samo s ekonomskog već i s ekološkog stanovišta, posebno odrazio na proučavanje gubitaka asinkronih motora, koji su najzastupljeniji u tim pogonima. Bitnu komponentu koja utječe na energetske korisnost asinkronih motora predstavljaju gubici energije u magnetskim materijalima. Gubici u statorskim i rotorskim paketima limova dijele se na osnovne gubitke, koji su uglavnom vezani uz temeljni harmonik indukcije u zračnom rasporu (gubici magnetiziranja), i na dodatne gubitke, koje uzrokuju viši ili niži harmonici indukcije u zračnom rasporu [1] i [2]. S obzirom na mjesta nastajanja osnovne i dodatne gubitke dijelimo na površinske gubitke u zubima, pulzacijske gubitke u zubima, te pulzacijske gubitke u jarmovima. Komponenta površinskih gubitaka obično se zanemaruje pri proračunu osnovnih gubitaka. Polje u asinkronim motorima je rotacijsko, pa se ipak smatra da gubici u zubima nastaju zbog izmjeničnog polja, dok se samo za gubitke u jarmovima smatra da nastaju zbog rotacijskog polja.

Raspodjela magnetskog polja električnih strojeva na mnogim mjestima je potpuno dvodimenzionalna i iskrivljena, pa je uobičajeno takvu raspodjelu magnetskog polja opisivati kao izobličeno eliptičko rotacijsko magnetsko polje. Zbog takvog kompliciranog mehanizma magnetiziranja, određivanje gubitaka u statorskim i rotorskim paketima limova asinkronih motora vrlo je teško izvesti analitičkim izračunima, jer se moraju napraviti određena zanemarivanja. Kao prvo, zbog nelinearnosti krivulje prvog magnetiziranja, pojave histereze i utjecaja obrade limova i paketa limova, proračun gubitaka u statorskim i rotorskim paketima limova ne može se provesti egzaktno. Nadalje, proračun otežava i složena geometrija limova i pojava površinskog učinka (skin efekta), a zbog nemogućnosti da se istodobno uračunaju svi navedeni utjecaji za točan proračun, u ovom će se radu istražiti samo utjecaj geometrije limova na gubitke vrtložnih struja u zubima. Konačno, ostali se utjecaji mogu uzeti u obzir naknadno s odgovarajućim faktorima, izborima odgovarajućih podataka (permeabilnost) i eksperimentalnim podacima (gubici histereze, obrada lima) [1], [3] i [4].

Tijekom posljednjih trideset godina razvoj numeričkih tehnika omogućio je da se rješavaju realistični numerički modeli kojima se može objasniti puno širi spektar problema nego korištenjem analitičkih metoda, koje su u pravilu ograničene na homogenost, linearnost i statična polja. Iz tog je razloga, u ovom radu analitički proračun

1 INTRODUCTION

Electromotor drives make up about 70 % of industrial use and so the trend of increase of energetic efficiency of electromotor drives, not only from the economic but also from the environmental standpoint, has an especially significant effect on the study of induction motor losses which are most present in these drives. Energy losses in magnetic materials represent an important component which influences the energetic efficiency of induction motors. Losses in the stator and rotor sheet metal packages are divided into basic losses, which are mostly related to the fundamental induction harmonic in the air gap (magnetizing losses), and additional losses caused by higher or lower induction harmonics in the air gap [1] and [2]. Depending on the locations where fundamental and additional losses occur, these are divided into surface losses in the teeth and the pulsation losses in the yokes. The surface losses component is usually disregarded at the calculation of fundamental losses. The field in induction motors is rotational, so it is considered nevertheless that losses in the teeth occur due to the alternate field, while the losses in the yokes are considered to be caused by the rotational field.

The distribution of the magnetic field of electric machines is two-dimensional and distorted in many places, so it is common practice to describe such distribution of the magnetic field as a distorted elliptic rotational magnetic field. Because of such a complicated magnetizing mechanism, the determination of losses in the stator and rotor sheet metal packages is very difficult to derive by analytic calculations because certain disregarding needs to be done. Firstly, due to the non-linearity of the first magnetizing curve, the occurrence of the hysteresis phenomenon and the influence of the processing of sheets and sheet packages, the calculation of losses in the stator and rotor sheet metal packages cannot be undertaken exactly. Furthermore, the undertaking of the calculation is additionally made difficult by the geometry of sheet metal and occurrences of the skin effect, and, because of the impossibility of simultaneous inclusion of all the mentioned influences for an accurate calculation, only the influence of the geometry on eddy current losses in the teeth will be studied in this work. Finally, the other influences may be taken into consideration afterwards with the adequate factors, choices of adequate data (permeability) and experimental data (hysteresis losses, sheet processing) [1], [3] and [4].

Over the last thirty years, the development of numerical techniques has enabled the resolution of realistic numerical models which may be used to explain a much broader spectre of problems than by the use of analytical methods which are usually limited to homogeneity, linearity and static fields.

gubitaka vrtložnih struja u zubima asinkronog motora vrednovan usporedbom s numeričkim rezultatima dobivenim metodom konačnih elemenata.

U radu su izvedeni izrazi za analitički proračun gubitaka. Međutim, iz razloga što se u dostupnoj literaturi [1], [2] i [3], bez dokaza da je to dopušteno, neovisno računaju površinski i pulzacijski gubici, potrebno je bilo dokazati da se površinski i pulzacijski gubici mogu računati neovisno prema manje ili više složenim izrazima, pošto je polje u zubima jedinstveno.

2 RASPODJELA MAGNETSKOG POLJA U ZUBIMA

Pri analitičkom proračunu kao prvo pojednostavljenje uzima se homogena raspodjela permeabilnosti i vodljivosti u željezu paketa limova, a proračun se zatim započinje od jednostavnih oblika zuba (pravokutni oblik), radi izbjegavanja matematičkih teškoća koje se javljaju pri proračunu polja i gubitaka u zubima složenijih oblika.

Polje u zubu može se odrediti iz Laplaceove jednadžbe za skalarni magnetski potencijal:

$$\Delta\varphi = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = 0, \quad (1)$$

gdje je:

φ – skalarni magnetski potencijal,
 x, y – komponente (vektora).

Na slici 1a) je prikazan zub i stvarna raspodjela polja u zračnom rasporu za jedan utorski korak, dok je na slici 1b) prikazan pravokutni zub i koordinatni sustav, te su naznačeni rubni uvjeti. Laplaceova jednadžba za skalarni magnetski potencijal, u ovom se slučaju, može riješiti separacijom varijabli, tako da se pretpostavi da je rješenje produkt funkcija samo jedne varijable.

For that reason, the analytic calculation of eddy current losses in the teeth of the induction motor is assessed in this work only by virtue of comparison with the numerical results obtained by the finite element method.

The work derives expressions for the analytic calculation of losses. However, as in the available literature [1], [2] and [3], without proof of the admissibility thereof, surface and pulsation losses are calculated independently, it is necessary to prove that surface and pulsation losses may be calculated independently according to more or less complex expressions because the field in the teeth is unique.

2 DISTRIBUTION OF THE MAGNETIC FIELD IN THE TEETH

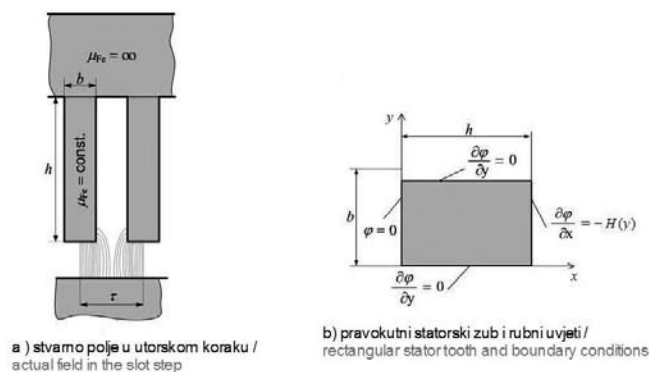
At analytical calculation, homogenous distribution of permeability and conductivity in the sheet metal packages is taken as the first simplification, and the calculation then starts from simple teeth forms (rectangular form) for the purpose of avoiding mathematical difficulties which arise at calculation of fields and losses in the complex-form teeth.

The field in the teeth can be determined from the Laplace equation for the scalar magnetic potential:

where it is as follows:

φ – scalar magnetic potential,
 x, y –(vector) components.

Figure 1a) shows the tooth and the actual distribution of the fields in the air gap for one slot step, while Figure 1b) shows the rectangular tooth and the coordinate system, as well as an indication of boundary conditions. The Laplace equation for the scalar magnetic potential, in this case, may be solved by a separation of the variables so as to assume that the solution is a product of functions of only one variable.



Slika 1 — Polje u zračnom rasporu i koordinatni sustavi
Figure 1 — The field in the air gap and the coordinate systems

Daljnji postupak određivanja vektora jakosti magnetskog polja kao rješenja Laplaceove jednačbe za skalarni magnetski potencijal detaljno je opisan u [5].

Iz definicije skalarnog magnetskog potencijala prema izrazu (2), dobiva se dvodimenzionalni vektor jakosti magnetskog polja (3), koji ima samo dvije komponente, jer je početna pretpostavka da se zanemaruje čeonni prostor stroja u proračunu gubitaka, što je u skladu sa 2D numeričkim proračunom magnetskog polja asinkronog motora korištenjem metode konačnih elemenata: Poznavanje struja statorskog namota i rotorskog

Further procedure for the determination of the magnetic field strength vector as a solution of the Laplace equation for the scalar magnetic potential is described in detail in [5].

The definition of the scalar magnetic potential according to the expression (2), a two-dimensional vector of magnetic field strength (3) is derived and has only two components because the initial assumption is that the frontal machine area is disregarded in the calculation of losses, which is in accordance with the 2D numerical calculation of the induction motor magnetic field by using the finite element method:

$$\mathbf{H} = -\nabla\varphi \quad (2)$$

$$\mathbf{H} = H_x + H_y = \mathbf{i} \cdot H_x + \mathbf{j} \cdot H_y \quad (3)$$

kaveza omogućava da se metodom konačnih elemenata proračuna raspodjela magnetskog polja u presjeku stroja. Ako se izračun načini za neopterećen motor, dovoljno je poznavati samo struju statorskog namota, na osnovi koje se onda mogu izračunati struje rotorskog kaveza. U slučaju opterećenog motora napajanog iz sinusoidalnog izvora sa skošenim rotorskim kavezom, skošenje se promatra odvojeno za proračun korištenjem metode konačnih elemenata, tj. rotor se segmentira na dijelove s neskošenim rotorom, koji su međusobno prostorno pomaknuti, te se kompletan rotorski kavez čini skošen [6].

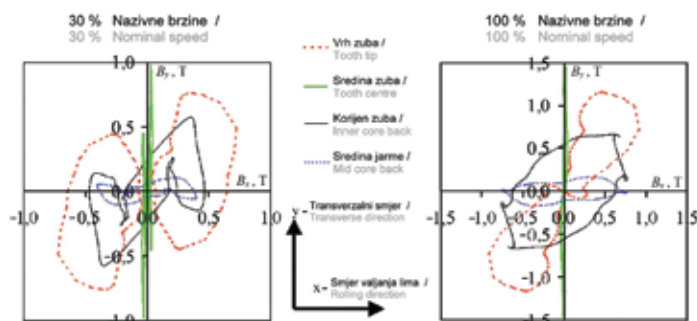
Izrazi za gubitke u željezu paketa limova neopterećenog motora korištenjem 3D modela magnetskog polja su izvedeni u [7]. Postupak određivanja gubitaka u željezu paketa limova statora i rotora asinkronog motora započinje rastavljanjem neodređenog oblika promjene vektora magnetske indukcije, korištenjem Fourierovog reda, na niz

Knowing the stator winding and rotor cage currents enables the calculation of the distribution of the magnetic field in the machine intersection by virtue of the finite element method. If the calculation is made for an unloaded motor, it suffices to know only the stator winding current based on which the rotor cage currents can be calculated. In the case of a loaded motor fed from the sinusoidal source with a skew rotor cage, the skewing is observed separately for the calculation by using the finite element method, that is, the rotor is divided in segments with non-skewed rotors, which are mutually spatially shifted, and so it all looks like a skew squirrel cage rotor [6].

Expressions for losses in the iron sheet metal package of the unloaded motor by using the 3D magnetic field model are derived in [7]. The procedure for the determination of the induction motor stator and rotor iron sheet metal package starts with the dissection of the undetermined change

eliptičkih harmonika vektora magnetske indukcije. Za svaki je harmonik određena komponenta magnetskog toka u smjeru glavne osi, poprečne osi, te je definiran omjer eliptičnosti. Ukupni su gubici zatim definirani kao zbroj gubitaka histereze (rotacijskih i izmjeničnih gubitaka), ukupnih gubitaka vrtložnih struja i dodatnih gubitaka. Takva složena definicija gubitaka zahtijeva vrlo složene postupke određivanja krivulje magnetiziranja lima u rotacijskom magnetskom polju što nije praktično u inženjerskoj praksi (slika 2).

type of the magnetic induction vector, by using the Fourier series, into a series of elliptic harmonics of the magnetic induction vectors. A magnetic flux component is determined for each harmonic in the direction of the main axis, the transverse axis, and the elliptic ratio is defined as well. Total losses are then defined as the aggregate of hysteresis losses (rotational and alternate losses), total eddy current losses and additional losses. Such a complex definition of losses demands very complex procedures for the determination of the sheet magnetizing curve in the rotational magnetic field which is not convenient in engineering practice (Figure 2).



Slika 2 – Prikaz deformiranog 2D valnog oblika raspodjele magnetske indukcije zbog slabljenja polja uzduž statorskog zuba pri promjeni brzine motora

Figure 2 – Overview of the deformed 2D waveform of the distribution of the magnetic induction due to fields weakening along the stator tooth at change in motor speed

3 ANALITIČKI MODEL IZRAČUNA GUBITAKA U ZUBIMA

3 ANALYTICAL MODEL FOR THE CALCULATION OF LOSSES IN THE TEETH

Za određivanje gubitaka u zubima paketa limova potrebno je prvo odrediti trenutni iznos gubitaka u jednom zubu jednog lima:

For the determination of the losses in the sheet package teeth, it is first of all necessary to determine the momentary amount of losses in one tooth of one sheet:

$$p_{gzil}(t) = \frac{1}{\kappa_{Fe}} \iiint_{V_{zi}} |J(x, y, z, t)|^2 dV \quad (4)$$

gdje je:

where it is as follows:

- p – trenutni iznos snage, W,
- J – vektor gustoće struje, A/m²,
- κ – vodljivost, S/m,
- V – volumen, m³,
- t – vrijeme, s,
- x, y, z – koordinate, m,
- Fe – željezo,
- g – gubici,
- z – zub,
- l – lim,
- μ – oznaka harmonika.

- p – momentary amount of power, W,
- J – current density vector, A/m²,
- κ – conductivity, S/m,
- V – volume, m³,
- t – time, s,
- x, y, z – coordinates, m,
- Fe – iron,
- g – losses,
- z – tooth,
- l – metal sheet,
- μ – harmonics symbol.

Ako se pretpostavi da je lim u x, y ravnini (debljina lima je u smjeru z osi), uz zanemarivanje utjecaja vrtložnih struja na raspodjelu indukcije u zubu [8], može se pretpostaviti da se y komponenta gustoće struje linearno mijenja s koordinatom z te naknadno uzeti u obzir efekt potiskivanja toka u limu. Također, može se pretpostaviti da je vektor \mathbf{J}_y proporcionalan s \mathbf{H}_x , ali vremenski pomaknut za 90° . Iz toga slijedi da za trenutnu vrijednost gubitaka u jednom zubu jednog lima vrijedi izraz (5), a srednja vrijednost tih gubitaka prikazana je izrazom (6):

$$p_{gzil}(t) = \frac{1}{\kappa_{Fe}} \iiint_{V_{zi}} |\mathbf{J}_x + \mathbf{J}_y|^2 dx dy dz, \quad (5)$$

$$P_{gzil} = \frac{\omega_\mu}{2\pi} \int_0^{\omega_\mu} p_{gzil} dt = \frac{1}{\kappa_{Fe}} \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \int_0^b \int_0^h \frac{\omega_\mu}{2\pi} \int_0^{\omega_\mu} (J_x^2 + J_y^2) dt dx dy dz, \quad (6)$$

Assuming that the metal sheet is in the x, y plain (sheet thickness is in the direction of the z axis), disregarding the impact of eddy currents on the distribution of the induction in the tooth [8], it can be assumed that the current density component y linearly changes with the z coordinate and the effect of flow suppression in the sheet metal can be taken into consideration subsequently. Moreover, it can be assumed that the \mathbf{J}_y vector is proportionate to \mathbf{H}_x , but temporally shifted by 90° . From this it ensues that for the momentary value of losses in one tooth of one sheet metal, the expression (5) applies and the average value of those losses is depicted by the expression (6):

gdje je:

P – srednja vrijednost trenutne vrijednosti snage, W,
 b – širina zuba, m,
 h – visina zuba, m,
 δ – debljina lima, m
 ω – kružna frekvencija, rad/s.

where it is as follows:

P – average value of the momentary power value, W,
 b – tooth width, m,
 h – tooth height, m,
 δ – metal sheet thickness, m,
 ω – circular frequency, rad/s.

Integriranjem prema (6) dobiva se konačan izraz za gubitke u jednom zubu jednog lima (7), gdje je faktor utjecaja otvora utora definiran izrazom (8), a koeficijent α_μ je definiran izrazom (9):

By integration according to (6), the final expression is gained for the losses in one tooth of one sheet (7), where the factor of the impact of the slot opening is defined by the expression (8), and the coefficient α_μ is defined by the expression (9):

$$P_{gzil} = \frac{\delta^3}{24\kappa_{Fe}} \left(\frac{\kappa_{Fe} B_\mu \omega_\mu}{k_{Fe}} \right)^2 \left\{ h \cdot b \left[\frac{\sin\left(\alpha_\mu \frac{\pi}{2}\right)}{\alpha_\mu \frac{\pi}{2}} \right]^2 + \frac{R \cdot b}{\mu} K(\mu) \right\}, \quad (7)$$

$$K(\mu) = \frac{8\alpha_\mu^3}{\pi^2} \left[\sum_{n=2,4,\dots}^{\infty} \frac{\sin^2\left(\alpha_\mu \frac{\pi}{2}\right)}{n(n^2 - \alpha_\mu^2)^2} \operatorname{th}\left(n\pi \frac{h}{b}\right) + \sum_{n=1,3,\dots}^{\infty} \frac{\cos^2\left(\alpha_\mu \frac{\pi}{2}\right)}{n(n^2 - \alpha_\mu^2)^2} \operatorname{th}\left(n\pi \frac{h}{b}\right) \right], \quad (8)$$

$$\alpha_\mu = \mu \frac{b}{R\pi}, \quad (9)$$

gdje je:

B – amplituda indukcije, T,

where it is as follows:

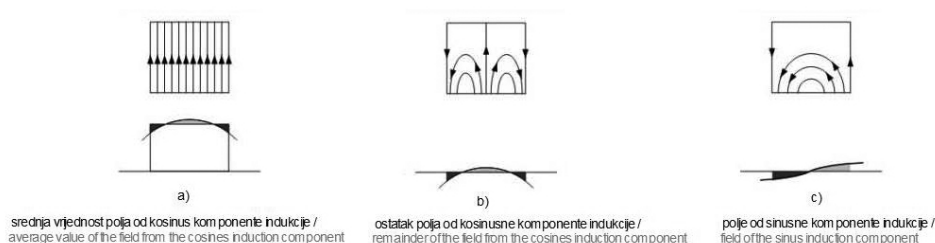
B – induction amplitude, T,

k_{Fe} – faktor punjenja željeza,
 R – polumjer provrta statorskog paketa limova, m,
 K – faktor utjecaja otvora utora na površinske gubitke,
 n – cijeli broj (redni broj harmonika).

k_{Fe} – iron filling factor,
 R – radius of the bore of the stator sheet metal package, m,
 K – factor of the slot opening impact on surface losses,
 n – full number (harmonics ordinal number).

Prvi član u vitičastoj zagradi u (7) potječe od homogenog polja prikazanog na slici 3a) koje je polazište je za proračun pulzacijskih gubitaka. Drugi član u vitičastoj zagradi u (7) potječe od polja prikazanog na slikama 3b) i 3c), koja su polazište za proračun površinskih gubitaka prema [2], [5] i [8].

The first term in convolute brackets in (7) originates from the homogenous field shown in Figure 3a) which is the starting point for the pulsation losses calculation. The second term in convolute brackets in (7) originates from the field shown in Figures 3a) and 3b) which are the starting point for the surface losses calculation according to [2], [5] and [8].



Slika 3 – Komponente magnetskog polja u zubu
 Figure 3 – The components of the magnetic field in the tooth

Ukupni gubici u zubima prema izrazu (10) dobiveni su množenjem srednje vrijednosti gubitaka u jednom zubu jednog lima s brojem zubi i omjerom duljine željeza prema debljini lima, pri čemu su pulzacijski gubici u zubima definirani izrazom (11), a površinski gubici izrazom (12):

Total losses in the teeth according to the expression (10) are obtained by multiplying the average value of losses in one tooth of one metal sheet with the number of teeth and iron length ratio according to sheet thickness, whereat the pulsation losses in the teeth are defined by the expression (11) and surface losses by the expression (12):

$$P_{gz\mu} = Q \frac{l_u}{\delta} P_{gz\mu l} = P_{gz\mu pul} + P_{gz\mu pov} \quad (10)$$

$$P_{gz\mu pul} = \frac{\kappa_{Fe}}{24 k_{Fe}} (\omega_{\mu} \delta)^2 \left[B_{\mu} \frac{\sin\left(\alpha_{\mu} \frac{\pi}{2}\right)}{\alpha_{\mu} \frac{\pi}{2}} \right]^2 Q l_e h b \quad (11)$$

$$P_{\mu pov} = \frac{\kappa_{Fe} k_C^2}{24 k_{Fe}} \left(\omega_{\mu} \beta \delta \right)^2 2\pi R_e l \frac{b R}{\tau \mu} K(\mu) \quad (12)$$

gdje je:

where it is as follows:

Q – broj zuba,
 l – duljina paketa limova, m,
 τ – utorski korak, m,
 pov – površinski,
 pul – pulzacijski,
 u – ukupna vrijednost,
 e – efektivna (idealna) vrijednost.

Q – number of teeth,
 l – metal sheet package length, m,
 τ – slot step, m,
 pov – surface,
 pul – pulsation,
 u – total value,
 e – effective (ideal) value.

Iz izvoda polja i gubitaka proizlazi da se za pravokutne zube, uz navedene pretpostavke $\mu_{Fe} = \text{konst.}$ i $k_{Fe} = \text{konst.}$, pulzacijski i površinski gubici vrtložnih struja mogu računati odvojeno.

Ta se spoznaja koristi i pri proračunima gubitaka u zubima složenijih oblika. Za određivanje konačnih izraza za gubitke potrebno je prvo načiniti korekcije, kojima se u obzir uzima stvarna raspodjela polja prema slici 1a). Izraz za proračun pulzacijskih gubitaka u pravokutnim zubima tada vrijedi izraz (13), gdje je k_C Carterov faktor za stator ili rotor, ovisno gdje se računaju gubici, a nadomjesni volumen zubi definiran je izrazom (14). Zbog vrlo složenog polja uz zračni raspored, za pravokutne zube i za složenije oblike zuba najbolje je površinske gubitke računati pojednostavljeno izrazom (15):

From the extraction of the fields and losses, the derivation is that for rectangular teeth, with the mentioned assumptions $\mu_{Fe} = \text{konst.}$ and $k_{Fe} = \text{konst.}$, pulsation and surface eddy current losses can be calculated separately.

That cognition is also used at calculation of losses in complex-form teeth. For the determination of final expressions for the losses, the first thing to do is to make corrections which take into consideration the actual distribution of the field according to Figure 1a). Then the expression (13) applies to the calculation of pulsation losses in rectangular teeth, and there k_C is the Carter factor for the stator or the rotor, depending on where the losses are calculated, and the equivalent number of teeth is defined by the expression (14). Because of the very complex field along the air gap, for rectangular teeth and more complex teeth forms, it is best to calculate surface losses in a simplified manner by the expression (15).

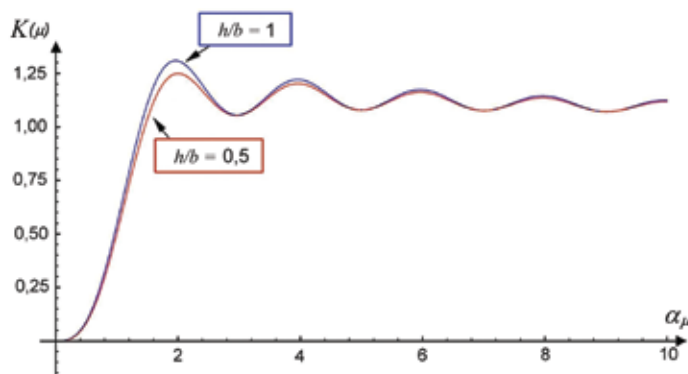
$$P_{gz\mu pul} = \frac{\kappa_{Fe}}{24 k_{Fe}} (\omega_{\mu} \delta)^2 \left[k_C B_{\mu} \frac{\sin\left(\frac{\tau}{k_C b} \alpha_{\mu} \frac{\pi}{2}\right)}{\alpha_{\mu} \frac{\pi}{2}} \right]^2 V_e, \quad (13)$$

$$V_e = Q l_c h b, \quad (14)$$

$$P_{gz\mu pov} = \frac{\kappa_{Fe} k_C^2}{24 k_{Fe}} (\omega_{\mu} B_{\mu} \delta)^2 2\pi R l_c \frac{b R}{\tau \mu} K(\mu). \quad (15)$$

Faktor utjecaja otvora utora $K(\mu)$ prema (8) prikazan je na slici 4, općenitiji je od odgovarajućeg faktora u [2], koji je identičan izrazu koji se dobiva iz (8) ako se u uzme da je $\text{th}(n\pi h/b) = 1$, što je u pravilu dopušteno zbog toga što omjer $h/b > 1$ vrijedi za obrađivanu grupu motora.

The slot opening impact factor $K(\mu)$ according to (8) is shown in Figure 4; it is more general than the equivalent factor in [2], which in turn is identical to the expression obtained from (8) if $\text{th}(n\pi h/b) = 1$ is implied, which is generally allowed because the ratio $h/b > 1$ applies to the studied motor group.



Slika 4 – Faktor utjecaja otvora utora na površinske gubitke
Figure 4 – Factor of slot opening impact on surface losses

Formule za proračun pulzacijskih i površinskih gubitaka vrtložnih struja (13) i (15) potrebno je korigirati utjecajem potiskivanja toka u limovima, utjecajem prigušenja toka u zubima, te utjecajem obrade lima i paketa limova. Da bi se dobili ukupni gubici u zubima treba dodati pojednostavljeno određene gubitke histereze [1] i [8].

Pri proračunu pulzacijskih gubitaka za korekciju utjecaja potiskivanja toka u limovima [3], treba za korigiranu širinu zuba [8] odrediti indukciju, a iz nje odgovarajuću relativnu permeabilnost. Korigirane vrijednosti permeabilnosti za proračun gubitaka magnetiziranja određuju se iz krivulje prvog magnetiziranja, za proračun dodatnih gubitaka uzrokovanih višim harmonicima polja u zračnom rasporu treba uzeti inkrementalnu ili diferencijalnu permeabilnost [3], dok je pri proračunima površinskih gubitaka za određivanje permeabilnosti mjerodavna širina zuba uz zračni raspor (kruna zuba).

Faktor prigušenja toka u zubima koristi se samo pri proračunu pulzacijskih gubitaka, te je za gubitke u zubima neskošenih rotora određen u [3]. Najteže je odrediti faktore korekcije zbog obrade lima i paketa limova, jer oni osim o svojstvima lima ovise o rezu lima i tehnologiji izrade paketa. Za određivanje tih faktora potrebno je raspolagati s mjerenjima utjecaja vrste obrade (tokarenje, brušenje) na probama lima [4], te poznavati tehnologiju izrade paketa limova. Na kraju je ipak potrebno, unatrag, iz mjerenja gubitaka (osnovnih i dodatnih) i proračuna po teorijskim formulama za gubitke, korigirati faktore određene iz mjerenja na probama lima.

4 NUMERIČKI POSTUPAK ODREĐIVANJA GUBITAKA U ZUBIMA

Za numerički proračun gubitaka u zubima statora i rotora asinkronog kaveznog motora korištena je metoda konačnih elemenata i iterativni proračun. Tako dobiveni rezultati uspoređeni su s rezultatima analitičkog proračuna gubitaka kako bi se odredio utjecaj pojednostavljenja u analitičkom modelu za određivanje gubitaka u zubima. Numerički proračun polja metodom konačnih elemenata temelji se na Maxwellovim jednadžbama, koje sačinjavaju temeljne zakone kojima se podvrgavaju sve električne i magnetske pojave. Iako se Maxwellove jednadžbe primjenjuju na cijeli prostor, praktični problemi su konačnih dimenzija, pa su rješenja Maxwellovih jednadžbi unutar tih područja povezana s

Formulas for the calculation of pulsation and surface eddy current losses (13) and (15) need to be corrected by the impact of the suppression of fluxes in the metal sheets, the impact of flux losses in the teeth and the impact of metal sheet and metal sheet package processing. In order to get total losses in the teeth, the hysteresis losses determined in a simplified manner need to be added [1] and [8].

At calculation of pulsation losses for the correction of impacts of flux suppressions in the sheets [3], for the corrected teeth width [8], induction needs to be defined and from this, the adequate relative permeability. Corrected permeability values for magnetizing loss calculation are determined from the first magnetizing curve; the calculation of additional losses caused by higher harmonics of fields in the air gap require the incremental or differential permeability, while teeth width along the air gap (tooth crown) is relevant for the calculation of surface losses for the determination of permeability.

The teeth flux attenuation factor is used only at calculation of pulsation losses and for the losses in the teeth of non-skewed rotors, it is determined in [3]. The hardest part is to determine the correction factors due to sheet and sheet packages processing because they depend not only on the sheet cutting but also on the package fabrication technology. The determination of those factors requires the availability of measurements of the impacts of types of processing (turning, grinding) on sheet tests [4], and knowing the sheet package fabrication technology. In the end, it is necessary, nevertheless, to use the measurements of the losses (fundamental and additional) and calculations according to theoretical loss formulas to make backward corrections of factors determined from the measurements on sheet tests.

4 NUMERICAL PROCEDURE FOR THE DETERMINATION OF THE LOSSES IN THE TEETH

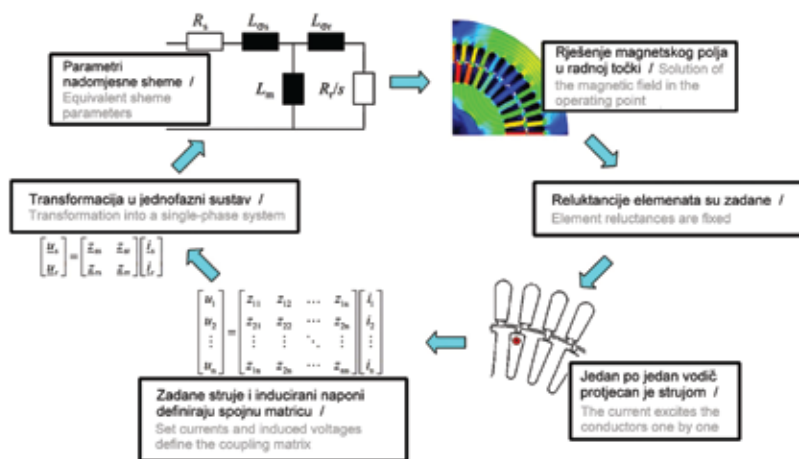
The finite element method and the iterative calculation were used for the numerical calculation of the losses in the teeth of the stator and rotor of the induction squirrel cage motor. The results gained in such a manner are compared with the results of the analytical calculation of losses in order to determine the impact of the simplification in the analytical model for the determination of losses in the teeth. The numerical calculation of fields by virtue of the finite element method is based on Maxwell's equations which constitute the basic laws to which all electrical and magnetic occurrences are subjected. Although Maxwell's equations apply to the entire area, practical problems are of final dimensions and so the solutions of Maxwell's equations within those areas are

okolnim prostorom korištenjem rubnih uvjeta i početnih vrijednosti.

Postupak definiranja mreže konačnih elemenata dijeli se na odvojeno mreženje dva prostora. Prvi je prostor od središta osovine do polovice zračnog raspora (rotorski prostor), a drugi od sredine zračnog raspora prema van (statorski prostor). Pri tome se dodirna kontura oba prostora dijeli na jednak broj dijelova (inkrementa), koji mora biti višekratnik broja utora statora i rotora. Iterativni postupak proračuna odnosi se na zakretanje rotorskog prostora za inkrement, te strujni izračun nadomjesne sheme asinkronog motora čiji parametri se računaju iz proračuna polja metodom konačnih elemenata (slika 5). Rezultat strujnog proračuna je raspodjela struja koja je potrebna za sljedeći korak proračuna polja metodom konačnih elemenata. Ovisno o brzini rotora u postupku proračuna (kratki spoj, zalet, nazivna radna točka, prazni hod) mijenja se trajanje zakreta rotorskog prostora za inkrement, što ulazi u proračun parametara nadomjesne sheme za strujni proračun.

connected to the surrounding area by using boundary conditions and initial values.

The procedure for the definition of the finite element network is divided into separate networking of two spaces. The first area is from the middle of the axis to the half of the air gap (rotor space) and the other is from the middle of the air gap outwards (stator space). Thereat, the boundary contour of both spaces is divided into the same number of parts (increments) which must be the multiple of the number of stator and rotor slots. The iterative calculation procedure relates to the rotation of the rotor space for the increment and the current calculation of the induction motor equivalent scheme with parameters that are calculated from the calculation of the field by virtue of the finite element method (Figure 5). The result of the current calculation is the distribution of currents necessary for the next step of the calculation by virtue of the finite element method. Depending on the rotor speed in the calculation process (short circuit, start up, nominal operating point, idle running), the duration of the rotation of the rotor space for the increment changes, and this is put into the calculation of the parameters of the equivalent scheme for the current calculation.



Slika 5 — Iterativni postupak za numerički proračun električnih parametara asinkronog motora na osnovi izračuna magnetskog polja metodom konačnih elemenata
Figure 5 — Iterative procedure for the numerical calculation of the asynchronous motor parameters based on the calculation of magnetic fields using the finite element method

5 VREDNOVANJE ANALITIČKOG MODELA IZRAČUNA GUBITAKA PRAZNOG HODA USPOREDBOM S REZULTATIMA MJERENJA

Za analizu modela proračuna gubitaka praznog hoda i njegovu usporedbu s mjerenjima, odabrana su dva asinkrona kavezna motora iz proizvodnog

5 VALUATION OF THE ANALYTICAL MODEL FOR IDLE RUNNING LOSS CALCULATION BY COMPARISON WITH THE MEASUREMENTS RESULTS

For the analysis of the model for the calculation of the idle running losses and its comparison with the measurements, two induction squirrel cage

programa poduzeća KONČAR. Podaci tih motora prikazani su u tablici 1.

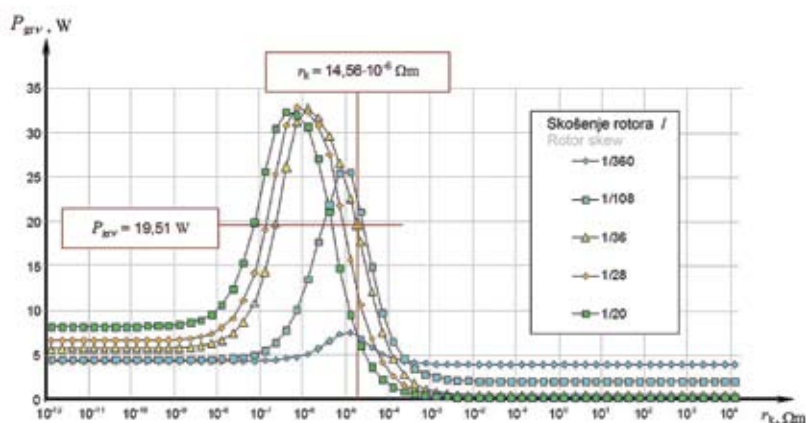
motors have been selected from the KONČAR company programme. Data on those motors are shown in Table 1.

Tablica 1 – Podaci motora na kojima su mjereni gubici praznog hoda
Table 1 – Data on motors on which idle running losses were measured

	MOTOR 1	MOTOR 2
Nazivna snaga / Nominal power, kW	7,5	11
Nazivni napon / Nominal voltage, V	380	380
Nazivna struja / Nominal current, A	15,2	22,7
Nazivni brzina vrtnje / Nominal rotation speed, mm^{-1}	1450	965
Faktor snage / Power factor	0,86	0,84
Korisnost / Efficiency	0,87	0,88
Broj pari polova / Number of pole pairs	2	3
Broj utora statora / Number of stator slots	36	54
Broj utora rotora / Number of rotor slots	28	40
Skošenje štapova kaveza rotora / Rotor cage bar skewing	1/36	1/54

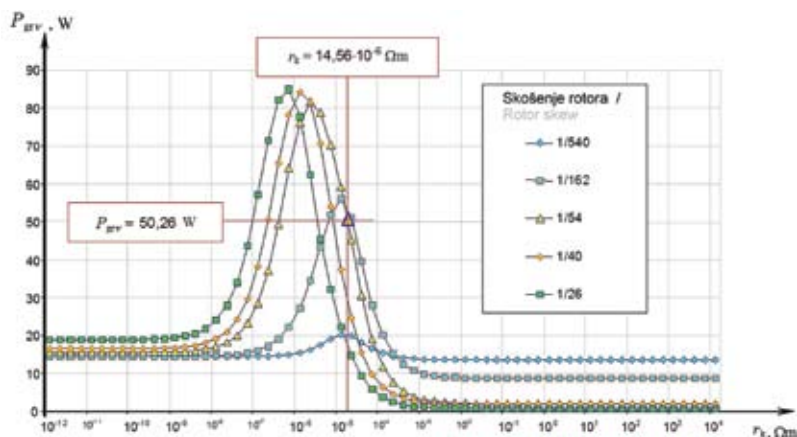
Postupak analitičkog proračuna počinje od proračuna struje magnetiziranja, a zatim se preko analitički određene vrijednosti nadomjesnog otpora željeza statorskog paketa limova dobiva vrijednost osnovnih gubitaka u statorskom paketu limova. Pri proračunu dodatnih gubitaka pored primarne u proračun je uzeta i sekundarna reakcija armature [8]. Primarna reakcija armature uključuje proračun gubitaka poprečnih struja (P_{grv}). Konačne vrijednosti kontaktnih otpora između štapova kaveza i paketa limova rotora dovede do povećanja dodatnih gubitaka. Izmjerena vrijednost kontaktnih otpora [8] za oba motora je $r_k = 14,56 \cdot 10^{-6} \Omega\text{m}$. Ovisnost gubitaka poprečnih struja o vrijednostima kontaktnog otpora uz različite vrijednosti skošenja za oba motora prikazana je dijagramima na slikama 6 i 7.

The analytical calculation procedure starts with the calculation of the magnetizing current and then, through the analytically determined value of the equivalent resistance of the stator metal sheet package the amount of the basic losses in the stator sheet metal package is obtained. At calculation of additional losses, besides the primary reaction, the secondary armature reaction is also taken into the calculation [8]. The primary armature reaction includes the calculation of the transverse current losses (P_{grv}). Final values of contact resistances between the squirrel cage bars and the rotor sheet metal packages give rise to an increase of additional losses. The measured value of contact resistances [8] for both motors is $r_k = 14,56 \cdot 10^{-6} \Omega\text{m}$. The dependency of losses of transverse current on the values of the contact resistance with different skew values for both motors is shown in diagrams in Figures 6 and 7.



Slika 6 — Ovisnost gubitaka poprečnih struja o vrijednostima kontaktnog otpora uz različite vrijednosti skošenja rotora za motor 1

Figure 6 — The dependency of transverse current losses on the values of the contact resistance with different values of the rotor skew for motor 1



Slika 7 – Ovisnost gubitaka poprečnih struja o vrijednostima kontaktnog otpora uz različite vrijednosti skošenja rotora za motor 2

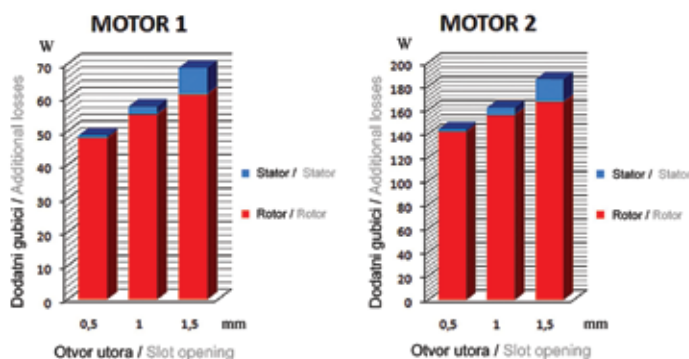
Figure 7 – The dependency of transverse current losses on the values of the contact resistance with different values of the rotor skew for motor 2

Uz gubitke poprečnih struja u proračunu su još uzeti u obzir i pulzacijski i površinski gubici. Pri proračunu dodatnih gubitaka u statorskom namotu proračun indukcija i parametara statorskog namota različit je za pojedine harmonike (temeljni harmonik, harmonici čiji je broj pari polova višekratnik broja tri i ostali harmonici).

Iz prethodnih se poglavlja vidi da dodatni gubici, pored ostalog, ovise i o širini otvora utora na rotoru. Na slici 8 prikazana je ovisnost gubitaka o širini otvora utora, gdje je vidljivo da povećanje širine otvora utora dovodi do povećanja gubitaka zbog poprečnih struja, tj. do povećanja dodatnih gubitaka u rotoru. Također, uz povećanje gubitaka zbog poprečnih struja dolazi i do povećanja pulzacijskih i površinskih gubitaka i u rotoru i u statoru, tj. do povećanja dodatnih gubitaka u statoru.

Besides the transverse current losses the calculation also took into consideration the pulsation and surface losses. At calculation of additional losses in the stator winding, the calculation of the inductions and parameters of the stator winding is different for certain harmonics (basic harmonic, harmonics with the number of pole pairs being the multiple of number three and other harmonics).

It is evident from the previous chapters that additional losses, inter alia, also depend on the rotor slot opening width. Figure 8 shows the dependency of losses on the slot opening width, where it is evident that the increase of the slot opening width brings about an increase of losses due to the transverse currents, that is, to the increase of additional rotor losses. Moreover, with the increase of losses due to transverse currents, an increase of pulsation and surface losses also occurs in the rotor and in the stator, that is, an increase of additional losses in the stator.



Slika 8 – Prikaz raspodjele dodatnih gubitaka u rotoru i statoru motora s obzirom na promjenu širine otvora utora

Figure 8 – Overview of the distribution of the losses in the motor rotor and stator considering the change of the slot opening width

Zbrajanjem dodatnih gubitaka u rotoru i statoru dobivaju se dodatni gubici u praznom hodu. Usporedba rezultata analitičkog proračuna i izmjenjenih vrijednosti za oba motora dana je u tablici 2.

By adding up additional losses in the rotor and the stator, additional idle running losses are gained. Comparison of the results of the analytical calculation and the measured values for both motors are given in Table 2.

Tablica 2 – Rezultati analitičkog proračuna gubitaka praznog hoda i njihova usporedba s mjerenim rezultatima
Table 2 – The results of the analytical calculation of the idle running losses and their comparison with the measured results

Veličina / Size	MOTOR 1			MOTOR 2		
	Izračunate vrijednosti / Calculated values, W	Izmjerene vrijednosti / Measured values, W	Relativna pogreška / Relative error, %	Izračunate vrijednosti / Calculated values, W	Izmjerene vrijednosti / Measured values, W	Relativna pogreška / Relative error, %
Osnovni gubici u željezu / Basic losses in the iron P_{Fe10}	124,3	138	-9,9	262,6	212	23,6
Dodatni gubici / Additional losses P_{dod0}	57,5	58	-0,9	162,0	132	22,7
Ukupni gubici praznog hoda / Total idle running losses P_0	317,8	332	-4,3	570,6	503,4	13,3

Razlika između proračuna i mjerenja u prvom je redu uvjetovana ispravnosti ulaznih podataka motora s kojima se ulazi u proračun. To se prvenstveno odnosi na kvalitetu lima i na stvarnu veličinu zračnog raspora, što bitno utječe na iznos gubitaka praznog hoda. Pritom pogreška u veličini zračnog raspora u iznosu od desetinke milimetra može dovesti do promjene konačne vrijednosti dodatnih gubitaka u iznosu od desetak wata. Razmatrani proračun gubitaka u praznom hodu ukazuje da:

- dodatni gubici mogu biti znatan dio ukupnih gubitaka praznog hoda,
- pretežiti dio gubitaka može nastati u statorskom i rotorskom paketu limova.

Dodatni gubici koje uzrokuju raspored namota u utorima i otvori utora, utječu na dodatno zagrijavanje motora, te smanjuju korisnost.

The difference between the calculation and the measurement is conditioned in the first place by the correctness of those input data on the motor which enter the calculation. This is primarily related to the sheet quality and the actual air gap size which significantly affects the amount of idle running losses. Thereat the error in the size of the air gap in the amount of one tenth of a millimetre may bring about a change of the final value of additional losses in the amount of some ten watts. The studied calculation of idle running losses points to the following:

- additional losses may be a significant part of the total idle running losses,
- most of the losses may occur in the stator and rotor metal sheet packages.

Additional losses caused by the spacing of the windings in the slots and the slot openings affect the additional motor heating and decrease the efficiency.

6 VREDNOVANJE ANALITIČKOG MODELA IZRAČUNA GUBITAKA VRTLOŽNIH STRUJA USPOREDBOM S REZULTATIMA NUMERIČKOG PRORAČUNA

Za vrednovanje analitičkog modela proračuna gubitaka vrtložnih struja u zubima asinkronog motora s numeričkim rezultatima dobivenih metodom konačnih elemenata odabran je šestopolni asinkroni kavezni motor (KONČAR): 30 kW, 380 V, 973 o/min.

6 VALUATION OF THE ANALYTICAL MODEL FOR EDDY CURRENT LOSS CALCULATION BY COMPARISON WITH THE RESULTS OF THE NUMERICAL CALCULATION

A six-pole induction squirrel cage motor (KONČAR) was selected for the valuation of the analytical model for the calculation of eddy current losses in the induction motor teeth with the numerical result obtained by virtue of the finite element method: 30 kW, 380 V, 973 o/min.

Pored osnovnih gubitaka u statorskom paketu limova potrebno je odrediti i dodatne gubitke u rotorskom kavezu i statorskom namotu (primarna i sekundarna reakcija armature [8]), te dodatne gubitke u rotorskom i statorskom paketu limova uzrokovanih harmonicima protjecanja statorskog namota i kaveza. Pri određivanju iznosa harmonika indukcije potrebno je uzeti u obzir nazubljenje statorskog i rotorskog paketa limova. U proračunu gubitaka u željezu za faktore obrade lima i paketa limova uzete su sljedeće vrijednosti:

- 1,6 – za proračun osnovnih gubitaka u statorskom paketu limova,
- 2,2 – za proračun pulzacijskih dodatnih gubitaka,
- 2,2 – za proračun površinskih gubitaka na statoru i
- 3,8 – za proračun površinskih gubitaka na rotoru [8].

Iz definicije protjecanja za m_s fazni namot, slijedi da u asinkronom motoru pored osnovnog harmonika nastaje i sustav viših harmonika koji se dijeli na direktni i inverzni sustav harmonika [1] i [9], a koji se može definirati izrazom:

$$v = (2g_s m_s + 1)p, \quad g_s = 0, \pm 1, \pm 2, \dots \quad (16)$$

Zbog reakcije armature za svaki harmonik statorskog protjecanja slijedi niz harmonika rotorskog protjecanja [8]:

$$\mu = g_r Q_r + v, \quad g_r = 0, \pm 1, \pm 2, \dots \quad (17)$$

Na temelju (16) i (17) za promatrani trofazni asinkroni motor s tri para polova dobiva se niz harmonika prikazanih u tablici 3.

Besides the basic losses in the stator sheet metal package, it is also necessary to determine additional losses in the rotor cage and the stator winding (primary and secondary armature reaction [8]), and additional losses in the rotor and stator metal sheet package caused by the magnetic flux harmonics of the stator winding and the squirrel cage. At determination of the amounts of the induction harmonics, it is necessary to take into consideration the serration of the stator and rotor sheet metal package as well. The following values for the sheet and sheet package processing factors were taken for the calculation of the losses in the iron:

- 1,6 – for the calculation of the basic losses in the stator sheet package,
- 2,2 – for the calculation of the additional pulsation losses,
- 2,2 – for the calculation of surface losses on the stator and
- 3,8 – for the calculation of surface losses on the rotor [8].

From the magnetic flux definition for the m_s phase winding, it ensues that, in the induction motor, besides the basic harmonic, also occurs the system of higher harmonics and is divided into the direct and the inverse harmonics system [1] and [9] which can be defined by the expression:

Due to the armature reaction, each stator magnetic flux harmonic is followed by a series of rotor magnetic flux harmonics [8]:

Based on (16) and (17), a series of harmonics shown in Table 3 is gained for the studied three-phase induction motor with three pole pairs.

Tablica 3 – Sustav nadharmonika magnetskog polja statora (v) i rotora (μ), za motor s tri para polova ($p = 3$)
Table 3 – System of higher stator (v) and rotor (μ) magnetic field harmonics for a three-pole pairs motor ($p = 3$)

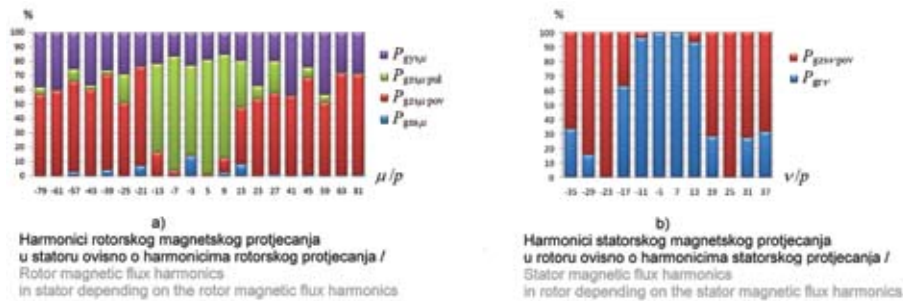
v/p	μ/p			
1	-21	23	-43	45
-17	-39	5	-61	27
19	-3	41	-25	63
-35	-57	-13	-79	9
37	15	59	-7	81

Nadalje, prema izrazima za gubitke iz trećeg poglavlja [5] za odabrani su motor izračunati dodatni gubici u statoru i rotoru, te su prikazani u slici 9. Pri tome su prikazani sljedeći gubici:

- $P_{gys\mu}$ – gubici u jarmu statora,
- $P_{gzs\mu\text{ pul}}$ – pulzacijski gubici u zubima statora,
- $P_{gzs\mu\text{ pov}}$ – površinski gubici u zubima statora,
- $P_{gns\mu}$ – gubici u statorskom namotu,
- $P_{gzs\text{ v pov}}$ – površinski gubici u zubima rotora,
- $P_{gr\text{ v}}$ – gubici u rotorskom kavezu.

Furthermore, according to the expression for the losses from the third chapter [5], for the selected motor, additional stator and rotor losses were chosen and shown in Figure 9. Thereat the following losses are shown:

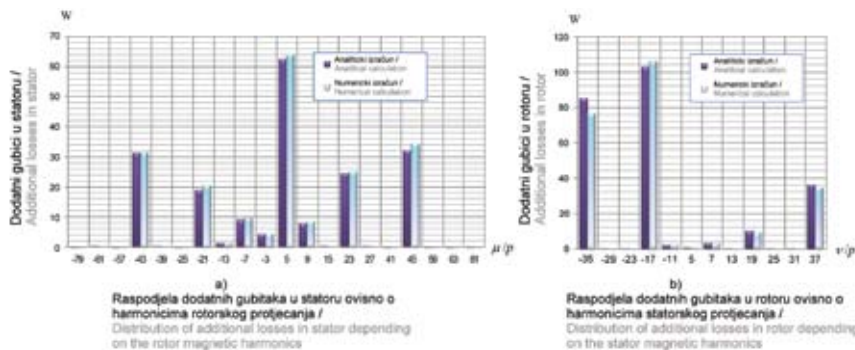
- $P_{gys\mu}$ – losses in the stator yoke,
- $P_{gzs\mu\text{ pul}}$ – pulsation losses in stator teeth,
- $P_{gzs\mu\text{ pov}}$ – surface losses in stator teeth,
- $P_{gns\mu}$ – stator winding losses,
- $P_{gzs\text{ v pov}}$ – surface losses in rotor teeth,
- $P_{gr\text{ v}}$ – losses in the rotor cage.



Slika 9 – Prikaz udjela pojedinih gubitaka u ukupnim dodatnim gubicima
Figure 9 – Overview of the share of certain losses in the total additional losses:

Numeričkim proračunom polja metodom konačnih elemenata i iterativnim postupkom opisanim u 4. poglavlju dobiva se raspodjela polja u motoru kao funkcija vremena, te se razvojem u Fourierov red izračunato polje rastavlja na komponente, prema harmonicima koji su prikazani u tablici 3. Usporedba ukupnih dodatnih gubitaka u statoru dobivenih analitičkim izračunom s vrijednostima dobivenih numeričkim postupkom za pojedine harmonike rotorskog protjecanja dana je na slici 10a), a usporedba ukupnih dodatnih gubitaka u rotoru dobivenih analitičkim izračunom s vrijednostima dobivenih numeričkim postupkom za pojedine harmonike statorskog protjecanja dana je na slici 10b).

The numerical calculation of the field by virtue of the finite element method and the iterative procedure described in Chapter 4, a distribution of the fields in the motor is obtained as a function of time and the field, calculated by elaboration into the Fourier series, is analysed into components, according to the harmonics shown in Table 3. The comparison of total additional losses in the stator gained by analytical calculation with the values gained by virtue of the numerical procedure for certain rotor flux harmonics is given in Figure 10a), and the comparison of total additional losses in the rotor gained by virtue of the analytical calculation with the values gained by virtue of the numerical procedure for certain stator flux harmonics is given in Figure 10b).



Slika 10 – Usporedba dodatnih gubitaka dobivenih analitičkim izračunom s vrijednostima dobivenih numeričkim postupkom za pojedine harmonike protjecanja
Figure 10 – Comparison of additional losses gained by analytical calculation with the values gained by virtue of the numerical procedure for certain flux harmonics

7 ZAKLJUČAK

Utjecaj geometrije lima na gubitke vrtložnih struja u pravokutnim zubima gotovo su egzaktno određeni iz rezultantnog polja u zubima, koje uzrokuje harmonik magnetske indukcije u zračnom rasporu. Izvod je načinjen uz pretpostavke $\mu_{Fe} = \text{konst.}$ i $\kappa_{Fe} = \text{konst.}$, iz čega je izvedeno da se ukupni gubici vrtložnih struja u pravokutnim zubima mogu razdvojiti na pulzacijske i površinske gubitke, što je i do sada rađeno u poznatoj literaturi [10] i [11], ali bez dokaza.

Na kraju analitičkog modela izračuna dane su upute kako se za zube svih oblika mogu odrediti ukupni gubici u zubima. Za to je potrebno uzeti u obzir histerezne gubitke u limovima, te korekcije zbog potiskivanja toka u limovima, prigušenja toka u zubima za pulzacijske gubitke i korekcije zbog obrade lima i paketa limova.

Vrednovanje analitičkog modela izračuna potvrđeno je usporedbom s mjerenjima gubitaka u praznom hodu kaveznih asinkronih motora. Za dva motora snage 11 kW i 7,5 kW, izračunati su gubici u praznom hodu, te uspoređeni s odgovarajućim mjerenjima. Pokazano je da se izračunati ukupni gubici praznoga hoda dobro slažu s mjerenjima, a odstupanja se mogu objasniti nedostatnim (ulaznim) podacima. To se u prvom redu odnosi na stvarne gubitke magnetiziranja ugrađenoga lima, jer se računalo s garantiranim iznosima za specifične gubitke lima proizvođača, a zatim i na stvarnu (izvedenu) veličinu zračnog raspora, zbog čega je prikazana promjena dodatnih gubitaka u ovisnosti o širini otvora utora. Također, rezultati mjerenja i računa pokazuju da su dodatni gubici u praznom hodu tzv. naponski ovisni dodatni gubici značajan dio ukupnih gubitaka u praznom hodu. To se najbolje može vidjeti na dijagramima koji prikazuju ovisnost gubitaka zbog poprečnih struja o kontaktnom otporu i skošenju kaveza.

Odstupanje numeričkog od analitičkog izračuna dodatnih gubitaka za promatrani asinkroni motor je oko 10 %, čime je potvrđeno stajalište da analitički pristup određivanja dodatnih gubitaka ima određenu prednost pred numeričkim metodama proračuna, s obzirom da taj pristup pretpostavlja poznavanje uzroka i mehanizama nastajanja gubitaka, a izvedene formule daju uvid u utjecajnost pojedinih podataka motora. Za kvalitetan numerički izračun potrebno je puno više vremena iz razloga konstrukcije motora, mreženja modela, te samog iterativnog postupka proračuna. Analitički izračun je u tome puno brži, ali glavni nedostatak analitičkog modela izračuna u odnosu na numerički izračun je da zahtijeva inženjersko iskustvo projektiranja, u smislu određivanja koe-

7 CONCLUSION

The impact of the sheet geometry on eddy current losses in the rectangular teeth is determined with almost exact precision from the resultant field in the teeth caused by the magnetic induction harmonic in the air gap. The extraction is done under the assumptions $\mu_{Fe} = \text{konst.}$ and $\kappa_{Fe} = \text{konst.}$, from which it is derived that the total eddy current losses in rectangular teeth may be divided into pulsation and surface losses, which has been done so far in the known literature [10] and [11], but without proof.

At the end of the analytical calculation model, instructions are given about teeth of all forms being apt for determination of total losses in the teeth. This requires taking into consideration the hysteresis losses in the sheets and the corrections because of the suppression of the flow in the sheets, the attenuation of the flow in the teeth for pulsation losses and corrections due to sheet and sheet package processing.

The valuation of the analytical calculation model is confirmed by the comparison with the measurements of losses in the idle running of the squirrel-cage induction motors. For two motors with 11 kW and 7,5 kW power, losses are calculated in idle running and compared with adequate measurements. It has been shown that total calculated losses of idle running agree well with the measurements, and the deviations may be explained by insufficient (input) data. This primarily concerns the actual magnetizing losses for the inbuilt sheet because guaranteed values for specific producer's sheet losses was counted on, as well as the actual (derived) size of the air gap, because of which the change of additional losses was shown depending on the slot opening width. Moreover, measurement and calculation results show that additional losses in idle running, so-called voltage-dependent additional losses, are a significant part of the total losses in idle running. This can be best seen in the diagrams which show the dependency of losses, due to transverse currents, on contact resistance and cage skew.

Deviation of the numerical from the analytical calculation of additional losses for the studied induction motor is about 10 %, which confirms the standpoints that the analytical approach to the determination of additional losses has certain advantage compared to the numerical calculation methods, considering that the approach assumes knowing the cause and mechanisms of loss occurrence, and the derived formulas provide insight into the impact of certain motor data. A quality numerical calculation requires much more time because of the motor construction, model networking and the iterative calculation process itself. The analytical calculation is much faster in this process, but the main disad-

ficijenata kojima se uzimaju u obzir da je izračun pravljen za pravokutni zub, kao i pripadajuće faktore obrade lima.

Mogućnost analize dodatnih gubitaka u ovisnosti o vrijednosti kontaktnog otpora za različite vrijednosti skošenja je velika prednost analitičkog pristupa pri istraživanju dodatnih gubitaka, koja osobito pomaže pri samom projektiranju motora, gdje se odluka o broju utora i skošenju rotorskog kaveza zasniva prvenstveno na iskustvima samog projektanta.

vantage of the analytical calculation model compared to the numerical calculation is that it requires engineering experience in designing, in the sense of determination of coefficients by which it is taken into consideration that the calculation was done for a rectangular tooth, as well as the pertaining sheet processing factors.

The possibility of analysis of additional losses depending on the contact resistance value for different skew values is a great advantage of the analytical approach when additional losses are researched, and it is especially helpful with the very motor design where the decision on the number of slots and the skew of the rotor cage is based primarily on the experiences of the designer himself/herself.

LITERATURA / REFERENCES

- [1] RICHTER, R., *Elektrische Maschinen I*, Basel/Stuttgart, Birkhäuser Verlag, 1967
- [2] DANILEVIĆ, J. B., KAŠARSKIJ, E. G., *Dobavočne poteri v električeskih mašinah*, Moskva/Leningrad, Gosudarstvennoe energetičeskoe izdateljstvo, 1963
- [3] TAEGER, F., *Zusatzverluste von Asynchronmaschinen*, *Acta Technica ČSAV*, 13 (1968), 1-31
- [4] BABIĆ, S., *Utjecaj tehnoloških postupaka prerade na magnetska svojstva neorijentiranog magnetskog lima*, V. savjetovanje Elektromotorni pogoni, Poreč, 1990.
- [5] ŠTEFANKO, S., HEDERIĆ, Ž., *Gubici u zubima u asinkronim strojevima*, *Tehnički vjesnik*, 14 (2007), No. 1, 2, 9-1
- [6] MCCLAY, C. I., WILLIAMSON, S., *The Variation of Cage Motor Losses with Skew*, *Record of IEEE-IAS-1998*, Vol.1, pp.79-86
- [7] ZHU, J. G., RAMSDEN, V. S., *Improved formulations for rotational core losses in rotating electrical machines*, *Magnetics, IEEE Transactions on*, Volume 34, Issue 4, Part 2, July 1998, pp. 2234-2242
- [8] HEDERIĆ, Ž., *Gubici u praznom hodu kaveznih asinkronih strojeva*, magistarski rad, Zagreb, Fakultet elektrotehnike i računarstva, 2002.
- [9] SCHUIJSKY, W., *Induktionsmaschinen*, Springer-Verlag, Wien, 1957
- [10] ITO, M., FUJIMOTO, N., OKUDA, H., TAKAHASHI, N., MIYATA, T., *Analytical Model for Magnetic Field Analysis of Induction Motor Performance*, *Power Apparatus and Systems, IEEE Transactions on*, Volume PAS-100, Issue 11, Nov. 1981, pp. 4582-4590
- [11] SATO, T., INUI, Y., Saito, S., *Solution of Magnetic Field, Eddy Current and Circulating Current Problems, Taking Magnetic Saturation and Effect of Eddy Current and Circulating Current Path into Account*, *IEEE Paper A77 168-8*, 1977

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